A METHOD FOR CALCULATING STRUCTURAL RELIABILITY OF MEMS SYSTEMS BY THE IMPLICIT FUNCTIONAL FUNCTION

by

Juan DU[a*], Haibin LI[b], and Yun HE[c*]

[a]College of Statistics and Mathematics, Inner Mongolia University of Finance and Economics, Hohhot, Inner Mongolia, China
[b]College of Science, Inner Mongolia University of Technology, Hohhot, Inner Mongolia, China
[c]College of Water Conservancy and Civil Engineering, Inner Mongolia Agricultural University, Hohhot, Inner Mongolia, China

Original scientific paper
https://doi.org/10.2298/TSCI2602961D

In this paper, a novel approach to calculate the structural reliability of micro-electro-mechanical systems systems is proposed. This approach utilizes the implicit functional function, which involves the acquisition of multiple sets of inputs and responses of the structure through numerical simulation or experimentation. Concurrently, the exponential function is employed as the activation function of the hidden layer of the neural network. A multi-layer neural network possesses the capacity to approximate the properties of arbitrary non-linear functions with arbitrary precision and construct a customized neural network structure. The training of this customized neural network enables the visualization and expression of the structural function. This approach has been demonstrated to enhance the precision of functional fitting. The proposed method for modeling complex structural systems reliability is substantiated by numerical examples. Micro-electro-mechanical systems play a pivotal role in modern engineering, and their integration has the potential to further enhance structural analysis and reliability assessment.

Keywords: *implicit function, custom neural network, response surface method, structural reliability, complex structure, micro-electro-mechanical systems*

Introduction

Micro-electro-mechanical systems (MEMS) [1, 2] have significant implications for structural reliability. The MEMS sensors can be embedded in structures for the purpose of monitoring various parameters, including but not limited to strain, temperature, and vibration. This real-time data offers early warning signs of potential failures and facilitates proactive maintenance [3]. To illustrate, in the domain of aerospace engineering, MEMS sensors have the capacity to discern micro-cracks in aircraft structures prior to their escalation into a critical state. Moreover, the incorporation of MEMS-based actuators facilitates active control of the structural response, thereby enhancing its stability and durability [4]. The integration of MEMS technology into diverse systems is instrumental in ensuring their reliable operation, thereby promoting safety and efficiency.

* Corresponding authors, e-mail: djsd2007@126.com; heyun1021@163.com

However, in engineering practice, complex structures often present challenges due to the non-linear nature of the relationship between variables and responses, which may not be fully expressible in explicit functions. The presence of implicit functions has the potential to further complicate the reliability of structural calculation and analysis [5-7].

In addressing these challenges, scholars have dedicated significant efforts to developing reliable calculation methods for complex implicit functional functions. Deng *et al.* [8] have proposed a novel approach that utilizes ANN technology to approximate the implicit functional function. This approach subsequently leverages the Monte Carlo simulation (MCS) method or the first order second moment method to solve the structural reliability. The potential support vector machine (SVM) has been adopted as a classification method for structural risk minimization inference rules, exhibiting strengths such as small volume, robust learning capability, and the capacity for effective generalization of data. Building upon the SVM, Li *et al.* [9] have proposed a calculation method for structural reliability, substantiated through a numerical example that demonstrates the method feasibility. The first-order inverse reliability method has been identified as deficient in addressing inverse reliability problems with implicit functional functions. Consequently, Cheng and Li [10] employed the polynomial response surface method to assess the reliability of steel structures and substantiate the efficacy of this approach. Zhang *et al.* [11] have developed a novel Kriging method for the effective analysis of structural reliability. This method involves the use of a simple random sampling method to generate a limited number of samples for the construction of an initial Kriging model. Subsequently, the probability classification function is employed to select the subsequent samples within this area, and the implicit functional function is approximated by the Kriging model. Finally, the MCS method is utilized for reliability analysis. Wen *et al.* [12] investigated an adaptive sampling area method, with the objective of mitigating the inflexible constraints imposed by the area on the sample. Subsequently, they refined the structural reliability analysis method based on Kriging. In the context of implicit functionalities, Su *et al.* [13] employed the response surface methodologies (RSM) based on Gaussian processes to approximate the functionalities. This method involves augmenting the initial training set with new design points to enhance the approximation accuracy of Gaussian processes around the design points.

