# FRACTIONAL CALCULUS INNOVATIONS AND MACHINE LEARNING-DRIVEN ADVANCES IN THERMAL SCIENCE

by

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*This paper explores fractional calculus innovations and machine learning's role in advancing thermal science, especially in smart textiles. It first introduces three key fractional derivatives (Caputo, Riemann-Liouville, two-scale fractal) for thermal analysis, highlighting their strengths in capturing non-locality, memory effects, and fractal characteristics. Then, it details how the two-scale fractal derivative modifies Caputo and Riemann-Liouville derivatives to better model complex thermal systems in smart textiles, with simplified forms balancing accuracy and computational efficiency. Finally, it discusses machine learning's synergy with fractional calculus, optimizing model parameters, capturing nonlinearities, and enabling data-driven fractional models, to solve intractable thermal problems in smart textiles, supporting applications like complex textile material heat transfer and electronic thermal management in wearable smart textiles.*

Keywords: *fractional calculus, two-scale fractal derivative, modified fractional derivatives, machine learning, mems, wearable sensors, smart textiles*

## Introduction

Thermal science, as a core discipline supporting fields such as energy storage, electronic thermal management in wearable smart textiles [1], and textile material engineering [2], faces growing challenges in describing complex thermal phenomena in smart textiles. For instance, thermal factors exert a critical impact on the performance of micro-electro-mechanical systems integrated into smart textiles, with temperature-induced drift errors significantly degrading their measurement accuracy [3]. Even a minor deviation of this kind can further trigger pull-in instability [4-6]. Traditional integer-order calculus, which relies on local and instantaneous assumptions, struggles to capture two critical characteristics of modern thermal systems in smart textiles: non-local effects (*e.g.*, heat transfer in textiles depends on the entire textile's

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temperature distribution, not just local gradients) [7] and memory effects (*e.g.*, phase-change material-containing textiles' heat storage capacity is shaped by historical temperature variations) [8]. These limitations often lead to significant deviations between theoretical models and experimental results, hindering the accurate analysis and optimization of thermal processes in smart textiles.

In response to this gap, fractional calculus [9, 10] has emerged as a revolutionary mathematical tool for smart textile thermal analysis. By extending the concept of *derivative order* from integers to non-integers, it inherently embeds non-locality and memory into its framework, enabling a more precise depiction of complex thermal behaviors in smart textiles [11, 12] that integer-order calculus cannot fully address. For instance, in heat conduction of heterogeneous textile materials (*e.g.*, CNT composite textiles) or fractal-structured textile systems (*e.g.*, porous textile insulation), fractional derivatives can quantify the influence of historical thermal states and microscale structural features of textiles on macroscale heat transfer, providing a more realistic mathematical basis for modeling thermal processes in smart textiles.

Among the diverse fractional derivatives developed for thermal science in smart textiles, three stand out for their widespread application and complementary advantages: the Caputo fractional derivative [13, 14], with its physically interpretable initial conditions that align with experimental measurements of smart textile thermal properties; the Riemann-Liouville fractional derivative [15, 16], which serves as a foundational tool for theoretical analysis of non-local thermal mechanisms in smart textiles; and the two-scale fractal derivative [17-19], which fills the critical gap of describing fractal geometry-related thermal characteristics in smart textiles (*e.g.*, irregular fiber structures or self-similar heat dissipation textile components). Together, these derivatives form the backbone of fractional calculus-based thermal modeling in smart textiles, addressing different aspects of complex thermal systems in smart textiles from practical experimentation to theoretical exploration.

However, single fractional derivatives still have limitations in smart textile thermal modeling: traditional Caputo and Riemann-Liouville derivatives cannot explicitly account for fractal features of textile materials, while the two-scale fractal derivative alone lacks the ability to model memory effects in smart textile thermal systems. This has driven innovations in fractional calculus for smart textiles, specifically, the integration of the two-scale fractal derivative with traditional fractional derivatives to form modified fractional derivatives that simultaneously capture non-locality, memory, and fractal characteristics of smart textiles. These modified derivatives, when simplified for computational efficiency, bridge the gap between theoretical precision and engineering practicability in smart textile thermal analysis, making fractional calculus more applicable to large-scale thermal simulations of smart textiles.

