CALCULATION MODEL FOR THE MONTHLY OPTIMAL TILT ANGLE OF FIXED PHOTOVOLTAIC PANELS

by

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In photovoltaic systems, the installation tilt angle of modules plays a critical role in determining both performance and economic returns. Existing methods for calculating the optimal tilt angle typically rely on complex solar radiation and anisotropic sky scattering models, which, despite their accuracy, involve intensive computational efforts. Based on the relationships between the proportion of direct radiation, latitude, and the optimal monthly tilt angle, this study proposed a method to calculate the monthly optimal tilt angle. Taking Zhangye City as an example, the accuracy and reliability of the proposed method were verified in comparison with the traditional method. Using this method, the monthly optimal tilt angles were calculated for three cities in China located at different latitudes, to reveal the annual variation patterns of these angles. The results showed that, at the same proportion of direct radiation, the monthly optimal tilt angle increases with latitude. Over the course of a year, the monthly optimal tilt angle first decreases and then increases. In low latitude cites, the monthly optimal tilt angle may even become negative.

Key words: *fixed photovoltaic panel, monthly optimal tilt angle, calculation model, proportion of direct radiation*

Introduction

Solar photovoltaic (PV) power generation has emerged as a crucial source of green energy in recent years [1, 2]. In PV systems, the tilt angle of the panels plays a pivotal role in determining both performance and economic viability [3, 4]. Currently, the optimal tilt angles for PV panels are generally classified into three categories: monthly, quarterly, and annual optimal tilt angles. These angles correspond to those that maximize the total solar radiation received by the panels over different time scales: monthly, quarterly, and annually. Despotović *et al*. [5] demonstrated that, for fixed PV systems, the monthly optimal tilt angle results in the highest annual energy yield. Benghanem [6] conducted the collected solar energy will be greater if choose the optimum panel tilt for each month.

In recent years, several models have been developed to estimate radiation on inclined surfaces, with some specifically designed for particular scenarios [7-9]. The process of determining the optimal tilt angle for PV panels typically involves calculating the total radiation received at various tilt angles and selecting the angle that maximizes this radiation [10].

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Although existing methods can determine the optimal tilt angle, they often involve extensive computational effort [11, 12]. This paper analyzes the relationship between the proportion of direct radiation, latitude, and the optimal tilt angle. Based on these findings, a new calculation model is proposed, which significantly improves calculation speed compared to traditional methods. The model incorporates latitude, tilt angle, and month, making it highly versatile and suitable for calculating the monthly optimal tilt angle across various regions. The comparison among existing calculation methods validated the accuracy of the proposed model. The model was further applied to calculate the monthly optimal tilt angles for cities at low, middle, and high latitudes in China, revealing the evolution patterns of the monthly optimal tilt angles throughout the year.

Calculating method for total radiation on inclined surfaces

Hay [8] identified that total radiation on an inclined surface, I_T, consists of three main components: the direct radiation, $I_{T,b}$, diffuse radiation, $I_{T,d}$, and reflected radiation, $I_{T,g}$. These can be expressed [2]:

$$I_T = I_{T,b} + I_{T,d} + I_{T,g} \tag{1}$$

The direct radiation received on an inclined surface can be expressed [13]:

$$I_{T,b} = I_b \times R_\mathrm{b} \tag{2}$$

where I_b is the direct radiation on the horizontal surface and R_b – the ratio of the average daily direction radiation on a tilted surface to that on a horizontal surface, which can be determined geometrically [14]:

$$R_b = \frac{\cos(\phi-\beta)\cos\delta\sin\omega_{sT} + \frac{\pi}{180}\omega_{sT}\sin(\phi-\beta)\sin\delta}{\cos\phi\cos\delta\sin\omega_s + \frac{\pi}{180}\omega_s\sin\phi\sin\delta} \tag{3}$$

where β is the tilt angle of the solar panel, ϕ – the latitude, δ – the declination of the Sun, ω_s – the sunset hour angle on a horizontal surface, and ω_T – the sunset hour angle on the inclined surface. The δ, ω_s, and ω_T can be determined [12]:

$$\delta = 23.45 \times \sin\left(360^\circ \times \frac{284+n}{365}\right) \tag{4}$$

$$\omega_s = \arccos(-\tan\phi\tan\delta) \tag{5}$$

$$\omega_T = \min\left\{\omega_s, \arccos\left[-\tan(\phi-\beta)\tan\delta\right]\right\} \tag{6}$$

where n is the day of the year.