In this paper, a reliability-based structural model (RBSM) is proposed, which is based on a custom neural network (CNN). The method utilizes numerical simulation or experimentation to obtain training samples of the structure. Subsequent to the completion of a sufficient training regimen, the neural network demonstrates the capacity to approximate the limit state equation of the structure with a high degree of precision. The RBSM, as derived from a CNN, offers an efficacious approach for conducting structural reliability analysis of implicit functional functions. The numerical examples presented in this chapter demonstrate the superior calculation accuracy of the method described herein.

Polynomial response surface method

The RSM have their origins in experimental design and constitute a fundamental method of experimental design. Subsequently, the RSM methodology was employed in the context of numerical simulation of structural reliability. The fundamental premise of this approach entailed the substitution of the actual functional function or limit state surface, which is often implicit or time-consuming to ascertain, with a response surface function or response surface, a strategy that is more expeditious. Subsequent to the implementation of a response surface over a series of sampling points, structural reliability analysis can be performed.

The fundamental tenet of RSM is its capacity to adequately align the sampling points, particularly in the proximity of the design checking point. Among the available methods, central composite design is the most frequently employed approach within the framework of response surface methodology. The response surface function should be formulated in a straightforward manner, with the objective of minimizing the coefficient of the undetermined function to alleviate the workload associated with structural analysis. The response surface function is most commonly expressed in polynomial form of the fundamental random variable. Quadratic polynomials are regarded as the preferred form, however, higher-degree polynomials are generally eschewed for computational reasons.

For the structure whose basic random variables are (X_1, X_2, …, X_n), the response surface function is set:

$$Z = g\left(X_1, X_2, \cdots, X_n\right) \approx \hat{g}\left(X_1, X_2, \cdots, X_n\right) = a + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n} c_i X_i^2 + \sum_{1 \le i \le j \le n} d_{ij} X_i X_j \quad (1)$$

where a, b_i, c_i, and d_{ij} are undetermined coefficients and $(n^2 + 3n)/2 + 1$ is the number of undetermined coefficients.

The objective of reliability analysis is to solve for the checking points and the reliability index. The response surface function is required to fit the functional function near the checking points. It is often difficult for response surface functions expressed by quadratic polynomials to fit real functional functions well in the entire space. Incomplete quadratic polynomials that ignore cross-product terms are sometimes used to address this issue:

$$Z_r = \hat{g}\left(X_1, X_2, \cdots, X_n\right) = a + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n} c_i X_i^2 \quad (2)$$

Within the formula, the number of undetermined coefficients is $2n + 1$. The response surface method is a valuable tool for solving problems related to reliability calculation through experimental design and regression methods. However, it should be noted that the traditional quadratic response surface method is not always capable of accurately approximating the structure of the original function. This can result in inaccuracies in the response. The structure and real value of the response surface are not consistent with the original surface structure. This discrepancy arises because the response surface is calculated using a function that does not accurately represent the original surface. Consequently, the reliability calculation is not precise, and the safety state of the structure is not adequately reflected.

The response surface methodologies based on custom neural network

The custom neural network model

In general, the MATLAB toolbox function is adequate for designing neural network structures. However, for specific, specialized requirements, such as networks that deviate from the conventional neural network structure or utilize unconventional learning and training algorithms, a distinct toolbox is required. Consequently, the neural network toolbox is expected to encompass the functionality of a customized neural network and a customized function. The implementation of a customized network necessitates the invocation of the network() function in MATLAB. The network object encompasses numerous attributes, and by assigning specific values to these attributes, the network structure, algorithm, and function can be modified.

The parameters of custom neural networks encompass a variety of properties, including structure, sub-object, network function, weight, and threshold. The structure attributes encompass the number of input vectors, the number of layers in the network, the threshold connection attribute, the input connection attribute, the layer connection attribute, the expected output vector connection attribute, the output vector number, the expected output vector number, the input delay number, and the layer delay number. Sub-object properties encompass input vector, network layer, output vector, expected output vector, threshold vector, input weights, and layer weights. The network function comprises the following: an adaptive training function, a gradient function, an initialization function, a performance function, and a training function. The weight and threshold attributes encompass the input weight matrix, the layer weight matrix, and the threshold vector. Subsequent to the completion of network design, the function init() is utilized to initiate the network, after which the training of the network can be initiated.

