# THE DEVELOPMENT AND OPTIMIZATION OF THE BUS DRIVING CYCLE FOR PUBLIC TRANSPORTATION AND MOBILITY SYSTEMS

by

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*The driving cycles developed for public transportation busses is crucial in terms of energy efficiency and emissions as well as enhancing fuel economy. Considering to the public bus transportation driving cycles, both international standardized like SORT, and regional driving cycles are created by researchers and authorities to evaluate the energy efficiency and also to compare the different vehicles with each other. To obtain results that are closest to real driving conditions in driving cycle tests, the driving cycles must reflect all the characteristics of the region where they were created. Otherwise, fuel consumption values obtained using driving cycles that do not include regional characteristics do not reflect real life and can lead to misleading results. This study focuses on creating driving cycles that are closest to real life for public transportation buses in Istanbul. As a different approach from the driving cycles currently used an developed, three classifications were made, and multiple driving cycles were created. Three driving cycles were obtained for specific times of the day. Four driving cycles were obtained to reflect traffic density across seasons. To create seasonal cycles, data from 15 bus lines collected over a month was correlated with the data from 500T bus lines covering a year using a new method. Additionally, to obtain results closest to real life, road profiles were designed considering gradient information. Significant differences were observed in the fuel consumption values calculated using the obtained driving cycles.*

Key words: *driving cycle, fuel consumption, test, SORT, bus*

## Introduction

Each driving cycle is the result of scientific work using specific methodologies. When creating a driving profile, care is taken to reflect all the characteristics of the target area. Driving cycles that do not reflect the characteristics of the region from which they are derived can provide misleading results in fuel consumption or emission calculations. Driving cycles are mainly divided into two categories: artificial and real. In artificial cycles, acceleration and deceleration profiles are constant, which fails to reflect real life conditions accurately. In real driving cycles, data is collected while vehicles are driven in traffic. Additionally, driving cycles are categorized based on vehicle class, power-to-weight ratio, and vehicle speed.

Currently developed driving cycles for big metropolitan cities which has a massive traffic congestion mainly do not include traffic density at specific times of the day, seasonal variations in traffic density, and road gradient simulation. This highlights the inadequacy of driving cycles in reflecting real life conditions of these cycles. Therefore, as an alternative ap-

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proach, driving cycles were developed by focusing on three factors that influence the driving cycle, creating separate driving cycles for each factor.

When examining the traffic density over a day, the completion times for public transport services vary in the morning, afternoon, and evening. In the morning, due to people going to work and school, traffic is heavy. In the afternoon, this congestion decreases, and the completion times for public transport services become shorter. In the evening, traffic is at its most intense due to people returning from work and school. Three separate driving cycles were prepared for different times of the day.

Similarly, due to the dynamics within each season (such as weather conditions, school being in session, vacation seasons*, etc.*), driving characteristics are affected. In the summer, with schools closed, less rainfall, and people going on vacation out of town, the completion times for public transport services are shorter. In the fall, the conditions affecting summer reverse, leading to increased completion times for public transport services. Four separate driving cycles were prepared for each season.

Driving cycles are typically conducted on a dynamometer or the road, with a particular emphasis on having a road gradient close to zero. Some cycles specify that tests cannot be performed if the road gradient exceeds 2%. However, fuel consumption values vary depending on whether the driving cycle is conducted on roads with slope or without slope. To address this, a new road profile was designed in the simulation program to account for road gradients based on the collected data, and fuel consumption calculations were performed using this profile.

In this study for the daily driving cycles, data was collected from 15 different routes over one month. For seasonal driving cycles, data from the 500T route, one of the longest and most used routes covering both sides of Istanbul, was used over a year. As a new approach, the data from the 15 routes collected in one month was correlated with the year long data from the 500T route. Completion times for the 500T route were extracted for each month and season. Seasonal coefficients were obtained by proportioning the completion times. Since the data covering 15 routes was collected in the spring, it was accepted as having the same coefficient as the spring season of the 500T data. The coefficients for the remaining three seasons were then used to relate to the full day driving cycle.

