ANALYTICAL SOLITON SOLUTIONS OF THE FRACTIONAL PERTURBED GERDJIKOV-IVANOV EQUATION USING THE JACOBI ELLIPTIC FUNCTION EXPANSION METHOD

by

Nauman AHMED[a], Shazia UMER[b], Dumitru BALEANU[c], Muhammad Zafarullah BABER[a,d], Ali AKGUL[e,f,g,h,i*], Ali RAZA[j,k*], and Hala H. TAHA[l]

[a] Department of Mathematics and Statistics, The University of Lahore, Lahore, Pakistan
[b] Department of Mathematics and Statistics, University of Lahore Sargodha Campus, Sargodha, Pakistan
[c] Department of Computer Science and Mathematics, Lebanese American University, Beirut, Lebanon
[d] Department of Mathematics, Shanghai University and Newtouch Center for Mathematics, Shanghai University, Shanghai, China
[e] Department of Electronics and Communication Engineering, Saveetha School of Engineering, SIMATS, Chennai, Tamilnadu, India
[f] Department of Mathematics, Art and Science Faculty, Siirt University, Siirt, Turkey
[g] Near East University, Mathematics Research Center, Department of Mathematics, Nicosia/Mersin, Turkey
[h] Department of Computer Engineering, Biruni University Topkapı, Istanbul, Turkey
[i] Applied Science Research Center, Applied Science Private University, Amman, Jordani
[j] IT4Innovations, VSB-Technical University of Ostrava, Ostrava, Czech Republic
[k] Center for Research and Development in Mathematics and Applications (CIDMA), Department of Mathematics, University of Aveiro, Aveiro, Portugal
[l] Department of Mathematical Sciences, College of Science, Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia

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The perturbed Gerdjikov-Ivanov equation is important and has several applications in photonic crystal fibers and fiber optics. In this work, the Jacobi elliptic function expansion approach has been used to solve the Gerdjikov-Ivanov equation. The Jacobi elliptic function expansion method is used to find the solutions for solitary waves and soliton waves. We develop hyperbolic and trigonometric functions for solitary wave solutions and other types of soliton solutions, including dark and bright. The physical behaviors of the solutions can be illustrated in 2-D and 3-D using contours. Simulations are utilized to confirm the exact soliton solutions. With a few closing thoughts, we concluded the investigation.

Key words: *Jacobi elliptic function expansion approach, Gerdjikov-Ivanov equation, exact solutions*

Introduction

Non-linear partial differential equations (NLPDE) [1] have significantly affected the applications in the real world. One prominent idea that belongs to the larger category of NLPDE is solitary wave theory. Lambert and Russell [2] discovered solitary waves, sometimes

* Corresponding authors, e-mail: aliakgul00727@gmail.com; ali.raza@ua.pt

known to be enormous waves, for the first time in 1834. As they circulate and interact with other solitons [3], they respond like particles and keep their speed and shape. Considering the various uses they have in solid-state physics [4], fluid mechanics [5], optical fibers [6], and plasma physics [7], solitary wave solutions have attracted the curiosity of multiple researchers. Multiple computational [8], and analytical techniques [9] to calculate the precise solutions of NLPDE have advanced significantly in the past decade. These encompass the extended rational sine-cosine method [10], Legendre homotopy method [11], residual power series method [12], homotopy analysis [13], the subequation method [14], the first integral method [15], the modified Kudryashov method [16], the G'/G expansion method [17], the modified simple equation method [18], the extended tanh method [19], the two variables (G'/G, $1/G$) expansion method [20], and the projective Reccati equation method [21]. Fractional calculus [22] has always been a popular topic among researchers and can be successfully employed to extract the solutions of fractional PDE [23]. There are countless scientific and engineering implications for it. The reproductive and memory behavioral characteristics of many structures and materials, including polymers [24], are usually described by Atangana-Baleanu, Beta M-truncated, and Riemann-Liouville fractional derivatives, among others. Current developments in fractional differential equations (FDE) have contributed to an expansion of the use of exact solutions of FDE in non-linear theory. A variety of FDE have been explored by numerous scholars, such as the Burgers' equation, the Navier-Stokes equation, the space-time fractional Schrodinger's equation, and the generalized Riccati equation. A spectrum problem and the corresponding perturbed Gerdjikov-Ivanov (GI) hierarchy of non-linear evolution equations are discussed in the present article. It demonstrates that the GI hierarchy has a bi-Hamiltonian structure and is integrable in the Liouville sense. Jacobi elliptic function expansion (JEFE) approach have been used for analyzing the perturbed GI equation. The recommended approach have been used to find solutions for the given equation that are hyperbolic, trigonometric, and rational functions. By choosing suitable figures for the free parameters revealed in the derived solutions, a broad spectrum of optical soliton solutions with complicated dynamical topologies can be effectively produced. For instance, power law linearity, dual-power law, and even log-law non-linearity cannot be controlled by the traditional inverse scattering methodology. Only bright soliton solutions may be extracted using the semi-inverse variational principle method.