The custom function can be based on specific needs during the initialization, simulation, and training processes, allowing for the application of various methods to achieve self-adjustment of the network. The function under discussion can be broadly categorized into four distinct types: the simulation function, the initialization function, the learning function, and the self-organizing mapping function. The simulation function encompasses the transfer function, the network input function, the weight function, and their respective derivative functions. The initialization function encompasses three distinct components: network initialization, layer initialization, and weight and threshold initialization. The learning function encompasses network training, network adaptation, network performance, weight, and threshold learning. The self-organizing mapping function encompasses both topological and distance functions.

He and Li [14] applied the CNN method to fit the relaxation modulus of viscoelastic materials and used it to construct a joint training algorithm for multiple neural networks to calculate the PDE [15]. A satisfactory calculation result was obtained through analysis of an example.

In the context of structural reliability analysis, the efficacy and precision of traditional RSM in modeling high-dimensional functional functions are hindered when the underlying implicit functional form is not known. The employment of a neural network method as a hidden layer activation function for a simple function has been demonstrated to circumvent these limitations. The proposed methodology involves the implementation of a customized neural network RSM, which is trained using CNN, with the objective of enhancing the accuracy of the fitting process. The specific formulation of the functional function is derived.

The construction of the functional function of the structure is achieved through the custom neural network, and its expression is:

$$z_a = \hat{g}\left(X_1, X_2, \cdots, X_n\right) = \mathbf{W} \cdot \exp\left(\mathbf{K} \cdot \mathbf{X}\right) + c \tag{3}$$

The independent variable is initially weighted by the index operation, and subsequently, the index is linear weighted superposition. The function form mirrors the input and output relationship of a feedforward neural network with a single hidden layer. Capitalizing on this characteristic, the network weights correspond to the weighting coefficients in the equation, the network thresholds correspond to the constants in the equation, and the exponential function is employed as the activation function. The specific structure of the CNN is outlined below and illustrated in fig. 1.

The network adopts a three-tier structure, the input vector is $\mathbf{X} = [X_1, X_2, \ldots, X_n]^T$, the weight matrix are:

$$\mathbf{K} = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \vdots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{m1} & k_{m2} & \cdots & k_{mn} \end{bmatrix},$$

and $\mathbf{w} = [w_1 \quad w_2 \quad \cdots \quad w_m]$

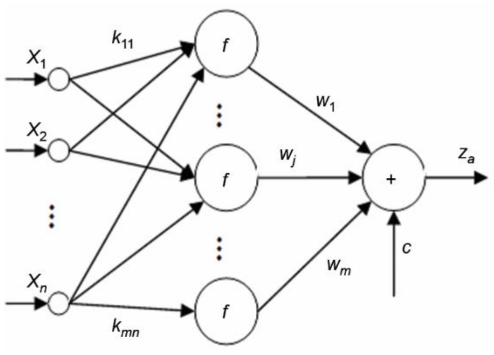

Figure 1. The CNN structure

the threshold is c. The $J(x) = e^x$ is activation function of hidden layer.

The custom neural network learning process and algorithm

Hidden layer unit output is:

$$y_j = e^{\sum_{i=1}^{n} k_{ji} X_i}, \quad j = 1, 2, \cdots, m \tag{4}$$

Output layer unit output is:

$$z_a = \sum_{j=1}^{m} w_j y_j + c \tag{5}$$

For q network output samples, the error function form is [7]:

$$\tilde{E} = \frac{1}{q} \sum_{p=1}^{q} [g(p) - z_a(p)]^2 \tag{6}$$

where $g(p)$ denotes the expected value of the output sample of the network. Network weights and thresholds can be converted into the coefficients of the functional functions in the reliability analysis. Once this conversion is complete, the functional function model can be established.