The diffuse radiation on an inclined surface can be calculated [2]:

$$I_{T,d} = I_d\left[\frac{I-I_d}{I_0}R_b + \frac{1}{2}(1+\cos\beta)\left(1-\frac{I-I_d}{I_0}\right)\right] \tag{7}$$

where I is total radiation on the horizontal surface and I_0 – the extraterrestrial radiation, given [2]:

$$I_0 = \frac{24}{\pi}I_\mathrm{se}\left[1+0.033\times\cos\left(\frac{360n}{365}\right)\right]\left(\cos\phi\cos\delta\sin\omega_s + \frac{\pi}{180}\omega_s\sin\phi\sin\delta\right) \tag{8}$$

where I_se is the solar constant, typically valued at 1367 W/m^2.

The reflected radiation can be calculated [15]:

$$I_{T,g} = \frac{1}{2}\rho I(1-\cos\beta) \tag{9}$$

where ρ is the Albedo of the Earth's surface, typically assumed to be 0.2.

By substituting equations , and into equation , the total solar radiation on the inclined surface can be obtained [2]:

$$I_T = I_b R_b + I_d\left[\frac{I-I_d}{I_0}Rb + \frac{1}{2}(1+\cos\beta)\left(1-\frac{I-I_d}{I_0}\right)\right] + \frac{1}{2}\rho I(1-\cos\beta) \tag{10}$$

Construction of the computational model

This section uses Zhangye City as an example to explain the model construction method proposed in this study. The method presented in this study first investigates the relationship between the proportion of direct radiation, latitude, and the deviation of the monthly optimal tilt angle, fig. 1. Ten different proportions of direct radiation (0%, 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, and 90%) were analyzed. Starting from January, the proportion of direct radiation was set to 0%, and the total radiation on PV modules with tilt angles ranging from –20° to 90° was calculated to determine the monthly optimal tilt angle for a direct radiation proportion of 0%. This process was then repeated for different proportions of direct radiation calculate the corresponding monthly optimal tilt angles.

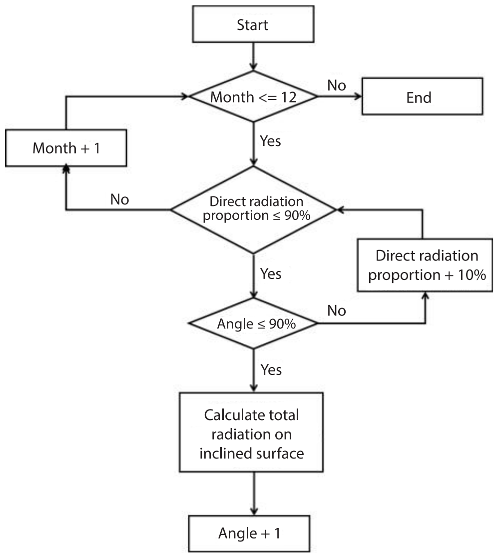

Figure 1. Iterative calculation flowchart

The monthly optimal tilt angles under different direct radiation proportions were calculated iteratively, as shown in fig. 2. These fitted curves can be obtained:

$$\phi - S = y + Ae^{-x/t} \tag{11}$$

where S is monthly optimal tilt angle and A, y, and t are parameters related to the month.