The results obtained in this study are grounded in principles from functional analysis, particularly in the treatment of fractional-order differential operators and solution spaces. The use of conformable derivatives involves mappings between function spaces where continuity, differentiability, and boundedness must be rigorously defined, often within the setting of Banach or Hilbert spaces. Moreover, the transformation of the non-linear fractional PDE into an ODE system and the subsequent derivation of solutions using Jacobi elliptic functions rely on properties of functionals and operators that are central topics in functional analysis. Key tools such as existence theorems, operator identities, and stability criteria derive from this theoretical foundation and justify the analytical validity of the constructed solutions.

Multiple important and productive techniques, in particular, have been suggested to figure out solutions to the GI equation. The GI equation, which originates in the study of complex systems, non-linear processes, and solitons, is a hybrid of the non-linear wave equation and the non-linear Schrodinger equation (NLSE) [25]. The basic structure of the perturbed GI equation:

$$iD_t^{\beta}S+\beta_1 D_x^{2\beta}S+\beta_2 |H|^4 H=i(\beta_3 S^2 D_x^{\beta}S^* +b_1 D_x^{\beta}S+b_2 D_x^{\beta}(|S|^2 S)+b_3 D_x^{\beta}(|S|^2 S)) \tag{1}$$

where an unknown function is represented by $S(x, t)$, D_t^{β} is notation for the R conformable derivative of t. The notation for a conformable derivative with respect to x is D_x^{β}. To show the fractional order, consider $\beta \in (0, 1)$. Intermodal dispersion, self-steepening, and non-linear dispersion can be represented by the real coefficients, b_1, b_2, and b_3, although the group velocity dispersion, quintic non-linearity, and self-steepening sections are characterized by the real coefficients β_1, β_2, and β_3, respectively. Equation (1) engages the consideration of numerous scholars and researchers.

The Jacobi elliptic function expansion method

Here, we describe the execution of the JEFE method. The following steps are part of the process. In general terms, the NLPDE are composed:

$$F\left(S, D_t^{\beta} S, D_x S, D_t^{2\beta} S, D_{xx} S, \ldots\right) = 0 \tag{2}$$

where F is one of S's polynomials. The transformation is now provided:

$$S\left(x,t\right) = \exp\left(i\psi\right) Z\left(\chi\right) \text{ where } \chi = \frac{x^{\beta}}{\beta} - \frac{ut^{\beta}}{\beta} \text{ and } \psi = \theta_0 + \frac{\omega t^{\beta}}{\beta} + \frac{\left(-\tau\right) x^{\beta}}{\beta} \tag{3}$$

The fractional order derivative in this case is β. When eq. (3) is entered into eq. (2), we obtain:

$$B\left(Z, Z', Z'', Z''', \ldots\right) = 0 \tag{4}$$

where B is the polynomial of Z.

