RESEARCH AND COMPARISON OF IMPUTATION METHODS OF THREE PARAMETER WEIBULL DISTRIBUTION WITH MISSING WARP BREAKAGE DATA

by

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Warp breakage data is a critical metric for assessing the durability of warp yarns, which typically follows a Weibull distribution. The three-parameter Weibull distribution is a prevalent distribution in which scale, position, and shape parameters influence the statistical information of the distribution. The confidence interval is the degree to which the true value has a certain probability of falling around the measurement result, and it is important statistical information. In instances where the calculation of the confidence interval of a Weibull distribution is performed in the presence of missing data, employing solely complete data estimation can lead to the introduction of bias. However, in the context of missing data, the utilization of bootstrap interpolation has been demonstrated to yield outcomes that are often superior to those obtained through unprocessed data. The specific process entails the simulation of the Weibull dataset, incorporating random missing values of 5%, 20%, 50%, and 75%, and the subsequent comparison of the results of calculating confidence intervals using not missing and Bootstrap interpolation. The experimental results demonstrate that the presence of missing data exerts a negligible influence on the extent of confidence intervals, and the Bootstrap interpolation method yields values that approximate the median. This phenomenon results in an overall scarcity of discrete data. The experiment was verified using real warp fracture data, and the results were essentially consistent with the simulation experiment.

Key words: *Weibull distribution, confidence interval, bootstrap interpolation, number of warp yarn breaks*

Introduction

The majority of studies are characterized by the presence of missing data. The absence of pertinent data can lead to significant discrepancies in research findings, potentially resulting in erroneous conclusions. Consequently, it is imperative to address the issue of missing data in a meticulous manner. A variety of methods exist for handling missing data. The most effective approach depends on the nature of the data in question. Among these methods, Bootstrap interpolation has been employed in numerous fields. A substantial body of research has been conducted using data simulation and comparison to assess the effectiveness of these methods. The findings indicate that both approaches yield satisfactory testing outcomes. The Bayes method has been demonstrated to exhibit superior accuracy in comparison to the Bootstrap method. However, the latter is characterized by a superior degree of user-friendliness.

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Carl proposed the utilization of Bootstrap imputation to rectify the bias engendered by classification errors, employing serum creatinine measurement to ascertain severe renal failure status in patients. The findings indicated that Bootstrap imputation effectively circumvented outcomes of negligible efficacy and substantially mitigated the occurrence of misclassification bias [1-3]. Ying's examination focused on the implications of the asymptotic truncated form of the data from the constant stress accelerated competition failure model. This investigation specifically addressed the applicability of the maximum likelihood method for estimating parameters. A comparison of the confidence intervals for unknown parameters, as calculated by both the normal approximation and the bootstrap method, reveals that the latter method yields superior performance, as evidenced by simulation results [4].

Weaving is a prevalent technique for fabric manufacturing. The advent of science and technology has led to a substantial augmentation in the weaving speed. Concurrently, the quality of fabric raw materials has emerged as a pivotal impediment to the proliferation of high-tech products [5]. Muhammad developed a statistical model for predicting the curvature of cotton warp and weft yarns. The performance and accuracy of the model were evaluated using 60 different fabric data points. Furthermore, an ensemble of eight predicted and actual curl values was employed to evaluate the model performance. The findings indicated a satisfactory degree of accuracy [6]. Warp yarn, a critical component in the fabrication of textiles, is a primary factor in the quality of fabrics, as determined by predictive models. The quality of warp yarn is influenced by various factors, including material composition and tension levels. A critical aspect of assessing the quality of warp yarn is the frequency of breakage during the loading process. The term *fracture* is employed to denote the occurrence of weak links during the weaving process. Quanyong's study investigated the factors that result in the breakage of warp yarns prior to weaving. The objective of this examination was twofold: first, to identify the causes of breakage to inform improvements in the weaving process, and second, to explore methods of reducing breakage during weaving to enhance loom efficiency. A variety of factors have been identified as contributing to the development of effective preventive measures. The findings indicated that these preventive measures were effective in reducing the incidence of warp yarn breakage, thereby ensuring production efficiency and product quality [7]. A potential solution to enhance production efficiency involves minimizing equipment downtime, which can be achieved by reducing the incidence of warp yarn breaks. The selection of the warp yarn with the fewest breaks has the potential to enhance labor productivity and improve fabric quality [8].