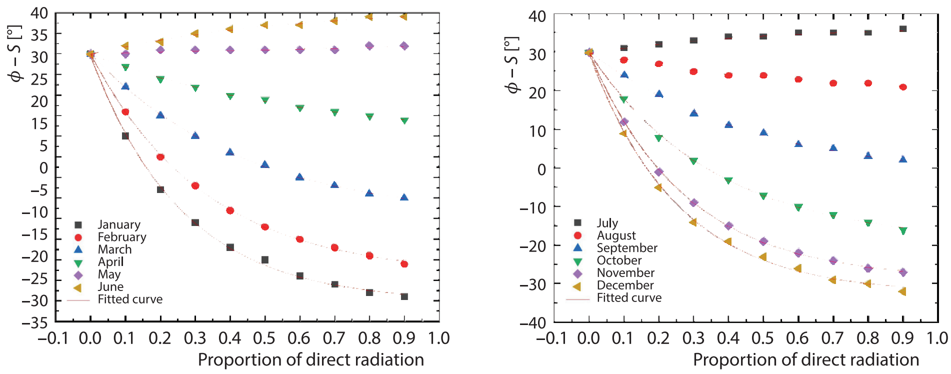

Figure 2. Curves of the proportion of direct radiation and the difference between latitude and installation tilt angle

The relationships between parameters (*A, y* and *t*) and month were investigated as illustrated in fig. 3. The fitted curves of these parameters can be obtained:

$$
\begin{aligned}
y &= -34.23 + 75.56\exp\left[-0.5\times\left(\frac{m-6.31}{w_y}\right)^2\right] \\
A &= 63.78 - 75.54\exp\left[-0.5\times\left(\frac{m-6.28}{w_A}\right)^2\right] \\
t &= 0.15 + 0.46\exp\left[-0.5\times\left(\frac{m-4.77}{w_t}\right)^2\right]
\end{aligned}
\tag{12}
$$

where m is the month and w_y,w_A and w_t are piecewise functions.

$$
w_A = \begin{Bmatrix} 2.14,\ m <= 6.3 \\ 2.03,\ m >= 6.3 \end{Bmatrix},\quad w_y = \begin{Bmatrix} 2.14,\ m <= 6.3 \\ 2.03,\ m >= 6.3 \end{Bmatrix},\quad w_t = \begin{Bmatrix} 2.10,\ m <= 4.8 \\ 4.28,\ m >= 4.8 \end{Bmatrix}
\tag{13}
$$

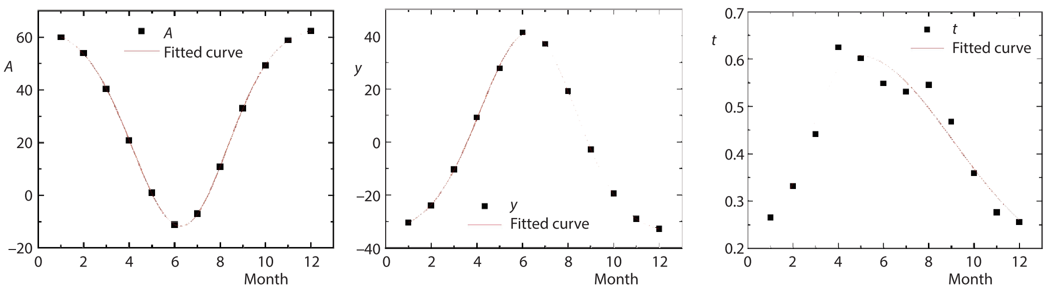

Figure 3. Fitted curves of the parameters *A*, *y* and *t* with month

Through eqs. (12) and (13), the monthly optimal tilt angle can be expressed as:

$$
\begin{aligned}
S = 73.16 &- 75.56\exp\left[-0.5\left(\left(m-6.28\right)/w_y\right)^2\right] - \\
&-63.78\exp\left[-x/\left(0.15+0.46\exp\left(-0.5\left(\left(m-4.77\right)/w_t\right)^2\right)\right)\right] + \\
&+75.54\exp\left[-0.5\left(\left(m-6.31\right)/w_A\right)^2 - x/\left(0.15+0.46\exp\left(-0.5\left(\left(m-4.77\right)/w_t\right)^2\right)\right)\right]
\end{aligned}
\tag{14}
$$

Validation of the calculation model

To validate the accuracy of the method proposed in this study, a comparative analysis was conducted between the monthly optimal tilt angles obtained using this method and those derived from the conventional calculation method. First, the total radiation on the inclined surface at different tilt angles was calculated, with data sourced from [2]. The comparison between the results of this study and those reported in [2] is shown in tab. 1. The maximum error is 1.24%, which is attributed to differences in the solar declination indices used in this study compared to those in [2]. The tilt angle that corresponds to the maximum total radiation on the inclined surface was considered the monthly optimal tilt angle derived from the conventional method. The comparison between the monthly optimal tilt angles obtained from the model established in this study and those determined by the conventional method is shown in fig. 4.