The fundamental idea behind this approach is to maximize the number of Jacobin elliptic solutions for the primary issue by increasing the likelihood of solving the auxiliary ODE. To demonstrate the auxiliary equation:

$$\psi'\left(\chi\right) = \sqrt{P\psi(\chi)^4 + Q\psi(\chi)^2 + R} \tag{5}$$

where R, Q, and P are the constants and $\psi' = \mathrm{d}\psi/\mathrm{d}\chi$ and $\chi = \psi(x, t)$.

Table 1. The $\psi(\chi)$ solution in eq. (5) for the specifically selected values of Q, R, and P

Sr. #	P	Q	R	G
	α^2	$-(1 + \alpha^2)$	1	sn, cd
	$-\alpha^2$	$2\alpha^2 - 1$	$1 - \alpha^2$	cn
	-1	$2 - \alpha^2$	$\alpha^2 - 1$	dn
	1	$-(1 + \alpha^2)$	α^2	ns, dc
	$\alpha^2 - 1$	$2 - \alpha^2$	-1	nd
	$1 - \alpha^2$	$2 - \alpha^2$	1	sc
	$-\alpha^2 (1 - \alpha^2)$	$2\alpha^2 - 1$	1	sd
	1	$2 - \alpha^2$	$1 - \alpha^2$	cs
	1/4	$(-2\alpha^2 + 1)/2$	1/4	$ns \mp cs$
	$(1 - \alpha^2)/4$	$(\alpha^2 + 1)/2$	$(1 - \alpha^2)/4$	$nc \mp sc$
	$\alpha^2/4$	$(\alpha^2 - 2)/2$	1/4	$sn/(1 \mp dn)$
	$(\alpha^2 - 1)/4$	$(\alpha^2 + 1)/2$	$(\alpha^2 - 1)/4$	$dn/(1 \mp \alpha sn)$
	$(1 - \alpha^2)/4$	$(\alpha^2 + 1)/2$	$(-\alpha^2 + 1)/4$	$cn/(1 \mp sn)$

In the tab. 1, where $i^2 = -1$, the solution eq. (5) can be found. The $\alpha(0 < \alpha < 1)$ is the modulus in the Jacobi elliptic technique: $sn\chi = sn(\chi, \alpha)$, $cn\chi = cn(\chi, \alpha)$, and, $dn\chi = \mathrm{d}n(\chi, \alpha)$. The following features of the double periodic elliptic functions are presented in the paper:

$$sn^2\chi + cn^2\chi = 1 \tag{6}$$

$$dn^2\chi + \alpha^2 sn^2\chi = 1 \tag{7}$$

$$\frac{d}{d\chi}sn\chi = cn\chi dn\chi,\ \frac{d}{d\chi}(cn\chi) = -sn\chi dn\chi,\ \frac{d}{d\chi}(dn\chi) = -\alpha^2 sn\chi cn\chi \tag{8}$$

The Jacobi elliptic functions become trigonometric and hyperbolic functions when α is restricted to $\alpha \to 0$ and $\alpha \to 1$, which tab. 2 mentions.

Table 2. Jacobi elliptic functions for $\kappa \to 1$ and $\kappa \to 0$

Sr. #	Case	$\kappa \to 1$	$\kappa \to 0$
	snS	tanh$S$	sin$S$
	cnS	sech$S$	cos$S$
	dnS	sech$S$	1
	sdS	sinh$S$	sin$S$
	ndS	cosh$S$	1
	ncS	cosh$S$	sec$S$
	scS	sinh$S$	tan$S$
	nsS	cosh$S$	sec$S$

Thus, we determine the various kinds of solutions to the eq. (9). The finite series of the Jacobi elliptic function is represented by the symbol $S(\chi)$. In this way, we obtain the subsequent expression:

$$S(\chi) = \sum_{j=0}^{N} (\kappa_j \psi\chi)^j \tag{9}$$

In this context, where $M(\chi)$ is the satisfies the non-linear ordinary eq. (5). Where ω_j (j = 0, 1, 2, ..., N) constitutes the constants to be obtained. The highest-order linear term in eq. (9) can be used to calculate the number, N:

$$O\left(\frac{d^v S}{dz^v}\right) = N + v,\quad v = 0,1,2,3,\cdots \tag{10}$$

Thus, eq. (11) represents the highest-order non-linear terms:

$$O\left(S^r \frac{d^v S}{dz^v}\right) = (r+1)N + v,\ v = 0,1,2,3,\cdots,\ r = 1,2,3,\cdots \tag{11}$$