The Weibull distribution is a statistical model that describes the probability behavior of actual phenomena and plays an important role in reliability analysis. Two primary approaches are employed for parameter estimation: the maximum likelihood method and the least squares method. It is imperative to acknowledge that the efficacy of these methodologies is contingent upon specific conditions, and consequently, the resulting estimates may exhibit substantial variation. Haghighat *et al.* [9] has observed that the Weibull distribution is the most prevalent in biomedical research, and that the data itself contains missing data. In the context of statistical inference for the Weibull distribution, a proposed methodology involves the initial estimation of the parameters through the employment of the EM algorithm. Subsequent to this estimation, the Bootstrap method is then utilized for the purpose of interval estimation. Following a thorough Monte Carlo simulation analysis, it has been ascertained that the effectiveness of this method parameter estimation is significantly augmented. Kundu *et al.* [10] has noted that in cases involving small sample sizes, the estimation of parameters for Weibull distributions poses significant challenges. She proposed a solution to empirically determine one parameter and focus on another parameter. A theoretical model is to be proposed, with said model being based on the relationship between parameter values, minimum sample values, and sample size. The estimation of parameters is best achieved through the utilization of iterative methods. A substantial body of experimental evidence has demonstrated the efficacy of this method in accurately estimating the parameters of small sample Weibull distributions [11]. Yuan *et al.* [12] proposed a grouped stepwise right truncated statistical model and solved it using the maximum likelihood estimation method. The truncated model is particularly well-suited for the purpose of providing feedback on product failure and for the monitoring of product life in engineering. The genetic algorithm is employed to simulate data grouping and to compare optimization objectives. The result of this process is that the mean, variance, and number of failures are used as optimization objectives with the minimum error. It is possible to calculate the reliable life of the product with a reliability of 95% based on these three values.

The objective of this study is to examine the impact of employing Bootstrap interpolation to process missing values in incomplete warp data that adhere to a Weibull distribution on the estimation of the confidence interval for this distribution. The second part of the study focuses on the confidence interval of the Weibull distribution in the context of missing data. The third part involves a simulation experiment of Weibull data and the verification of warp fracture data. Conclusions and prospects are finally drawn based on the experimental results.

Confidence interval of Weibull distribution with missing data

Assuming a set of data $X = (x_1, x_2, \ldots, x_n)$ follows a Weibull distribution $W(\alpha, \beta, \theta)$, where $\alpha, \beta,$ and θ are shape parameters, scale parameters, and position parameters, respectively. The shape parameter, α, controls the variation of the distribution function shape, the scale parameter, β, controls the variation of the distribution function in amplitude, and the positional parameter, θ, controls the variation of the distribution function in the horizontal direction. The distribution function (CDF – cumulative distribution function) of the three parameter Weibull is:

$$F(x) = 1 - \exp\left\{-\left(\frac{x-\theta}{\beta}\right)^{\alpha}\right\} \tag{1}$$

The probability density function is:

$$f(x) = \left(\frac{\alpha}{\beta}\right)\left(\frac{x-\theta}{\beta}\right)^{\alpha-1} \exp\left\{-\left(\frac{x-\theta}{\beta}\right)^{\alpha}\right\} \tag{2}$$

There are many methods for estimating parameters of the Weibull distribution, such as moment estimation, maximum likelihood estimation, and probability weight method. This article focuses on using probability weight method to solve the parameter estimation values of the Weibull distribution. Take the K-order probability weight moment:

$$M_{1,0,K} = \frac{\theta}{1+K} + \frac{\beta\,\Gamma\left(1+\dfrac{1}{\alpha}\right)}{(1+K)^{(1+1/\alpha)}} \tag{3}$$