Table 1.Total radiation error between this study and reference [2] under different tilt angles

Month	Angle							
	0°	ϕ – 5°	ϕ	ϕ + 5°	ϕ + 10°	ϕ + 15°	ϕ + 20°	ϕ + 25°
1	10.2 (0%)	16.3 (0.25%)	16.85 (0.24%)	17.29 (0.23%)	17.63 (0.23%)	17.87 (0.23%)	17.98 (0.28%)	17.99 (0.22%)
2	12.3 (0%)	17.2 (0.93%)	17.55 (0.97%)	17.8 (1.01%)	17.95 (1.11%)	17.97 (1.11%)	17.9 (1.17%)	17.72 (1.24%)
3	17.3 (0%)	19.41 (–0.05%)	19.38 (0.1%)	19.2 (0.1%)	18.94 (0%)	18.59 (0%)	18.15 (0%)	17.62 (0%)
4	20.7 (0%)	20.88 (0.14%)	20.5 (0.1%)	20.03 (0.10%)	19.47 (–0.35%)	18.82 (0.10%)	18.08 (0.11%)	17.26 (0.06%)
5	24.8 (0%)	23.07 (–0.22%)	22.41 (–0.22%)	21.65 (–0.22%)	20.82 (–0.28%)	19.89 (–0.19%)	18.89 (–0.25%)	17.83 (–0.26%)
6	25.7 (0%)	22.84 (0.09%)	22.00 (0.05%)	21.07 (0.05%)	20.06 (0.05%)	18.96 (0.05%)	17.8 (0.05%)	16.59 (0.06%)
7	24.6 (0%)	22.32 (0.18%)	21.73 (0.92%)	20.72 (0.92%)	19.79 (0.19%)	18.77 (0.25%)	17.68 (0.21%)	16.52 (0.28%)
8	22.7 (0%)	22.56 (0.40%)	22.06 (0.45%)	21.44 (0.45%)	20.71 (0.47%)	19.88 (0.48%)	18.94 (0.50%)	17.92 (0.48%)
9	19.0 (0%)	21.57 (0.46%)	21.48 (0.51%)	21.26 (0.51%)	20.92 (0.52%)	20.47 (0.53%)	19.90 (0.59%)	19.21 (0.65%)
10	15.5 (0%)	21.14 (0.52%)	21.50 (0.51%)	21.73 (0.51%)	21.83 (0.55%)	21.79 (0.60%)	21.62 (0.60%)	21.32 (0.65%)
11	10.7 (0%)	16.80 (–0.18%)	17.33 (–0.23%)	17.76 (–0.23%)	18.08 (–0.23%)	18.28 (–0.17%)	18.36 (–0.22%)	18.34 (–0.22%)
12	9.1 (0%)	14.94 (0.13%)	15.49 (0.13%)	15.94 (13%)	16.29 (0.13%)	16.53 (0.06%)	16.68 (0.00%)	16.73 (0.00%)

It was observed that in February, March, August, and November, the results calculated in this study closely align with those obtained using the conventional method. However, the largest discrepancy in the optimal tilt angle occurs in May, with a difference of 4.8°. According to the method described in [2], the calculated tilt angle for May is 5°, corresponding to a total radiation on the inclined surface of 24.85 MJ/per day. In comparison, the method proposed in this study calculates the tilt angle as 10°, with a corresponding total radiation of 24.82 MJ per day. The difference in total radiation between the two methods is only 0.03 MJ per day, which is negligible.