Considering eqs. (2) and (9), and setting all power coefficients $\psi(\chi)$ to zero. This resulted in the collection of non-linear algebraic equations for κ_j(j = 0, 1, 2, 3,..., G). We use the Mathematica software to solve the system of equations and equate all of the values for Q, R, and P, in eq. (5) in tab. 1. This leads to the exact solution, eq. (2).

Fractional-order derivative

An important derivative of fractional order that is conformable is described in this section. It enables us to comprehend physical acts' effects more clearly.

Definition 1. For example, let *d:* [0, ∞] → *R* be a function of β^{th} order. The term *conformable functional derivative* means:

$$D^{\beta}\left(d\right)\left(t\right)=\frac{\lim\limits_{\varsigma\to 0}\left(d\left(\zeta t^{1-\gamma}+t\right)-ht\right)}{\varsigma},\ \ \forall\ \ t>0,\ \text{and}\ \ \beta\in(0,1] \tag{12}$$

Definition 2. The conformable integral of function, *g*, is defined as:

$$D_{\beta}^{\beta}\left(g\right)\left(w\right)=\int\limits_{0}^{w}\frac{\mathrm{d}t}{t^{1-\beta}}\mathrm{d}t,\ \ \forall t>0,\ \text{and}\ \ \beta\in(0,1] \tag{13}$$

Theorem 1. Given a function ϱ and ϑ, and β ∈ (0, 1] being β differentiable at a point $t > 0$:

$$D^{\beta}\left(\varrho*p+\vartheta*q\right)=p*D^{\beta}\left(\varrho\right)+q*D^{\beta}\left(\vartheta\right),\ \forall p,\ q\in R$$

$$D^{\beta}\left(t^{u}\right)=u*t^{r-\beta},\ \ \forall u\in R$$

$$D^{\beta}\left(\varrho.\vartheta\right)=\varrho*D^{\beta}\left(\vartheta\right)+\vartheta*D^{\beta}\left(\varrho\right)$$

$$D^{\beta}\frac{\varrho}{\vartheta}=\frac{\eta*D^{\beta}\left(\varrho\right)-\varrho*D^{\beta}\left(\vartheta\right)}{\vartheta^{2}}$$

$$D^{\beta}\left(p\right)=0,\ \forall\ \text{constant}\ \ \text{function}\ \ q\left(t\right)=p$$

Solution of the model

Now transform PDE of eq. (1) into total differential equations using transform:

$$S\left(x,t\right)=\exp\left(i\psi\right)Z\left(\chi\right)\ \text{where}\ \chi=\frac{x^{\beta}}{\beta}-\frac{ut^{\beta}}{\beta}\ \text{and}\ \psi=\theta_{0}+\frac{\omega t^{\beta}}{\beta}+\frac{\left(-\tau\right)x^{\beta}}{\beta} \tag{14}$$

by substituting eq. (14) into eq. (1) we get ODE:

$$q_{1}Z''Z\left(\chi\right)-Z'^{2}+q_{4}Z(\chi)^{4}+q_{3}Z(\chi)^{3}+q_{2}Z(\chi)^{2} \tag{15}$$

Now applying balancing procedure to eq. (15) and put in eq. (9):

$$Z\left(\chi\right)=\kappa_{0}+\kappa_{1}\psi\left(\chi\right) \tag{16}$$

where κ_0 and κ_1 are the constants.