The process and realization of structural reliability calculation

Through the previous analysis, the steps of the RSM based on CNN to calculate the reliability problem of implicit functional function structure are:

- Random sampling is obtained X_i (i = 1, 2, …, n).
- The responses $g(X_i)$ of the structure under different basic random variables X_i were obtained through numerical analysis or experiment, and the training samples $\{X_i, z_a(X_i)\}$ of the neural network were generated.
- Design the neural network structure.
- Using data $\{X_i, z_a(X_i)\}$ to train network structure.
- Extract the weights and thresholds of the neural network and construct the functional function expression.
- The dual neural network method is used to calculate the structural reliability [16, 17].

The example analysis

Example 1

A consideration of the undamped single-DoF system [18-20] is warranted, as it arises in many practical applications. For instance, it is relevant to circular sector vibration systems [21], non-linear oscillators [22, 23], and MEMS systems [24, 25]. In this example, we consider a spring system as shown in fig. 2, which contains six basic random variables. The statistical characteristics of the system are shown in tab. 1 to calculate the structural reliability of the system. Its functional function expression (the functional function directly given to the structure, which is used in place of numerical analysis or experiment) is:

$$g\left(c_1, c_2, m, r, t_1, F_1\right) = 3r - \left|z_{\max}\right| = 3r - \left|\frac{2F_1}{m w_0^2}\sin\left(\frac{w_0 t_1}{2}\right)\right|$$

In the formula, the maximum response displacement of system is $|z_{\max}|$, $w_0 = [(c_1 + c_2)/m]^{1/2}$, c_1 and c_2 – the stiffness coefficients of spring 1 and 2.

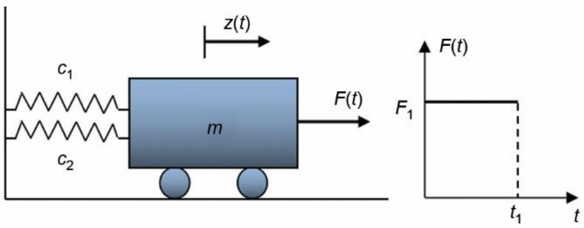

Figure 2. Non-linear oscillator

Table 1. Distribution parameters of basic random variables in *Example 1*

Random variable	c_1	c_2	m	r	t_1	F_1
Distribution type	Normal	Normal	Normal	Normal	Normal	Normal
Mean value	1.0	0.1	1.0	0.5	1.0	1.0
Standard deviation	0.10	0.01	0.05	0.05	0.20	0.20

In order to facilitate representation, let the vector:

$$\mathbf{X} = \left[X_1 \quad X_2 \quad X_3 \quad X_4 \quad X_5 \quad X_6\right]^{\mathrm{T}} = \left[c_1 \quad c_2 \quad m \quad r \quad t_1 \quad F_1\right]^{\mathrm{T}}$$

which is taken as the input vector of the CNN, and the number of hidden layer units are set as 3. The 1000 samples were sampled by Monte Carlo sampling method and used as training samples for the custom neural network.

The weight and threshold values of the neural network are extracted, and the functional function of the structure is obtained

$$z_a = \hat{g}\left(c_1, c_2, m, r, t_1, F_1\right) = \mathbf{W}\cdot\exp\left(\mathbf{K}\cdot\mathbf{X}\right) + c$$

where

$$c = -13.0614, \quad \mathbf{W} = \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} = \begin{bmatrix} 4.3725 & 4.2976 & 4.2976 \end{bmatrix},$$

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \end{bmatrix} = \begin{bmatrix} 0.0116 & 0.0134 & 0.0594 & 0.2264 & -0.3483 & 0.1823 \\ 0.0442 & 0.0416 & 0.0161 & 0.2170 & 0.0794 & -0.1883 \\ 0.0439 & 0.0416 & 0.0160 & 0.2170 & 0.0805 & -0.1872 \end{bmatrix}$$

In order to verify the fitting accuracy of the RSM based on CNN, a comparison is made between the calculated functional values and the real functional values of the structure and the functional values fitted by the polynomial RSM when the same data samples are input. The Monte Carlo sampling method was employed to obtain 500 samples, which were utilized as input samples and were respectively calculated by the RSM and PRSM of CNN. The results of this study are presented in figs. 3 and 4. As demonstrated by the figure, the CNN method is a superior approach for approximating the actual function of the non-linear oscillator.