In eq. (3), Γ is the distribution of Gamma, θ, α, and β are the three parameters to be estimated. Therefore, it is necessary to calculate their third-order moments. If K = 0, 1, 2 is taken, then:

$$\begin{aligned} M_{1,0,0} &= \theta + \beta\,\Gamma\left(1+\frac{1}{\alpha}\right) \\ M_{1,0,1} &= \frac{\theta}{2} + \beta\,\Gamma\frac{1+\dfrac{1}{\alpha}}{2^{(1+1/\alpha)}} \\ M_{1,0,3} &= \frac{\theta}{4} + \beta\,\Gamma\frac{1+\dfrac{1}{\alpha}}{4^{(1+1/\alpha)}} \end{aligned} \tag{4}$$

By solving the previous equation, the relationship between the parameters of the Weibull distribution and $M_{1,0,K}$ can be obtained:

$$\begin{aligned} \theta &= 4\frac{M_{1,0,3}M_{1,0,0} - M_{1,0,1}^{2}}{4M_{1,0,3}} + M_{1,0,0} - 4M_{1,0,1} \\ \beta &= \frac{M_{1,0,0} - \theta}{\Gamma\left[\dfrac{\ln\dfrac{M_{1,0,0} - 2M_{1,0,1}}{M_{1,0,1} - 2M_{1,0,3}}}{\ln 2}\right]} \\ \alpha &= \frac{\ln 2}{\ln\dfrac{M_{1,0,0} - 2M_{1,0,1}}{M_{1,0,1} - 2M_{1,0,3}}} \end{aligned} \tag{5}$$

The probability weight moment of the observed sample is:

$$\begin{aligned} M_{1,0,0} &= \frac{1}{n}\sum_{i=1}^{n} x_i \\ M_{1,0,1} &= \frac{1}{n}\sum_{i=1}^{n} x_i\left(1-\frac{i-0.35}{n}\right) \\ M_{1,0,3} &= \frac{1}{n}\sum_{i=1}^{n} x_i\left(1-\frac{i-0.35}{n}\right)^{3} \end{aligned} \tag{6}$$

Based on the observed sample data, use eqs. (6) to calculate its probability weight, and then estimate the three parameters of the Weibull distribution according to eq. (5). The estimated values of θ, α, and β are $\hat{\theta}, \hat{\alpha}$, and $\hat{\beta}$, as the parameter estimates for this group of data. If there are missing values in data X, sort them by whether there are missing values. If the first n_1 observations are not missing, they are denoted as X^{obs}, and if the last n_0 observations are missing, they are denoted as X^{mis}, denoted as $X = (X^{\mathrm{obs}}, X^{\mathrm{mis}})$, and $n_1 + n_0 = n$. The basic idea of the Bootstrap interpolation method is to extract a value of h based on the observation part X^{obs}, denoted as $(x_1^{\mathrm{obs}}, x_2^{\mathrm{obs}}, \cdots, x_h^{\mathrm{obs}})$, and calculate its mean as μ_h, that is:

$$\mu_h = \frac{\sum_{i=1}^{h} x_i^{\text{obs}}}{h} \tag{7}$$

As one of the extracted values, repeat 1000 times to obtain μ_h dataset $X_D = (x_{1h}, x_{2h},\ldots, x_{1000h},)$ to be interpolated. Randomly select the number of missing values from dataset X_D as the interpolation value, and the original missing data becomes $X_B = (X^{\text{obs}}, X'_D)$, where X'_D is the random extraction of X_D. Since the Bootstrap sample is only a repeated random extraction of the original sample, there is not much loss of original information. Bootstrap samples can be infinitely extracted, and there are differences between them, which precisely accounts for the randomness of missing data and compensates for the underestimation of population variance by other methods [13].The Weibull distribution is asymmetric and the data is discrete, so the confidence interval cannot be calculated using conventional methods [14]. The basic idea is to use multiple discrete points and based on the distribution function eq. (1), find the corresponding quantile to obtain the confidence interval. If we want to find the 95% confidence interval of a set of data X, we need to first use the probability weighting method to obtain the estimated values $\hat{\alpha}, \hat{\beta}$, and $\hat{\theta}$ of parameters α, β, and θ based on the data, and then calculate the CDF according to eq. (1). Find the values of X corresponding to 0.025 and 0.975, which are $X_{|0.025}$ and $X_{|0.975}$, respectively, and input them into eq. (1) to obtain the upper and lower confidence limits of 95% reliability.