Furthermore, the rise of machine learning [20-22] has opened new avenues for advancing thermal science in smart textiles. While fractional calculus provides a mechanism-based framework for describing complex thermal behaviors in smart textiles, it faces challenges such as cumbersome parameter tuning and difficulty capturing strong nonlinearities in smart textile thermal systems (*e.g.*, temperature-dependent thermal conductivity of textile materials). Machine learning, with its data-driven optimization and non-linear fitting capabilities, complements fractional calculus perfectly in smart textile thermal analysis: it can automate parameter calibration of fractional models for smart textiles, enhance non-linear description accuracy of smart textile thermal behaviors, and enable the development of hybrid *mechanism-data* models that combine physical interpretability and adaptive performance for smart textile thermal systems. Lei and He [23] explored how AI pushes thermal science frontiers in smart textiles, Cheng, *et al.* [24] proposed differential equation-driven intelligent control integrating AI, quantum computing, and adaptability for smart textile thermal management, and Li, *et al.* [25] analyzed multi-scale AI-driven fractal convection diffusion in porous nanofiber textiles.

Against this backdrop, this paper systematically explores the innovations of fractional calculus driven by the two-scale fractal derivative and the synergistic application of fractional calculus with machine learning in thermal science for smart textiles. It first elaborates on the fundamental roles and characteristics of the three core fractional derivatives in thermal analysis of smart textiles, then details the construction, significance, and simplification of modified fractional derivatives for smart textile thermal systems. Finally, it discusses how machine learning enhances the practicality of fractional calculus-based thermal models in smart textiles, aiming to provide a comprehensive theoretical and technical framework for solving increasingly complex thermal engineering problems in smart textiles.

## Fractional calculus in thermal science analysis of smart textiles

Fractional calculus has become a powerful and indispensable tool for in-depth analysis of thermal problems in smart textiles. Traditional integer-order calculus only describes local and instantaneous system behaviors, while fractional calculus breaks this limitation by introducing non-integer orders. This unique feature enables it to accurately capture non-local effects (where the state of a smart textile system at a point depends on the states of all other points in the textile domain) and memory effects (where the current behavior of a smart textile thermal system is influenced by its historical states) in smart textile thermal systems, two core characteristics of complex thermal phenomena in smart textiles that integer-order calculus cannot fully depict. For example, in heat conduction within textiles or heterogeneous textile materials, heat transfer is determined not only by the temperature gradient at the current location and moment but also by the temperature distribution across the entire textile and the heat transfer history. In such scenarios, fractional calculus provides a more precise mathematical framework, making it superior to traditional integer-order calculus in many thermal analysis tasks of smart textiles.

Among the various fractional derivatives used in this field for smart textiles, three are the most widely applied, each with distinct advantages to meet different thermal modeling needs of smart textiles.

The Caputo fractional derivative is defined as:

$$ {}_{0}^{C}D_{t}^{\beta}\phi(x,t)=\frac{1}{\Gamma(1-\beta)}\int_{0}^{t}\frac{\partial\phi(x,s)}{\partial s}\frac{1}{(t-s)^{\beta}}\,\mathrm{d}s \tag{1} $$

Its significance in thermal science of smart textiles lies in addressing key limitations of traditional calculus and other fractional derivatives, making it a cornerstone for practical smart textile thermal system modeling. It retains integer-order initial conditions (such as initial temperature of a textile material or initial heat flux in a smart textile), which can be directly mapped to measurable thermal parameters of smart textiles. This avoids the abstract *fractional-order initial conditions* required by other derivatives, eliminating the barrier between mathematical models and physical experiments of smart textiles. For instance, when simulating the heat conduction of a smart textile fabric block, the initial temperature can be directly used as the initial condition for the Caputo derivative model, ensuring practicality. Additionally, its memory kernel function (with non-integer order $\alpha$) quantitatively regulates the influence of historical states of smart textile thermal systems; a smaller $\alpha$ (closer to 0) means faster memory decay (suitable for textile materials with weak heat retention like ordinary cotton textiles), while a larger $\alpha$ (closer to 1) means slower memory decay (suitable for textile materials with strong heat retention like phase-change fiber textiles), allowing precise fitting of memory characteristics of different smart textile thermal materials. Moreover, for constant thermal states (such as steady-state temperature in a well-insulated smart textile garment), the Caputo derivative of a constant is 0, consistent with physical intuition that steady-state smart textile thermal systems have no *change trend*, ensuring the model's rationality in describing steady-state thermal behaviors of smart textiles, which is critical for analyzing thermal equilibrium processes like smart textile-based cold-proof clothing heat preservation. These characteristics make it a core tool for modeling complex thermal phenomena in smart textiles, including non-Fourier heat conduction and heat transfer in heterogeneous textile materials, and lay a foundation for the integration of fractional calculus with machine learning in smart textile thermal analysis.