In order to further illustrate the capacity of CNN to approximate the functional value, fig. 5 presents the discrepancy between the functional value obtained by the RSM and PRSM based on CNN and the actual functional value. A comparison of the functional values obtained through the CNN and the real functional value reveals a negligible discrepancy. The functional function obtained after each sample input demonstrates remarkable stability, with a minimum difference of 0.0001 and a maximum difference of 0.2018. The mean error is calculated to be 4.1833×10^{-4}. However, the discrepancy generated by the polynomial response surface method can be both substantial and negligible, thereby exhibiting relative instability. The minimum observed difference was 0.0002, the maximum difference was 0.3822, and the average error was –0.0561.

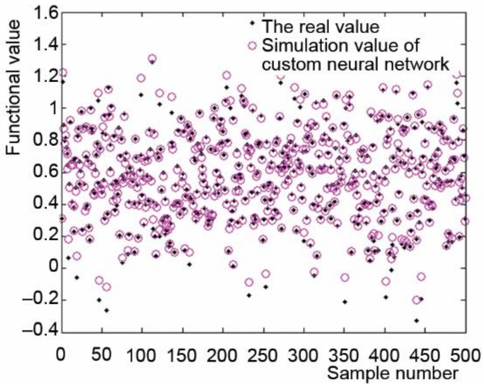

Figure 3. The CNN simulation values

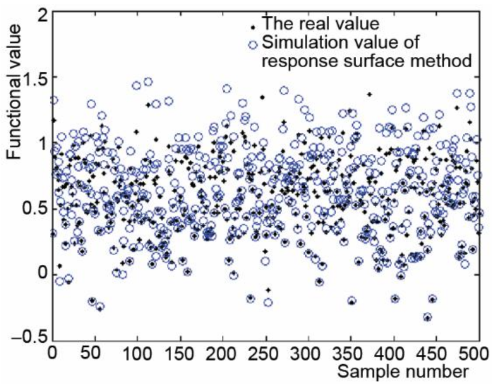

Figure 4. Simulation values of PRSM

Based on the structural function constructed by the response surface method of the custom neural network, combined with the direct integration method of the dual neural network, the reliability of the non-linear oscillator structure was calculated to be 0.9711. At the same time, it was compared with the results of MCS method and PRSM for 10^6 sampling times, as shown in tab. 2.

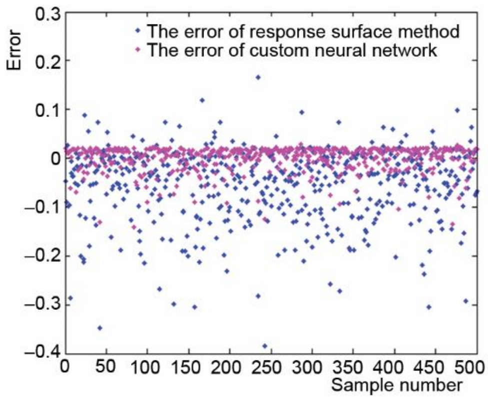

Figure 5. The error between the two methods and the true value

Table 2. Comparison of reliability calculation results in *Example 1*

Methods	MCS method	PRSM	The proposed method
Reliability	0.9715	0.9689	0.9711
The relative error	0	0.26%	0.04%

As can be seen from tab. 2, the error between the structural reliability obtained by the RSM based on CNN and the results of MCS method, traditional PRSM is very small, so the proposed method in this paper can be used to construct implicit structural functional functions.

Example 2

A high-dimensional non-linear limit state function problem with cross terms is used to verify the algorithm, and the form of limit state function is:

$$g(X)=\sum_{i=1}^{10}x_i+10x_1^2x_2^2+x_2^2x_3^2+x_4^2x_5^2+x_5^2x_6^2+x_7^2x_8^2+x_8^2x_9^2+x_9^2x_{10}^2-16$$

The random variables $X = (x_1, x_2, \ldots, x_{10})$ are normally distributed, mean value is $\mu_x = 1.0$, standard deviation is $\sigma_x = 0.2$.