There are many different driving cycles, both international and local. The European cysles (ECE) cycle is an urban driving cycle (UDC). It is designed to represent the driving conditions of cities like Paris or Rome. Low vehicle speed, low engine load, and low gas temperature characterize it. After the fourth ECE cycle, an extra urban driving cycle (EUDC) segment was included for more aggressive and high speed driving modes. Representing rural driving, it involves steady speed driving at high speeds with minimal gear changes. A new cycle was created by combining four UDC and one EUDC. At the beginning of the cycle, there is a 40 seconds idle period. In the year 2000, the idle period was removed from the cycle. With the removal of the idle period, the cycle was named NEDC. The NEDC is a combined chassis dynamometer test used for European emission testing and certification. The JP 10-15 mode cycle is used in Japan for light commercial vehicles' emission and fuel economy certification. The section called the 10 mode in the cycle represents urban driving, while the 15 mode represents extra urban driving. In the US, a standard UDC called federal test procedure (FTP 75) was implemented in 1975 and is still used today. This test method is used in every state of America, including California and different countries. [1]. The WLTP driving cycle, which takes light commercial vehicles into account, is widely used today. It is employed in a dynamometer environment to determine vehicle fuel consumption and emissions. Driving cycles are classified into three categories based on the power-to-weight ratio of the test vehicle, and different driving cycles have been performed for each category [2]. Measurement values such as average speed, load, time lost in stops, and distance between stops were collected on various European passenger transport routes. Based on these data, the standardized on-road test cycles (SORT) standard was developed by the International Association of Public Transport and leading European commercial vehicle and transmission manufacturers [3]. Kaymaz [4] focused on the Istanbul Metrobus line. The data collected from the Metrobus line was divided into micro cycles. These micro cycles were categorized into four groups, and these groups were used to create the driving cycles. Dinc [5] conducted a study to develop driving cycles. Four different routes were determined when creating the driving cycles. The data was collected from a passenger vehicle. Sanghpriya *et al*. [6] conducted a driving cycle study for Pune City in India. The proposed methodology utilized micro trips based on real world data. Tong [7] conducted a driving cycle study for Hong Kong City in China. The data collection process followed a circular route with ten bus stops. Fotouhi and Montazeri [8] conducted a driving cycle study for Tehran in Iran. Micro trips were then divided into four groups based on driving characteristics using the *k*-means clustering method. Mallouh *et al*. [9] conducted a driving cycle study for Amman City in Jordan. When creating the cycle, the average values of the 20 datasets and the total data were taken, and the dataset closest to these values was selected as the Amman Driving Cycle. Wang *et al*. [10] conducted a study in 2008 covering 11 cities in China. The vehicle tracking technique was used in each city to collect speed time data on highways, arterials, and local roads during both congested and non-congested periods. Micro trips and cycles were created using a random selection method. Nesamani and Subramanian [11] conducted a driving cycle study for Chennai city in India. The vehicle mounted measurement method was used for data collection. Zhao *et al*. [12] conducted a driving cycle study for the Xian region of China. Electric vehicles were selected for the cycle. The method for creating the proposed driving cycle was based on Markov and Monte Carlo simulation methods. Yang *et al*. [13] conducted new driving cycles were developed based on classified usage scenarios for heavy duty trucks. To verify the validity of the cycles, driving data from three thousand heavy duty trucks were collected. Ye *et al*. [14] proposed the use of two different cycles for electric vehicles and internal combustion engine vehicles instead of using the same cycle for both types of vehicles. Salihu *et al.* [15] investigated the effect of road gradient on UDC parameters. In this study, vehicles were driven on roads with different gradients in Prizren. Micro trips were extracted using a Python based clustering algorithm. Bhatti *et al*. [16] developed a driving cycle taking into account the inclined roads of Islamabad. The driving cycle was created using a Markov chain Monte-Carlo method that took into account the weights of different road types in the studied geographic area. Qin *et al*. [17] utilized data collected during travel in Xiamen city. Natural driving data is more accurate and easily obtainable compared to data collected through traditional methods. Hung *et al*. [18] conducted a driving cycle study for Hong Kong City in China in 2007. The route selection was based on the average daily traffic between major residential areas and commercial and industrial areas on the road network. Hang *et al*. [19] conducted a driving cycle study for military areas. Data were collected from military areas with numerous rugged roads. The percentage of cruising time is significantly different from other driving cycles because military vehicles used on military routes are almost unaffected by traffic. Amirjamshidi and Roorda [20] conducted a driving cycle study for Toron, Canada. It was prepared for light, medium, and heavy duty trucks traveling in the Toron Waterfront area.