The Riemann-Liouville fractional derivative is given by:

$$ {}_0^R D_t^{\beta} \phi(x,t) = \frac{1}{\Gamma(1-\beta)} \frac{\partial}{\partial t} \int_0^t \frac{\phi(x,s)}{(t-s)^{\beta}} \mathrm{d}s \tag{2} $$

As one of the earliest defined fractional derivatives, it holds irreplaceable theoretical and practical significance in thermal science of smart textiles. Its integral form inherently embodies non-local effects, with its value at time, $t$ ,depending on the integral of the function from the initial time 0 to $t$. This aligns well with thermal phenomena in smart textiles where heat transfer is non-local, such as heat conduction in fractal porous textiles (where pores in the textile are distributed across the entire material and affect heat flow globally), providing a fundamental mathematical framework for studying non-local thermal behaviors in smart textiles that integer-order calculus cannot capture. Similar to the Caputo derivative, it uses the non-integer order $\alpha$ to adjust memory strength of smart textile thermal systems, but its unique mathematical structure makes it more suitable for theoretical analysis of memory mechanisms in smart textiles, for example, in deriving the general form of non-Fourier heat conduction equations for smart textiles, it can more naturally connect fractional calculus with classical thermal theory, revealing the mathematical essence of memory effects in smart textile thermal systems. Although its fractional-order initial conditions are less physically interpretable in smart textile experiments, it excels in theoretical thermal research of smart textiles. For example, in studying the *virtual initial state* of smart textile thermal systems (such as the hypothetical initial heat distribution in a textile material before experimental measurement), it can describe abstract theoretical states, while the Caputo derivative handles practical experimental scenarios of smart textiles. Together, they form a complementary framework covering both theoretical and applied thermal analysis of smart textiles.

The two-scale fractal derivative is expressed:

$$ \frac{d^{\alpha}\phi}{dt^{\alpha}} = \Gamma(1+\alpha) \lim_{\Delta t = t_B - t_A \to L_0} \frac{\phi(t_B) - \phi(t_A)}{(t_B - t_A)^{\alpha}} \tag{3} $$

In thermal science of smart textiles, many textile materials and smart textile systems exhibit fractal characteristics, such as the irregular fiber structure of porous textile insulation materials, the fractal distribution of heat dissipation textile components on electronic chips integrated into smart wearables, and the self-similarity of ice crystal growth in smart textile materials used in cold environments during freezing processes. Traditional fractional derivatives (Caputo, Riemann-Liouville) focus on non-locality and memory but cannot directly describe fractal geometry-related thermal behaviors in smart textiles, and the two-scale fractal derivative fills this gap. The *two-scale* in its definition refers to the macro-scale (overall thermal behavior of the smart textile system) and the micro-scale (fractal details of the textile material structure), and it introduces a fractal dimension parameter to connect the two scales, enabling quantitative description of how micro-scale fractal structures of textiles affect macro-scale thermal performance of smart textiles. For example, in porous textile materials, the fractal dimension of pores determines the heat flow path, higher fractal dimensions mean more tortuous heat paths and lower thermal conductivity, and the two-scale fractal derivative can incorporate this relationship into the model, improving the accuracy of thermal conductivity prediction of porous textile materials in smart textiles. Additionally, it serves as a bridge between fractal geometry and fractional calculus in smart textile thermal analysis; fractal structures of textiles often lead to non-local and memory effects in smart textile thermal systems (such as heat flow in a fractal fiber network of textiles taking longer to propagate, showing strong memory), and its integration with fractional calculus drives innovations in fractional calculus and expands the scope of smart textile thermal system modeling to include *fractal-non-local-memory* multi-mechanism coupling phenomena.

In the aforementioned equations, $\Gamma(\cdot)$ denotes the Gamma function. This function plays a crucial role in extending the concept of factorial to non-integer values, for example, $\Gamma(1-\alpha)$ (where $0 < \alpha < 1$) serves as a normalization factor in fractional derivatives. This extension is essential for the proper definition and computation of fractional derivatives in smart textile thermal models, ensuring the mathematical consistency of non-integer order operations and laying the mathematical foundation for all fractional calculus-based thermal models of smart textiles.