We get:

$$q_{1}=\frac{3\left(\sqrt{q_{3}^{4}q_{4}^{4}\left(Q^{2}-4PR\right)}+q_{3}^{2}q_{4}^{2}Q\right)}{128Pq_{4}^{3}R},\ \ \kappa_{1}=-\frac{3\sqrt{-\dfrac{\sqrt{q_{3}^{4}q_{4}^{4}\left(Q^{2}-4PR\right)}+q_{3}^{2}q_{4}^{2}Q}{q_{4}^{4}R}}}{8\sqrt{2}}$$
$$\kappa_{0}=-\frac{3q_{3}}{8q_{4}},\ \ q_{2}=-\frac{3\left(q_{3}^{2}q_{4}^{2}\left(Q^{2}-12PR\right)+Q\sqrt{q_{3}^{4}q_{4}^{4}\left(Q^{2}-4PR\right)}\right)}{128Pq_{4}^{3}R} \tag{17}$$

When $P = \alpha^2$, $Q = -(\alpha^2 + 1)$, $R = 1$, $\psi(\chi) = sn(\chi, \alpha)$, if $\alpha = 1$, then $sn(\chi, \alpha) = \tanh(\chi)$:

$$S_1(x,t)=\left(\frac{3}{8}\sqrt{\frac{q_3^2}{q_4^2}}\tanh\left(\frac{ut^\beta}{\beta}-\frac{x^\beta}{\beta}\right)-\frac{3q_3}{8q_4}\right)e^{i\left(\theta_0+\frac{\omega t^\beta}{\beta}-\frac{\tau x^\beta}{\beta}\right)} \tag{18}$$

When $P = \alpha^2$, $Q = -(\alpha^2 + 1)$, $R = 1$, $\psi(\chi) = sn(\chi, \alpha)$, if $\alpha = 0$, then $sn(\chi, \alpha) = \sin(\chi)$:

$$S_2(x,t)=\left(\frac{3\sqrt{-\frac{\sqrt{q_3^4q_4^4}-q_3^2q_4^2}{q_4^4}}\sin\left(\frac{ut^\beta}{\beta}-\frac{x^\beta}{\beta}\right)}{8\sqrt{2}}-\frac{3q_3}{8q_4}\right)e^{i\left(\theta_0+\frac{\omega t^\beta}{\beta}-\frac{\tau x^\beta}{\beta}\right)} \tag{19}$$

When $P = 1 - \alpha^2$, $Q = 2\alpha^2 - 1$, $R = -\alpha^2$, $\psi(\chi) = nc(\chi, \alpha)$, if $\alpha = 1$, then $nc(\chi, \alpha) = \cosh(\chi)$:

$$S_3(x,t)=\left(-\frac{3\sqrt{\frac{q_3^2q_4^2+\sqrt{q_3^4q_4^4}}{q_4^4}}\cosh\left(\frac{ut^\beta}{\beta}-\frac{x^\beta}{\beta}\right)}{8\sqrt{2}}-\frac{3q_3}{8q_4}\right)e^{i\left(\theta_0+\frac{\omega t^\beta}{\beta}-\frac{\tau x^\beta}{\beta}\right)} \tag{20}$$

When $P = -\alpha^2(1 - \alpha^2)$, $Q = 2\alpha^2 - 1$, $R = 1$, $\psi(\chi) = sd(\chi, \alpha)$, $\alpha = 1$, then $sd(\chi, \alpha) = \sinh(\chi)$:

$$S_4(x,t)=\left(\frac{3\sqrt{\frac{q_3^2q_4^2+\sqrt{q_3^4q_4^4}}{q_4^4}}\sinh\left(\frac{ut^\beta}{\beta}-\frac{x^\beta}{\beta}\right)}{8\sqrt{2}}-\frac{3q_3}{8q_4}\right)e^{i\left(\theta_0+\frac{\omega t^\beta}{\beta}-\frac{\tau x^\beta}{\beta}\right)} \tag{21}$$