Let the vector $\mathbf{X} = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9\ x_{10}]^T$ be the input vector of CNN, and the number of hidden layer units is 3. The 1000 samples were sampled by Monte Carlo sampling method, and the training samples of neural network were obtained. Extract the weights and thresholds of network, and the structure function obtained is:

$$z_a=\hat{g}(x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9,x_{10})=\mathbf{W}\cdot\exp(\mathbf{K}\cdot\mathbf{X})+c$$

where

$$c=-21.1537,\quad \mathbf{W}=[w_1\ \ w_2\ \ w_3]=[-181.0814\ \ 182.7610\ \ 1.4348],$$

$$K=\begin{bmatrix}k_{11}&k_{12}&k_{13}&k_{14}&k_{15}&k_{16}&k_{17}&k_{18}&k_{19}&k_{110}\\k_{21}&k_{22}&k_{23}&k_{24}&k_{25}&k_{26}&k_{27}&k_{28}&k_{29}&k_{210}\\k_{31}&k_{32}&k_{33}&k_{34}&k_{35}&k_{36}&k_{37}&k_{38}&k_{39}&k_{310}\end{bmatrix}=$$

$$=\begin{bmatrix}1.6138&0.4983&0.2039&\cdots&0.0618&0.0240\\1.6088&0.5043&0.2031&\cdots&0.0616&0.0241\\-0.3574&-0.3718&0.2576&\cdots&0.4175&0.2357\end{bmatrix}$$

A comparison of the proposed method fitting accuracy with that of the real functional function is warranted. Such a comparison is made possible by the PRSM functional function. The MCS method was employed to sample 500 samples, which were subsequently utilized as input samples for the CNN model and the polynomial model. The calculation results of the functional values are displayed in figs. 6 and 7.

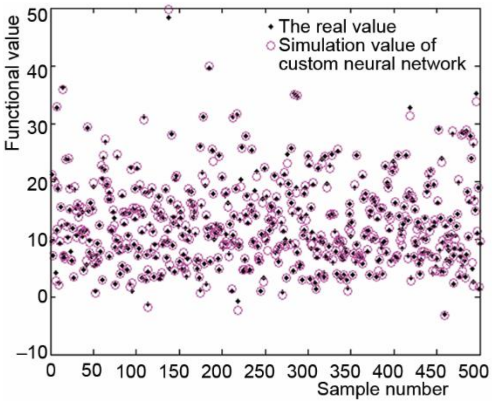

Figure 6. The CNN simulation values

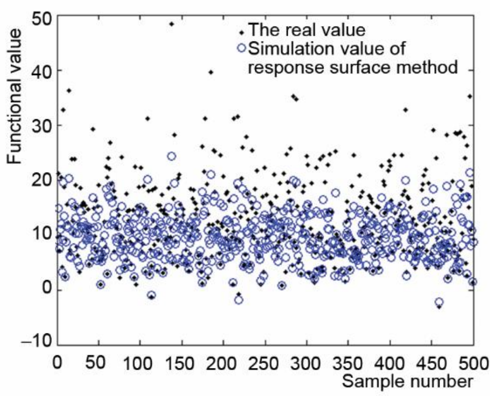

Figure 7. Simulation value of PRSM

The functional values calculated by the RSM based on CNN are essentially consistent with the actual functional values, as illustrated in fig. 6. The minimum error recorded was 0.0009, the maximum error was 3.1386, and the mean error was –0.0202, as demonstrated in fig. 8. As demonstrated in fig. 7, the functional values of the polynomial response surface method exhibit significant deviation from the actual functional values. The minimum error recorded was 0.005, the maximum error was 19.5153, and the average error was 2.9748, as demonstrated in fig. 8. A comparison of the RSM with the PRSM reveals that the RSM based on CNN demonstrates a superior ability to align with the actual value of the functional function. The discrepancy between the calculated result and the actual value is minimal, and the error value exhibits minimal variability.