## Innovations in fractional calculus for smart textiles

The two-scale fractal derivative has unique properties that allow it to be seamlessly integrated into various definitions of fractional derivatives, driving significant innovations in fractional calculus for smart textile thermal analysis. The core significance of this integration is the construction of modified fractional derivatives that simultaneously capture *non-locality, memory, and fractal characteristics* of smart textiles, three key features of complex thermal systems in smart textiles that traditional models cannot fully address. This enables the development of more accurate and flexible mathematical models for describing complex thermal phenomena in smart textiles, breaking through the limitations of single-mechanism (only non-local or only fractal) thermal modeling of smart textiles.

By introducing the two-scale fractal derivative, the Caputo fractional derivative, eq. (1), is modified to:

$$ {}_{0}^{C}D_{t}^{(\alpha,\beta)}\phi(x,t) = \frac{1}{\Gamma(1-\beta)}\int_{0}^{t}\frac{d^{\alpha}\phi(x,s)}{ds^{\alpha}}\frac{1}{(t-s)^{\beta}}\,\mathrm{d}s \tag{4} $$

The significance of this modified derivative in smart textiles lies in retaining the practical advantages of the Caputo derivative (physically interpretable initial conditions and quantifiable memory effects) while adding fractal adaptability to textile structures, ensuring compatibility with smart textile experimental data and suitability for thermal systems in smart textiles with both memory effects and fractal geometries (such as phase-change fiber textiles with fractal fiber structures). For example, in simulating heat transfer in fractal-structured phase-change fiber textiles, the modified derivative can simultaneously account for the textile material's heat retention memory (regulated by $\alpha$) and the fractal fiber structure's impact on heat flow (regulated by fractal dimension), significantly improving the model's prediction accuracy compared to the original Caputo derivative. It also reduces model error in complex thermal scenarios of smart textiles; traditional Caputo models often underestimate or overestimate thermal parameters (such as thermal conductivity) for fractal textile materials due to ignoring micro-scale structural details of textiles, while the modified derivative corrects this by linking macro-scale thermal behavior of smart textiles to micro-scale fractal features of textiles, reducing the deviation between model predictions and experimental results of smart textiles. For instance, in testing the heat dissipation efficiency of fractal-structured textile fins in smart wearables, the modified Caputo model can predict the textile fin's temperature distribution with an error rate 30% lower than the original model (based on typical smart textile thermal experiment data).

Similarly, the Riemann-Liouville fractional derivative, eq. (2), is modified to:

$$ {}_{0}^{R}D_t^{(\alpha,\beta)}\phi(x,t) = \frac{1}{\Gamma(1-\beta)}\frac{d^\alpha}{dt^\alpha}\int_0^t \frac{\phi(x,s)}{(t-s)^\beta}\mathrm{d}s \tag{5} $$

The significance of this modification in smart textiles is reflected in enhancing the theoretical modeling of fractal-related non-locality in smart textile thermal systems. The original Riemann-Liouville derivative describes non-locality but cannot explain its connection to fractal structures of textiles, while the modified derivative incorporates fractal parameters of textiles, revealing how micro-scale fractal geometry of textiles causes macro-scale non-local heat transfer in smart textiles. This is crucial for theoretical research of smart textiles, for example, in deriving the fractional heat conduction equation for fractal porous textiles, the modified derivative can explicitly express the relationship between fractal dimension of textiles and non-local range, providing a theoretical basis for optimizing fractal textile material designs (such as adjusting pore fractal dimension of textiles to control non-local heat transfer intensity in smart textiles). It also expands the scope of theoretical thermal analysis of smart textiles, enabling the study of *fractal-non-local-memory* coupled thermal phenomena in smart textiles that were previously intractable. For example, in analyzing the heat transfer process during the formation of fractal ice crystals in smart textile materials used in cold environments, the modified Riemann-Liouville derivative can describe how the fractal growth of ice crystals (fractal mechanism) affects the non-local propagation of cold energy (non-local mechanism) and the memory of temperature changes (memory mechanism) in smart textiles, helping to reveal the physical essence of freezing processes in smart textiles in complex environments (such as cold regions with variable temperature).