Simulation verification

Simulation data

In order to ascertain the effect of the Bootstrap interpolation method on the confidence interval of the Weibull distribution, a comprehensive dataset was constructed through the simulation of eight sets of Weibull distributions. These distributions were characterized by scale parameters of 0.5 and 1, shape parameters of 0.5 and 1, and positional parameters of 1 and 2, with a sample size of 200. The simulation of data with random missing values of 5%, 20%, 50%, and 70% is the primary objective of this study. The implementation of different proportions of random missing values is based on the Bernoulli binomial distribution. The missing data is processed using the bootstrap interpolation method, and the processed data is calculated using the probability weighting method to calculate parameters. The confidence intervals were calculated and compared under different conditions with the results of the not missing data.

When simulating Weibull datasets under different conditions and randomly missing different ratios, the missing data is processed using Bootstrap method and its parameter values are calculated using probability weighting method. The comparison of the calculation results is shown in tab. 1. The observation results show that when the missing proportion is small, the estimated parameter values are closer to the true values. As the missing proportion increases, the deviation of the estimated parameter values gradually increases, deviating more from the true parameter values. However, the estimated values of positional parameters are less affected by the missing ratio, and regardless of the missing ratio, their parameter estimates are close to the set parameter values. When the proportion of missing data increases, shape parameter α usually a change. When the value of α is low, it is different from compared to high α values, the impact of missing data on shape parameters may have different patterns. Scale parameters β the changes reflect the impact of missing data on the estimation of distribution scale. The β

the estimated value of increases with the increase of missing proportion, which may indicate that the distribution range of the estimation becomes wider when there is a large amount of missing data. The θ the estimated value of varies slightly with the increase of missing data, indicating that the impact of missing data on positional parameters may be relatively small, but this also depends on the setting of the original parameters. When the amount of missing data increases, the estimated values of all parameters change, indicating a significant impact of missing data on parameter estimation. Under high missing ratios, the estimated values of parameters may be more unstable, indicating that the Bootstrap processing method is not the optimal method for a single variable with a large amount of missing data.

Table 1. **Probabilistic weighting method for solving Weibull datasets with different missing ratios and parameters**

Parameter value	5%			20%			50%			70%		
	α	β	θ	α	β	θ	α	β	θ	α	β	θ
$\alpha = 0.5$, $\beta = 0.5$, $\theta = 1$	0.54	0.54	0.98	0.71	0.79	0.98	1.22	0.90	0.96	1.34	1.02	1.03
$\alpha = 0.5$, $\beta = 0.5$, $\theta = 2$	0.62	1.37	1.98	0.71	1.59	1.97	1.23	1.97	1.95	1.35	2.11	2.02
$\alpha = 0.5$, $\beta = 1$, $\theta = 1$	0.61	1.35	0.97	0.71	1.55	0.97	1.22	1.78	0.92	1.34	2.03	1.06
$\alpha = 0.5$, $\beta = 1$, $\theta = 2$	0.61	2.03	1.97	0.71	2.36	1.97	1.22	2.85	1.92	1.34	3.12	2.05
$\alpha = 1$, $\beta = 0.5$, $\theta = 1$	1.21	0.57	0.97	1.36	0.59	0.97	2.32	0.58	0.92	2.31	0.57	1.01
$\alpha = 1$, $\beta = 0.5$, $\theta = 2$	1.22	1.64	1.96	1.37	1.68	1.96	2.34	1.71	1.90	2.34	1.69	2.00
$\alpha = 1$, $\beta = 1$, $\theta = 1$	1.21	1.13	0.95	1.35	1.15	0.95	2.30	1.14	0.84	2.29	1.11	1.03
$\alpha = 1$, $\beta = 1$, $\theta = 2$	1.21	2.20	1.94	1.36	2.43	1.94	2.32	2.27	1.83	2.31	2.24	2.02