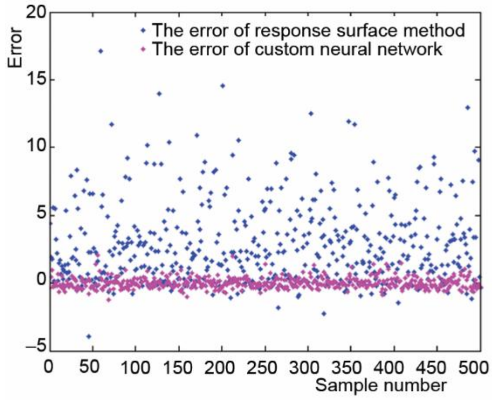

Figure 8. The error between the two methods and the true value

Based on the functional function constructed by the RSM based on CNN and the direct integration method of dual neural network, the structure reliability is calculated to be 0.9861. At the same time, it was compared with the calculation results of MCS method for 10^6 sampling times and the polynomial response surface method, and the results were listed in tab. 3.

Table 3. Comparison of Reliability Calculation Results in Example 2

Methods	MCS method	PRSM	The proposed method
Reliability	0.9896	0.9807	0.9861
The relative error	0	0.89%	0.35%

As can be seen from tab. 3, the error between the reliability obtained by the RSM based on CNN and the results of the MCS method, the traditional RSM is very small. Therefore, the proposed method in this paper can be applied to analyze and solve the structural reliability of implicit functional function.

Conclusion

The present paper proposed a methodology for assessing the structural reliability of MEMS systems by means of the implicit functional function. The exponential function was employed as the activation function for the hidden layer of the neural network. The training of the CNN yielded the specific expression of the functional function. In the calculation examples, problems with high-dimensional variables and a relatively high degree of non-linearity were selected for analysis and compared with the traditional PRSM. It is evident that the methodology outlined in this paper offers a superior approach for approximating the actual structure functional function, characterized by minimal error and fluctuation. This approach is noteworthy for its clear advantages over existing methods.

In contemplating prospective research endeavors, the integration of MEMS into this methodology promises significant advancements. The MEMS sensors have been demonstrated to provide real-time data on structural parameters, thereby enhancing the accuracy and reliability of the analysis. Future research could explore the combination of MEMS technology with the proposed RSM-CNN method to further improve structural reliability assessment and enable more effective monitoring and maintenance of complex structures. Furthermore, the investigation of optimization algorithms for the training of neural networks in the context of MEMS-enhanced structural analysis has the potential to result in the development of more efficient and accurate models.

Acknowledgment

This work was supported by Inner Mongolia Natural Science Foundation Project (grant number: 2023QN01006), Inner Mongolia Autonomous Region Higher Education Institutions Science Research Project (grant number: NJZY23053), Inner Mongolia Autonomous Region Directly Affiliated Higher Education Institutions Basic Science Research Fund Project (grant number: NCYWT23027), High Quality Research Achievement Cultivation Fund Project of Inner Mongolia University of Finance and Economics in 2024 (grant number: GZCG24215), Inner Mongolia Natural Science Foundation Project (grant number: 2023LHMS01012), The Program for Improving the Scientific Research Ability of Youth Teachers of Inner Mongolia Agricultural University (grant number: BR240101), First Class Discipline Research Special Project (YLXKZX-NCD-010), National Natural Science Foundation of China (11962021).

References

[1] Nadeem, M., *et al.*, Analysis of Nanobeam-Based Microstructure in N/MEMS System Using van der Waals Forces, *Facta Universitatis, Series: Mechanical Engineering*, *22* (2024), 4, pp. 673-688
[2] He, J.-H., *et al.*, Piezoelectric Biosensor Based on Ultrasensitive MEMS System, *Sensors and Actuators A: Physical*, *376* (2024), 115664
[3] Ozdemir, K., Mutlu, A. K., Cost-Effective Data Acquisition Systems for Advanced Structural Health Monitoring, *Sensors, 24* (2024), 4269
[4] He, J.-H., Periodic Solution of a Micro-Electromechanical System, *Facta Universitatis, Series: Mechanical Engineering, 22* (2024), 2, pp. 187-198