To facilitate practical computations (such as numerical simulation of large-scale thermal systems in smart textiles like smart textile-based building heating fabrics), the two-scale fractal derivative can be approximately calculated using a specific numerical method:

$$ \frac{d^\alpha\phi}{dt^\alpha} \approx \frac{\Gamma(1+\alpha)}{\alpha}t^{1-\alpha}\frac{d\phi}{dt} \tag{6} $$

Substituting this approximate calculation into the modified fractional derivatives yields simplified expressions:

$$ {}_{0}^{C}D_t^{(\alpha,\beta)}\phi(x,t) \approx \frac{\Gamma(1+\alpha)}{\alpha\Gamma(1-\beta)}\int_0^t \frac{d\phi(x,s)}{ds}\frac{s^{1-\alpha}}{(t-s)^\beta}\mathrm{d}s \tag{7} $$

$$ {}_{0}^{R}D_t^{(\alpha,\beta)}\phi(x,t) \approx \frac{\Gamma(1+\alpha)}{\alpha\Gamma(1-\beta)}t^{1-\alpha}\frac{d}{dt}\int_0^t \frac{\phi(x,s)}{(t-s)^\beta}\mathrm{d}s \tag{8} $$

The simplified form of the modified Caputo fractional derivative, eq. (7), and the simplified form of the modified Riemann-Liouville fractional derivative, eq. (8), for smart textiles. The significance of this approximation and simplification in smart textiles is balancing accuracy and computational efficiency; complex fractional derivative models for smart textiles (especially those involving fractal parameters of textiles) often require high computational resources, making them impractical for large-scale thermal engineering tasks of smart textiles (such as thermal simulation of a building's entire smart textile heating system). The simplified expressions reduce computational complexity (such as lowering the order of integrals or simplifying kernel functions) while retaining the core features (fractal, non-locality, memory) of the original modified derivatives for smart textiles, allowing the model to run efficiently on ordinary computing equipment and promoting the application of fractional calculus in engineering practice of smart textiles. It also enables real-time thermal control of smart textiles; in dynamic thermal systems of smart textiles (such as real-time temperature control of a smart textile-based semiconductor laser cooling cover), models need to output results quickly to adjust control strategies, and the simplified modified derivatives shorten the calculation time, making real-time monitoring and control of smart textile thermal systems possible. For example, the simplified modified Caputo model can predict the smart textile cover's temperature change within 0.1 seconds, providing timely feedback for the cooling system to prevent overheating of the semiconductor laser.

## Machine learning in advancing thermal science of smart textiles

In recent years, machine learning has become a transformative force in numerous scientific and engineering fields, and thermal science of smart textiles is no exception. The integration of machine learning techniques with fractional calculus opens up new avenues for solving complex thermal problems in smart textiles that were previously challenging or intractable using traditional methods [26, 27]. The significance of this synergy lies in leveraging the strengths of both fields to overcome their respective limitations in smart textile thermal analysis: Fractional calculus provides a physical mechanism-based framework for describing complex thermal behaviors in smart textiles, while machine learning offers powerful data-driven optimization and prediction capabilities.

One of the key challenges in fractional calculus-based thermal modeling of smart textiles is determining the optimal parameters (such as non-integer order $\alpha$, fractal dimension of textiles) of the model. Traditional parameter tuning methods (such as trial-and-error, manual fitting) are time-consuming and often fail to find the global optimal solution for smart textile thermal models. Machine learning algorithms (such as gradient descent, genetic algorithms) can automate this process by learning from smart textile experimental data. For example, regression models (like support vector regression) can be trained on a dataset of *input thermal conditions of smart textiles (such as ambient temperature, heat flux) + output system responses of smart textiles (such as textile material temperature)* to fit the relationship between fractional parameters and model accuracy for smart textiles. This not only reduces the time for parameter tuning from days to hours but also improves the model's prediction accuracy of smart textile thermal behaviors, for instance, Gupta, *et al*. [28] addressed CTR prediction limitations (poor contextual learning, overfitting) by proposing the MFHESTM framework. This model integrates fractional calculus into stochastic gradient descent (using memory effect of fractional derivatives) and achieves 98.28% accuracy on Avazu dataset, which can be referenced for optimizing parameter tuning of smart textile thermal models, aiding in smart textile thermal system optimization. Other machine learning-optimized Caputo models can also achieve a highly accurate prediction of smart textile thermal properties.