### Data collection method

In this study, data flow is provided through two different channels:

*Data collected bus lines*: Data was collected from 15 bus lines across Istanbul to create a driving cycle. This included eight lines from Asia, five lines from Europe, and one line each passing through the 15 July Bridge and the Fatih Sultan Mehmet Bridge, fig. 1.

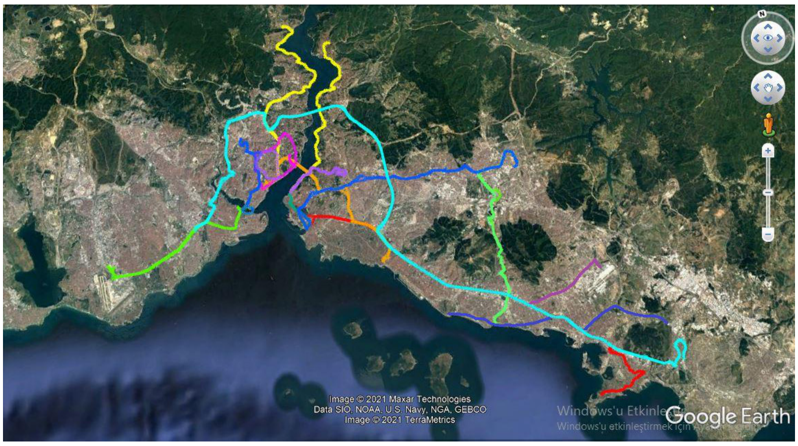


**Figure 1. Bus lines map on Google Earth [21]**

*Bus numbers which data is collected*: 30M, 40B, 46C, 82, DT1, 129T, 500T, 12, 14, 15, 15C, 130A, 132S 133KT, E10

Data collection was conducted at three different times of the day: Morning (08:00-12:00), Noon (12:00-16:00), and Evening (16:00-20:00)

The total length of the 15 routes from which data was collected is 320 km. Since the routes were collected in both outbound and inbound directions and at three different times of the day, a total of 1920 km of route data was collected from the 15 routes.

*The Istanbul Public Transportation Company (IETT) data information*: The location and speed information of the public transport vehicle is recorded within its system. To use the data in the study, discussions were held with IETT officials. Data for 40 buses was collected over one year, considering the door numbers of the 500T buses obtained from the Sifa Mahallesi Garage. The 500T route bus data was obtained under the Academic Research Data Sharing Agreement with IETT. After the data was processed, it was grouped by months, tab. 1. The average duration of 100 trips was calculated for each month. Approximately 88,000 km of route data were examined, comprising 1200 trips for 12 months. When looking at the route completion times by month, the most extended duration was observed in December. Conversely, the shortest completion time for the route was recorded in July.

**Table 1. The distribution of trip duration for IETT 500T route by months**

| Month | Average duration of 100 trips [second] | Month | Average duration of 100 trips [second] |
|---|---|---|---|
| January | 7426 | July | 6795 |
| February | 7730 | August | 7569 |
| March | 7519 | September | 8012 |
| April | 7605 | October | 8188 |
| May | 7497 | November | 8285 |
| June | 7925 | December | 8499 |

When examined by seasons, the 500T route takes the most extended trips during the autumn season. On the other hand, the shortest trip duration for the route occurs during the summer period.