When $P = \alpha^2/4$, $Q = 1/2(\alpha^2 - 2)$, $R = \alpha^2/4$, $\psi(\chi) = sn(\chi, \alpha) + icn(\chi, \alpha)$, if $\alpha = 1$, then $sn(\chi, \alpha) + icn(\chi, \alpha) = \text{sech}(\chi) + \tanh(\chi)$:

$$S_5(x,t)=e^{i\left(\theta_0+\frac{\omega t^\beta}{\beta}-\frac{\tau x^\beta}{\beta}\right)}\left(-\frac{3q_3}{8q_4}-\frac{3}{8}\sqrt{\frac{q_3^2}{q_4^2}}\left(-\tanh\left(\frac{ut^\beta}{\beta}-\frac{x^\beta}{\beta}\right)+i\,\text{sech}\left(\frac{ut^\beta}{\beta}-\frac{x^\beta}{\beta}\right)\right)\right) \tag{22}$$

When $P = 1/4$, $Q = 1/2(1 - 2\alpha^2)$, $R = 1/4$, $\psi(\chi) = cn(\chi, \alpha)\alpha + idn(\chi, \alpha)$, if $\alpha = 1$, then $cn(\chi, \alpha)\alpha + idn(\chi, \alpha) = \text{sech}(\chi) + \text{sech}(\chi)$.

$$S_6(x,t)=\left(-\frac{3q_3}{8q_4}+\left(-\frac{3}{8}-\frac{3i}{8}\right)\sqrt{\frac{q_3^2}{q_4^2}}\,\text{sech}\left(\frac{ut^\beta}{\beta}-\frac{x^\beta}{\beta}\right)\right)e^{i\left(\theta_0+\frac{\omega t^\beta}{\beta}-\frac{\tau x^\beta}{\beta}\right)} \tag{23}$$

When $P = 1/4$, $Q = 1/2(1 - 2\alpha^2)$, $R = 1/4$, $\psi(\chi) = cn(\chi, \alpha)\alpha + idn(\chi, \alpha)$, if $\alpha = 0$, then $cn(\chi, \alpha)\alpha + idn(\chi, \alpha) = \cos(\chi) + 1$:

$$S_7(x,t)=\left(-\frac{3q_3}{8q_4}+\frac{1}{8}(-3)i\sqrt{-\frac{q_3^2}{q_4^2}}\cos\left(\frac{ut^\beta}{\beta}-\frac{x^\beta}{\beta}\right)\right)e^{i\left(\theta_0+\frac{\omega t^\beta}{\beta}-\frac{\tau x^\beta}{\beta}\right)} \tag{24}$$

When $P = \alpha^2/4$, $Q = 1/2(\alpha^2 - 2)$, $R = 1/4$, $\psi(\chi) = sn(\chi, \alpha)/[dn(\chi, \alpha) + 1]$, if $\alpha = 0$, then $dn(\chi, \alpha) = 1$, $sn(\chi, \alpha) = \sin(\chi)$:

$$S_8\left(x,t\right) = \mathrm{e}^{i\left(\theta_0 + \frac{\omega t^\beta}{\beta} - \frac{\tau x^\beta}{\beta}\right)}\left(\frac{3\sqrt{-\frac{\sqrt{q_3^4 q_4^4} - q_3^2 q_4^2}{q_4^4}}\sin\left(\frac{ut^\beta}{\beta} - \frac{x^\beta}{\beta}\right)}{4\sqrt{2}\left(1\left(\frac{x^\beta}{\beta} - \frac{ut^\beta}{\beta}\right) + 1\right)} - \frac{3q_3}{8q_4}\right) \tag{25}$$

When $P = 1/4\ (1 - \alpha^2)$, $Q = 1/2(\alpha^2 + 1)$, $R = 1/4$, $\psi(\chi) = sn(\chi, \alpha)/[cn(\chi, \alpha) + dn(\chi, \alpha)]$, $dn(\chi, \alpha) = 1$, if $\alpha = 0$, then $cn(\chi, \alpha) = \cos(\chi)$, $sn = \sin(\chi)$:

$$S_9\left(x,t\right) = \mathrm{e}^{i\left(\theta_0 + \frac{\omega t^\beta}{\beta} - \frac{\tau x^\beta}{\beta}\right)}\left(\frac{3\sqrt{-\frac{q_3^2}{q_4^2}}\sin\left(\frac{ut^\beta}{\beta} - \frac{x^\beta}{\beta}\right)}{8\left(1\left(\frac{x^\beta}{\beta} - \frac{ut^\beta}{\beta}\right) + \cos\left(\frac{ut^\beta}{\beta} - \frac{x^\beta}{\beta}\right)\right)} - \frac{3q_3}{8q_4}\right) \tag{26}$$