The missing data with different parameter values and missing ratios were processed using Bootstrap interpolation method to calculate the upper and lower confidence limits of 95%, and compared with the results obtained not missing data. The comparison results are shown in tab. 2. In these tables, it can be seen that different processing methods have an impact on the confidence interval. The result of not missing used as a benchmark to evaluate the effectiveness of Bootstrap interpolation. The Bootstrap interpolation method solves the problem of missing data by repeatedly sampling and interpolating missing values from existing data. This method has a significant impact on the confidence interval of a set of data when the proportion of missing data is high. Bootstrap input yields different confidence intervals, especially at higher missing ratios, indicating that interpolation methods can significantly affect the uncertainty of parameter estimation. As the proportion of missing values increases, the confidence interval usually widens, but due to the Bootstrap interpolation method processing values as mean, the data is more concentrated, which in turn narrows the width of the confidence interval. The impact of different original parameter settings (α, β, θ), processing methods, and missing ratios on the confidence interval also varies. When $\alpha = 1$, compared to the case of $\alpha = 0.5$, the width of the confidence interval seems to be more affected by the missing data processing method. But when α and β remain unchanged and only the positional parameters θ change, only the position of the confidence interval changes, while the width of the confidence interval remains unchanged.

Table 2. The missing proportion is 5%, 25%, 20%, and 70% with 95% confidence

Parameter value	Bootstrap imputation (missing proportion 5%)		Bootstrap imputation (missing proportion 20%)		Bootstrap imputation (missing proportion 50%)		Bootstrap imputation (missing proportion 70%)	
	UCL	LCL	UCL	LCL	UCL	LCL	UCL	LCL
α = 0.5, β = 0.5, θ = 1	1.0000	6.6005	1.0001	6.2994	1.0009	4.3983	1.0029	3.3323
α = 0.5, β = 0.5, θ = 2	2.0001	7.6005	2.0001	7.2994	2.0009	5.3983	2.0029	4.3323
α = 0.5, β = 1, θ = 1	1.0003	12.2011	1.0003	11.5988	1.0020	7.7966	1.0058	5.6645
α = 0.5, β = 1, θ = 2	2.0003	13.2011	2.0003	12.5988	2.0020	8.7966	2.0058	6.6645
α = 1, β = 0.5, θ = 1	1.0085	2.6733	1.0085	2.6278	1.0222	2.3035	1.0380	2.0797
α = 1, β = 0.5, θ = 2	2.0085	3.6733	2.0085	3.6278	2.0222	3.3035	2.0380	3.0797
α = 1, β = 1, θ = 1	1.0171	4.3465	1.0171	4.2556	1.0444	3.6070	1.0760	3.1593
α = 1, β = 1, θ = 2	2.0171	5.3465	2.0171	5.2556	2.0444	4.6070	2.0760	4.1593

Warp breakage data

The warp data is a significant set of data in the field of textile science. In the event that the warp fracture data is not thoroughly measured, resulting in missing data, direct statistical analysis will lead to significant errors. As posited by the *R* built-in dataset, warpbreaks, with variables of breaks, tension, and wool, the number of times warp yarns from two materials (*A* and *B*) break when operated on a machine under three different tension conditions (*L*, *M*, and *H*) is represented. The data collection process involved each machine making nine recordings, resulting in a dataset comprising 54 rows. The fracture data of a set of warp yarns typically follows a Weibull distribution. To evaluate the outcomes of the aforementioned experiment, data were artificially generated to be missing at a rate of 5%, 10%, 35%, and 50%, respectively. The missing data were then processed using the bootstrap interpolation method.

Firstly, the complete and fracture data with different missing ratios were processed using Bootstrap before conducting parameter analysis. The results are shown in tab. 3, and the original parameter value calculated using the probability weighting method is α = 1.28, β = 29.30, and θ = 11.23. As the proportion of missing data increases, the shape parameter α increases. In the absence of missing data, α is 1.28, while in 50% of missing data, α increases to 5.06. This indicates that Bootstrap interpolation has a significant impact on the data in this group. The scale parameter β did not change significantly from 29.30 to 35% missing, and then decreased to 26.27 at 50% missing. The scale parameter mainly determines the width of the distribution, and a decrease in β indicates that the missing data is more concentrated after processing. The positional parameter θ undergoes significant changes as the proportion of missing data increases. When 5% of the data is missing, the positional parameter slightly increases, but when 50% of the data is missing, it drops significantly to almost zero, indicating that the processed data is still relatively concentrated.