## New driving cycle creation method

The steps followed to create the new cycles are given: Data collected from 15 routes was classified into morning, noon, and evening categories. Each category was further divided into micro trips. Micro trips consist of acceleration, cruising, deceleration, stopping, and waiting time at the stop. Micro trips were added successively until the target values were reached. The duration of the driving cycles was limited to between 15-25 minutes. All ten control parameters should have an error margin of less than 5%. Ten features were considered when creating the cycles, tab. 2. The characteristics of the data collected from 15 routes were analyzed to determine the goals of the cycles. The features of the cycles were compared to the target features, and the error percentage was calculated. The driving cycles were created using MATLAB R2023a.

**Table 2. Error comparison parameters**

| Average speed | Percentage of positive acceleration in driving mode (speed > 5 kph, acceleration > 0.1 m/s$^2$) |
|---|---|
| Average speed (driving) | Percentage of constant speed in driving mode (speed > 5 kph, acceleration < ±0.1 m/s$^2$) |
| Average acceleration (positive) | Percentage of negative acceleration in driving mode (speed >5 kph, acceleration > –0.1 m/s$^2$) |
| Average acceleration (negative) | Percentage of low speed (crawling) in driving mode (0 < speed < 5 kph) |
| Acceleration RMS value | Idle percentage in driving mode (speed: 0 kph) |

### *Driving cycle at specific times of the day*

The full day cycle lasts 21.6 minutes, with an average error of 2.79% calculated for the ten compared parameters. The total number of micro trips collected for the full day driving cycle is 3844. The 17 micro trips were used to create the cycle, fig. 2.

The morning cycle lasts 16.4 minutes, with an average error of 1.72% calculated for the ten compared parameters. The total number of micro trips collected for the full day driving cycle is 1299. The 14 micro trips were used to create the cycle.

The noon cycle lasts 21.5 minutes, with an average error of 1.45% calculated for the ten compared parameters. The total number of micro trips collected for the full day driving cycle is 1081. The 14 micro trips were used to create the cycle.

The evening cycle lasts 19.5 minutes, with an average error of 1.84% calculated for the ten compared parameters. The total number of micro trips collected for the full day driving cycle is 1464. The 14 micro trips were used to create the cycle.

### *Seasonal Istanbul driving cycle*

When creating seasonal driving cycles for Istanbul, a new method was used by correlating the data collected from 15 routes over one month with the year long data from the 500T route. Completion times for each month and season were extracted from the year long 500T data and proportioned accordingly. The following steps were applied to obtain the seasonal effect on driving cycles: The completion times of the data collected from the 500T route were classified seasonally. The data used to create the cycles was collected in the spring. Therefore,