When $P = \alpha^2/4$, $Q = 1/2(\alpha^2 - 2)$, $R = 1/4$, $\psi(\chi) = [cn(\chi, \alpha) + dn(\chi, \alpha)]/[dn(\chi, \alpha) - \alpha^2 + 1]^{1/2}$, if $\alpha = 0$, then $cn(\chi, \alpha) = \cos(\chi)$, and $dn(\chi, \alpha) = 1$:

$$S_{10}\left(x,t\right) = \mathrm{e}^{i\left(\theta_0 + \frac{\omega t^\beta}{\beta} - \frac{\tau x^\beta}{\beta}\right)}\left(-\frac{3\sqrt{-\frac{\sqrt{q_3^4 q_4^4} - q_3^2 q_4^2}{q_4^4}}\cos\left(\frac{ut^\beta}{\beta} - \frac{x^\beta}{\beta}\right)}{4\sqrt{2}\sqrt{2}\left(\frac{x^\beta}{\beta} - \frac{ut^\beta}{\beta}\right)} - \frac{3q_3}{8q_4}\right) \tag{27}$$

Graphical behavior

This section examines the graphical behavior of the exact solutions by solving the GI equation using the JEFE method. For the required solutions, we have to draw 3-D, 2-D, and their contours in an attempt at describing various physical features. We can get more reliable information about the solutions' behaviors from these graphs. Figure 1 indicates the graphical behavior of dark solitons, while figs. 2 and 4 illustrate the graphical behavior of bright soliton solutions and kink soliton solutions, respectively. Figure 3 illustrate the graphical behavior of solitary wave solutions.

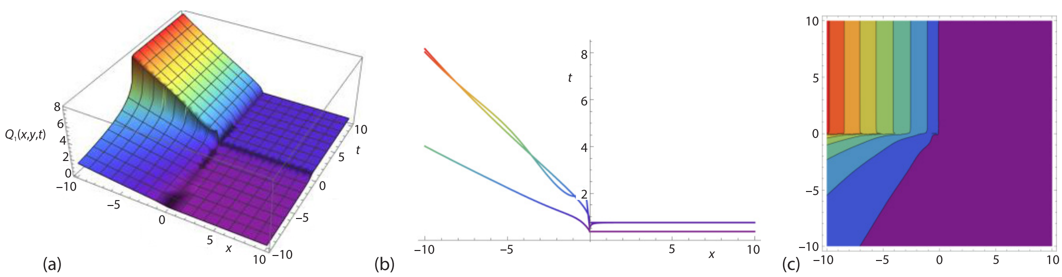

Figure 1. Above 3-D, 2-D and contour figures show the graphically behavior of $S_1(x, t)$ with parameters: $\beta = 0.4$, $\theta_0 = 0.1$, $q_3 = 1$, $q_4 = 1$, $\tau = 0.4$, $u = -1$, and $\omega = 0.3$

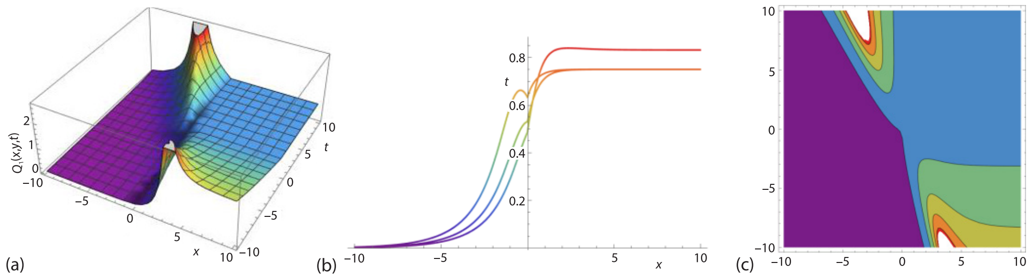

Figure 2. Above 3-D, 2-D and contour figures show the graphically behavior of $S_6(x, t)$ with parameters: $\beta = 0.9$, $\theta_0 = 0.1$, $q_3 = 1$, $q_4 = 1$, $\tau = 1$, $u = -0.4$, and $\omega = -0.3$