[5] Babaeimorad, S., *et al.*, An Integrated Optimization of Production and Preventive Maintenance Scheduling in Industry 4.0, *Facta Universitatis, Series: Mechanical Engineering*, *22* (2024), 4, pp. 711-720
[6] Mudasir, S., *et al.*, A Dual Approach to Parameter Estimation: Classical vs. Bayesian Methods in Power Rayleigh Modelling, *Thermal Science, 28* (2024), 6B, pp. 4877-4894
[7] Darwish, J. A., Statistical Properties and Applications for Exponentiated Exponential Rayleigh Distribution, *Thermal Science*, *28* (2024), 6B, pp. 4855-4006
[8] Deng, J., *et al.*, Structural Reliability Analysis for Implicit Performance Functions Using Artificial Neural Network, *Structural Safety, 27* (2025), 1, pp. 25-48
[9] Li, H. S., *et al.*, Support Vector Machine for Structural Reliability Analysis, *Applied Mathematics and Mechanics, 27* (2006), 10, pp. 1295-1303
[10] Cheng, J., Li, Q. S., Application of the Response Surface Methods to Solve Inverse Reliability Problems with Implicit Response Functions, *Computational Mechanics*, *43* (2009), 4, pp. 451-459
[11] Zhang, L. G., *et al.*, Efficient Structural Reliability Analysis Method Based on Advanced Kriging Model, *Applied Mathematical Modelling, 39* (2015), 2, pp. 781-793
[12] Wen, Z. X., *et al.*, A Sequential Kriging Reliability Analysis Method with Characteristics of Adaptive Sampling Regions and Parallelizability, *Reliability Engineering & System Safety, 153* (2016), Sept., pp. 170-179
[13] Su, G. S., *et al.*, A Gaussian Process-Based Response Surface Method for Structural Reliability Analysis, *Structural Engineering and Mechanics*, *56* (2015), 4, pp. 549-567
[14] He, Y., Li, H. B., A Novel Numerical Method for Heat Equation, *Thermal Science*, *20* (2016), 3, pp. 1018-1021
[15] He, Y., *et al.*, Fitting Methods Based on Custom Neural Network for Relaxation Modulus of Viscoelastic Materials, *International Journal of Performability Engineering, 15* (2019), 1, pp. 107-115
[16] Li, H. B., *et al.*, Structural Reliability Calculation Method Based on the Dual Neural Network and Direct Integration Method, *Neural Computing and Applications*, *29* (2018), 7, pp. 425-443
[17] Li, H. B., *et al.*, Dual Neural Network Method for Solving Multiple Definite Integrals, *Neural Computation, 31* (2019), 1, pp. 208-232
[18] Echard, B., *et al.*, Ak-Mcs: An Active Learning Reliability Method Combining Kriging and Monte Carlo Simulation, *Structural Safety*, *33* (2011), 2, pp. 145-154
[19] Dai, H., *et al.*, Wavelet Density-Based Adaptive Importance Sampling Method, *Structural Safety, 52* (2015), Part B, pp. 161-169
[20] Huang, X., *et al.*, Assessing Small Failure Probabilities by Ak–Ss: An Active Learning Method Combining Kriging and Subset Simulation, *Structural Safety, 59* (2016), Mar., pp. 86-95
[21] Feng, G. Q., A Circular Sector Vibration System in a Porous Medium: a Fractal-Fractional Model and He's Frequency Formulation, *Facta Universitatis Series: Mechanical Engineering*, *23* (2025), 2 pp. 377-385
[22] He, J.-H., *et al.*, Frequency-Amplitude Relationship in Damped and Forced Nonlinear Oscillators with Irrational Nonlinearities, *Journal of Computational Applied Mechanics*, *56* (2025), 2, pp. 307-317
[23] He, J.-H. Frequency Formulation for Nonlinear Oscillators (Part 1), *Sound & Vibration*, *59* (2025), 1, 1687
[24] He, J.-H., *et al.*, Modeling and Numerical Analysis for an MEMS Graphene Resonator, *Front. Phys. 13* (2025), 1551969
[25] He, J.-H., *et al.*, Variational Approach to Micro-Electro-Mechanical Systems, *Facta Universitatis Series: Mechanical Engineering*, *23* (2025), 4, pp. 649-665

Paper submitted: October 29, 2024
Paper revised: August 12, 2025
Paper accepted: August 12, 2025