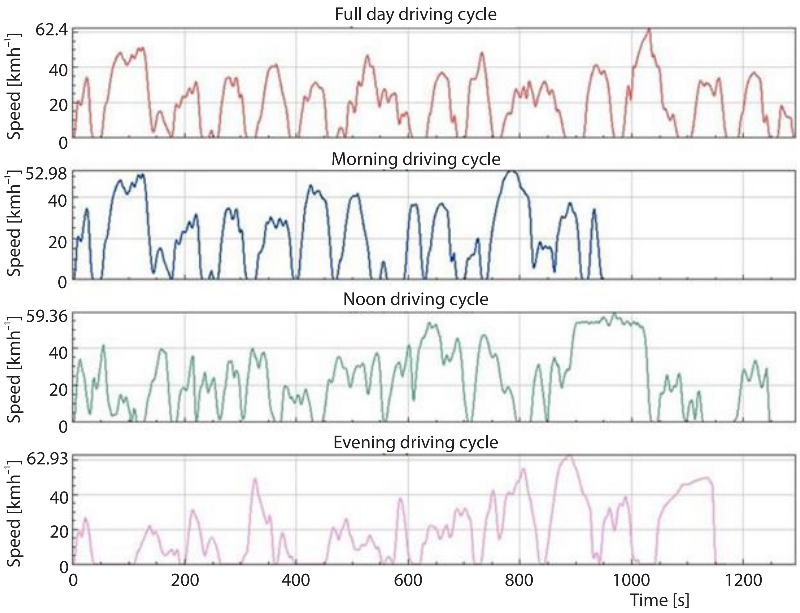


**Figure 2. Istanbul driving cycle at specific time of the day**

the spring route completion time was considered as 100 units. The route completion times for other seasons were proportioned relative to the spring route completion time, tab. 3. The seasonal route completion times obtained were correlated with the full day Istanbul driving cycle. For this, speed values were kept constant while time values were revised. The full day Istanbul driving cycle is 7.03 km in length. The distances of the seasonal cycles were revised with the new ratios changed. As a second step, to adjust the distances, time was kept constant and speed values were revised with the seasonal ratios. As a result of the second step, the lengths of the seasonal driving cycles were adjusted back to 7.03 km, fig. 3.

**Table 3. Seasonal completion times for the 500T route**

| Season | Duration [s] | Ratio [%] |
|---|---|---|
| Spring | 7540 | 100.0 |
| Summer | 7430 | 98.5 |
| Autumn | 8161 | 108.4 |
| Winter | 7885 | 104.2 |

### *Istanbul driving cycle with added gradient effect*

Steps to obtain the effect of road gradient on driving cycles: The gradients of the 17 micro trips used for the full day driving cycle were extracted. Roads were designed using gradient information in the IPG Truck Maker program. The full day driving cycle was run on roads with gradients, and fuel consumption was calculated, fig. 4.

## Fuel consumption simulation

The fuel consumption of the driving cycles obtained for public transport vehicles used in Istanbul was calculated using the IPG Truck Maker program. The IPG Truck Maker program calculates fuel consumption once the vehicle model is created and driving scenarios are loaded, fig. 5. To achieve accurate results, it is important to input realistic vehicle models, roads, and driving scenarios. For vehicle modelling, a validated low floor bus model equipped with diesel

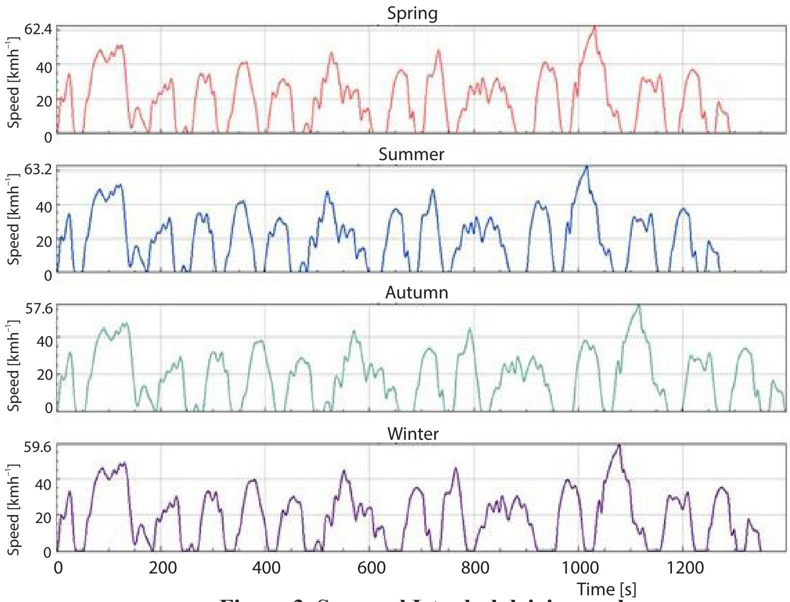


**Figure 3. Seasonal Istanbul driving cycle**

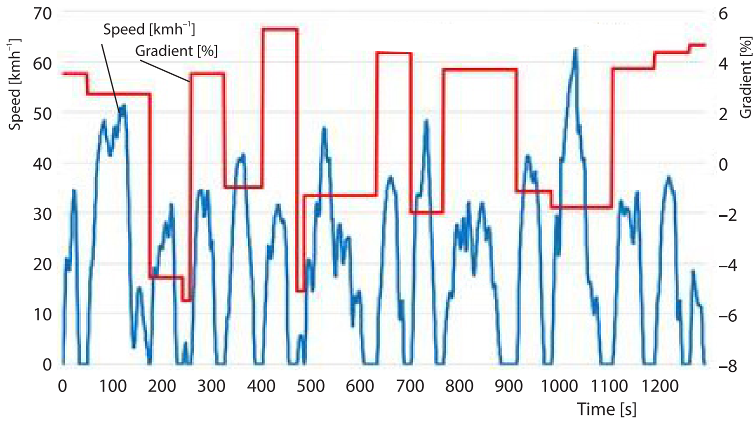


**Figure 4. Full day Istanbul driving cycle gradient information**

internal combustion engine coupled with automatic transmission with a torque converter was used [22].