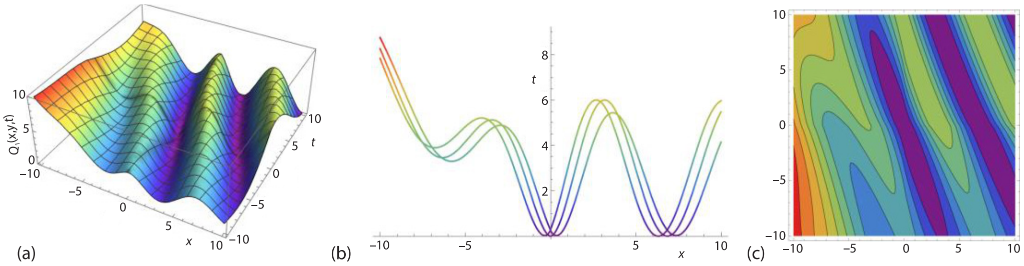

Figure 3. Above 3-D, 2-D and contour figures show the graphically behavior of $S_8(x, t)$ with parameters: $\beta = 0.9$, $\theta_0 = 0.1$, $q_3 = 0.8$, $q_4 = 0.1$, $\tau = -0.4$, $u = -0.4$, and $\omega = 0.3$

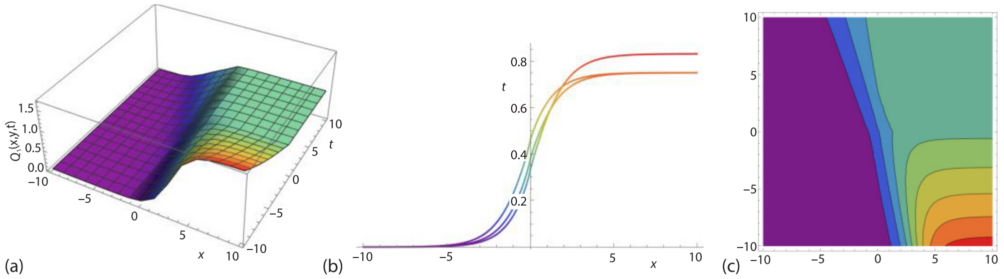

Figure 4. Above 3-D, 2-D and contour figures show the graphically behavior of $S_5(x, t)$ with parameters: $\beta = 0.9$, $\theta_0 = 0.1$, $q_3 = 1$, $q_4 = 1$, $\tau = -0.4$, $u = -0.4$, and $\omega = -0.3$

Conclusion

The present research is focused on finding solitary wave solutions of a particular kind of Schrodinger equation named the conformable perturbed GI equation. To do this, the proposed equation is reduced to a non-linear ODE employing the traveling wave transformation. The solitary wave solutions are then extracted using the JEFE approach and the full discriminating model. With the most suitable parameter selections, multiple 2-D, 3-D, and contour plots have been used to make it easier to understand the solutions physical illustration. Because of their distinct and reliable propagation characteristics, these soliton solutions play an essential role in the field of optics. Solitons have made optical communication feasible by permitting high capacity data transmission and all optical signal processing. Additionally, these results may be useful instruments for analyzing coastal, marine, and photonic crystal fibers. There is still a lot of work to be done on this model which might be explored in the future using numerous methods.

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Data availability

The references for the data used to support the findings of this study are -cited within the article.

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Conflicts of interest

The authors affirm they have no conflicts of interest to disclose concerning the current study.

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