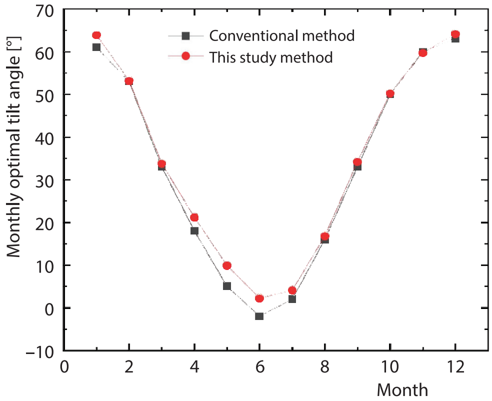

Figure 4. Comparison of monthly optimal tilt angles between the proposed method in this study and the conventional method

Thus, it can be concluded that the method proposed in this study is accurate and significantly faster in determining the monthly optimal tilt angle compared to the conventional method.

Model application

Using the method proposed in this study, the monthly optimal tilt angles were calculated for three cities in China located at different latitudes (Fuzhou, Hangzhou, and Shijiazhuang). The solar radiation data for horizontal planes in these cities were sourced from Solargis, and the results for the proportion of direct radiation are shown in tab. 2. Based on the established calculation models, the monthly optimal tilt angles for these cities are illustrated in fig. 5.

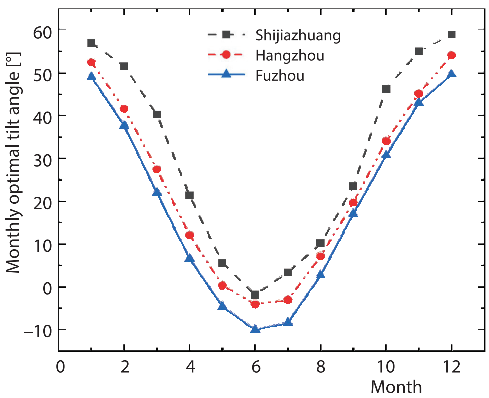

Figure 5. Monthly optimal tilt angles for three cites

Table 2. Proportion of direct radiation for each month

Month	January	February	March	April	May	June	July	August	September	October	November	December
Shijiazhuang	46.55%	45.16%	49.30%	49.19%	49.83%	42.01%	37.27%	41.58%	46.84%	48.53%	47.86%	50.44%
Hang Zhou	38.23%	35.86%	37.83%	39.38%	41.44%	33.45%	47.87%	44.30%	38.44%	40.07%	39.80%	43.38%
Fu Zhou	42.08%	38.09%	36.51%	36.64%	38.88%	42.91%	54.32%	50.34%	46.56%	43.78%	45.77%	44.66%

Figure 5 illustrates that in March, Fuzhou, and Hangzhou demonstrate similar proportions of direct solar radiation. However, due to Hangzhou's higher latitude, its corresponding monthly optimal tilt angle is larger than that of Fuzhou. This finding highlights that, for same proportions of direct radiation, the monthly optimal tilt angle increases with latitude. Moreover, the variation in the monthly optimal tilt angle typically exhibits a pattern of an initial decline followed by an increase, reaching its minimum values in June. Notably, in low latitude regions, the monthly optimal tilt angle during this period can even become negative.

Conclusion

This study proposed a method to calculate the monthly optimal tilt angle, and the accuracy and reliability of this proposed method have been verified. Based on this, this study explored the variation patterns of the monthly optimal tilt angle in China at different latitudes. With the same proportion of direct radiation, the monthly optimal tilt angle increases with latitude. Over the course of a year, the monthly optimal tilt angle initially decreases and then increases, reaching its minimum value in June. In low latitude regions, the monthly optimal tilt angle during this period may even become negative.

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Nomenclature

I – total radiation, [MJ per day]
I_b – direction radiation, [MJ per day]
I_{se} – solar constant, [Wm^{-2}]
I_T – total radiation, [MJ per day]
$I_{T,b}$ – direction radiation, [MJ per day]
$I_{T,d}$ – diffuse radiation, [MJ per day]
$I_{T,g}$ – reflected radiation, [MJ per day]
I_0 – extraterrestrial radiation, [MJ per day]

Greek symbols

β – tilt angle of the solar panel, [°]
ω_s – sunset hour angle, [°]
ω_T – sunset hour angle, [°]
ϕ – latitude, [°]

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