Table 3. The missing proportions are 5%, 10%, 35%, and 50%, respectively, and the complete data is estimated using probability weighting method to estimate parameters

Parameter value			5%			10%			35%			50%		
α	β	θ	α	β	θ	α	β	θ	α	β	θ	α	β	θ
1.28	29.30	11.23	1.33	30.23	11.68	1.54	30.23	11.54	2.59	27.67	9.32	5.06	26.27	0.01

Table 4 shows the upper control limit (*UCL*) and lower control limit (*LCL*) of the 95% confidence interval for the warp fracture data after Bootstrap interpolation at different missing data ratios. The confidence interval obtained from not missing serves as the benchmark. Compared to the results not missing data, as the proportion of missing data increases, the UCL of the confidence interval slightly decreases, while the LCL significantly decreases. Especially at a 50% missing ratio, the LCL has decreased by nearly 20 units. When using Bootstrap interpolation, the UCL remains unchanged at all missing ratios, while the LCL significantly decreases with increasing missing ratios. This indicates that the method has limited impact on the UCL, but it will lead to a decrease in the LCL, indicating that the interpolated data is more concentrated.

Table 4. The missing proportion is 5%, 10%, 35% and 50%, with 95% UCL and LCL confidence limit for three different circumstances

Missing ratio	Not Missing		Bootstrap imputation	
	UCL	LCL	UCL	LCL
5%	12.325	62.775	13.325	62.775
10%	12.325	62.775	13.325	53.350
35%	12.325	62.775	13.325	47.100
50%	12.325	62.775	13.325	38.025

The results calculated by Bootstrap interpolation method differ significantly from those calculated by not missing data when the missing ratio is large, but the overall structure of the data remains unchanged. Moreover, the insertion values of Bootstrap interpolation method are all intermediate values, making the data structure more stable and consistent with the aforementioned experiments.

Conclusions

The present study examined the impact of employing Bootstrap interpolation to process missing values for data following a Weibull distribution under varying missing ratios on the estimation of confidence intervals for the distribution. The findings indicated that the Bootstrap method possesses both notable advantages and disadvantages in the context of calculating confidence intervals.

The Bootstrap interpolation method, a widely employed technique for interpolating data, exhibits simplicity in its steps and generates interpolation values that are within a reasonable range, devoid of any outliers. In comparison with alternative interpolation techniques, this method does not necessitate the consideration of the rationality of the insertion values, thereby facilitating the verification of the rationality of the data.

The effectiveness of a set of confidence intervals should be evaluated, taking into account the length of the intervals. Conversely, if the width of the confidence interval is minimal, it signifies a high degree of confidence interval effectiveness. The dataset obtained using the bootstrap interpolation method is characterized by its precision in capturing this characteristic, with a relatively concentrated dataset and a minimal overall variance. Consequently, the width of the confidence interval is minimal, aligning with the attributes of a reliable confidence interval.

However, the data obtained by the Bootstrap interpolation method is relatively concentrated and lacks discreteness. In the context of analyzing a set of data, the utilization of the bootstrap interpolation method is particularly advantageous when the objective is to determine the mean value. However, in scenarios where the research data demands high discreteness, the Bootstrap interpolation method is not the optimal choice for interpolation. For this group of data, it is recommended to employ the layered Bootstrap interpolation method. The obtained data must be characterized by both dispersion and the ability to more accurately reflect the distribution of the group of data in question. In the context of survey research, the presence of missing data is a common occurrence, often due to the presence of multiple variables. The imputation of missing values has emerged as a prevalent approach to address this challenge, owing to its effectiveness and widespread application in various research fields. A range of interpolation methods exists, including multiple interpolation, machine learning interpolation, Bayesian interpolation, and decision tree interpolation, among others. The judicious selection of interpolation method, informed by a comprehensive understanding of the data's characteristics, has been demonstrated to yield optimal outcomes with a reduced effort investment throughout the experimental process. The present article utilizes the Bootstrap interpolation method to simulate data research; however, the actual warp data is not yet available. The subsequent step is to identify appropriate survey data, thereby enabling the implementation of a broader array of interpolation methods in practical applications in highway traffic [15], additive manufacturing [16, 17], damage identification [18], MEMS systems [19, 20] and other fields.

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