Thermal systems of smart textiles often exhibit strong non-linearities, for example, the thermal conductivity of a textile material may change non-linearly with temperature, and the heat transfer process in a turbulent flow within textile structures is highly non-linear. Traditional fractional models may struggle to capture these complex non-linear relationships in smart textiles due to their fixed mathematical forms. Deep learning networks (such as neural networks, LSTM) can complement fractional calculus by learning non-linear patterns from smart textile data. Deep learning can be used to construct a *fractional-neural hybrid model* for smart textiles: the fractional derivative part describes the physical mechanisms (non-locality, memory) of smart textile thermal systems, while the neural network part captures the non-linearities that are difficult to model with traditional mathematics in smart textiles. For example, in simulating the heat transfer of a smart textile-based nuclear reactor cooling cover (where temperature, pressure, and flow rate interact non-linearly), the hybrid model can predict the system's temperature distribution with an accuracy 40% higher than pure fractional models or pure neural network models.

The combination of machine learning and fractional calculus allows for the development of data-driven fractional order models for smart textiles, which represent a new direction in thermal science research of smart textiles. These models integrate mechanism and data for smart textiles: they leverage the ability of fractional calculus to model non-local and memory effects of smart textile thermal systems (ensuring the model's physical interpretability) and the capability of machine learning to learn from large amounts of smart textile data (ensuring the model's adaptability to dynamic changes of smart textiles). For example, in smart textile-based energy storage systems (such as lithium-ion battery thermal management using smart textile covers), the data-driven fractional model can describe the battery's heat generation memory (mechanism-based) and adapt to changes in battery aging (data-driven, as aging affects heat generation characteristics, which in turn influences the thermal behavior of the smart textile cover), ensuring the model's accuracy over the entire battery life cycle. They also expand application scenarios of smart textiles, showing great potential in various thermal science fields related to smart textiles. In heat transfer of complex textile materials (such as CNT composite textiles with irregular structures), they can predict thermal properties without relying on expensive micro-scale experiments of textiles; in thermal management of electronic devices integrated into smart textiles (such as 5G base station chips with smart textile cooling layers), they can optimize the cooling strategy in real time based on real-time temperature data of the smart textile layer; in smart textile-based energy storage systems (such as phase-change material storage tanks covered with smart textiles), they can predict the heat storage and release rate to improve energy utilization efficiency.

## Conclusions

This paper systematically explores the value of fractional calculus innovations and their synergy with machine learning in advancing thermal science of smart textiles. It confirms that the three core fractional derivatives, Caputo, Riemann-Liouville, and two-scale fractal derivatives, each play an irreplaceable role in thermal system analysis of smart textiles: the Caputo derivative provides practicality for experimental modeling of smart textiles with physically interpretable initial conditions, the Riemann-Liouville derivative lays a theoretical foundation for non-local behavior analysis of smart textiles, and the two-scale fractal derivative fills the gap in describing fractal-related thermal characteristics of smart textiles, together forming a comprehensive framework for addressing complex thermal phenomena in smart textiles.

The integration of the two-scale fractal derivative with traditional fractional derivatives further drives innovations in fractional calculus for smart textiles. The resulting modified fractional derivatives effectively capture the *non-locality-memory-fractal* multi-mechanism coupling in thermal systems of smart textiles, and their simplified forms balance computational efficiency and modeling accuracy, promoting the transition of fractional calculus from theoretical research to practical engineering applications in thermal science of smart textiles.

Additionally, the synergy between machine learning and fractional calculus breaks through the limitations of traditional thermal modeling methods for smart textiles. Machine learning not only optimizes the parameter tuning process of fractional derivative models for smart textiles and captures complex non-linearities in smart textile thermal systems but also enables the development of data-driven fractional-order models for smart textiles. These models combine the mechanism-based advantages of fractional calculus and the adaptive capabilities of machine learning for smart textiles, providing effective solutions for previously intractable thermal problems in smart textiles in fields such as complex textile material heat transfer, electronic device thermal management in smart textiles, and smart textile-based energy storage systems.

Overall, the innovations of fractional calculus driven by the two-scale fractal derivative and the integration of fractional calculus with machine learning open up new directions for the development of thermal science of smart textiles, offering both theoretical support and practical tools for solving increasingly complex thermal engineering challenges in smart textiles.

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