Fuel consumption is calculated based on the engine speed and torque output of the diesel engine. When entering the vehicle weight into the program, the SORT standard was considered for comparison. In the SORT standard, the test weight of the vehicle is calculated by taking into account vehicle dimensions, number of seats, fuel tank capacity, number of personnel involved in the test, air conditioning, and additional equipment.

The SORT test plays a critical role in reducing the environmental impact of public transportation and ensuring fuel efficiency in urban bus fleets. By using standardized driving cycles like SORT, authorities can evaluate the effectiveness of buses in real world conditions and ensure that buses meet environmental and fuel efficiency standards. In the EU, SORT is a widely accepted testing method, and it helps manufacturers optimize their designs for better

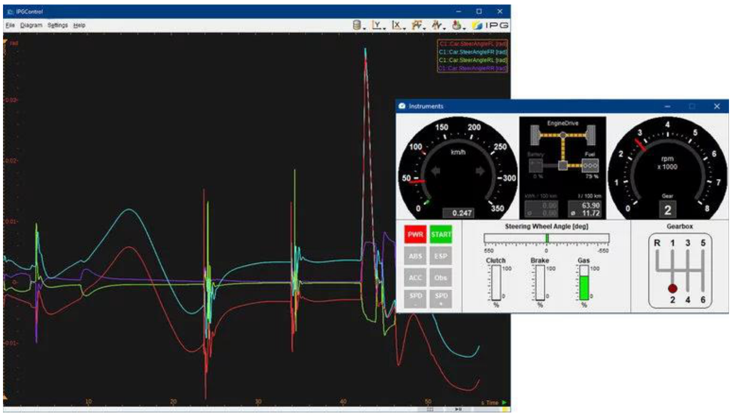


**Figure 5. Fuel consumption simulation with IPG Truck Maker program [23]**

fuel economy and lower emissions. For cities and transportation authorities, the test provides crucial data to implement sustainable transportation solutions and reduce the carbon footprint of public transit systems. The test involves three trapezoidal speed phases, with the bus accelerating and decelerating through each phase. The cycle simulates real life conditions where buses frequently change speeds, often speeding up and slowing down as they move through urban streets.

*SORT 1 (Heavy Urban)*: Typically used for smaller buses operating in urban areas with frequent stops. This version is suited for short distance routes with high stop-and-go traffic. The constant speeds are 20 km/h, 30 km/h, and 40 km/h. The average speed is 12.1 km/h, and the total distance is 1040 m.

*SORT 2 (Easy Urban)*: Designed for medium sized buses used in urban settings but operating over slightly longer distances or routes. These buses might be used for more extended city routes, still within the urban environment. The constant speeds are 20 km/h, 40 km/h, and 50 km/h. The average speed is 18 km/h, and the total distance is 920 m, fig. 6.

*SORT 3 (Easy Suburban)*: Intended for larger buses or those operating on suburban or intercity routes. These vehicles generally travel longer distances and might face different driving conditions compared to those in the city center. The constant speeds are 30 km/h, 50 km/h, and 60 km/h. The average speed is 25.3 km/h, and the total distance is 1450 m.

Once these values are entered, the calculated weight information is used for all test cycles. For the road design used in the simulation program, real life gradient information of the micro trips forming the driving cycle was input. For this, the height and distance of the road were defined in the program to cover each micro trip. The driving cycles obtained in this study were used as driving scenarios. Data was collected from public transport vehicles for the driving cycles, target values were maintained with an error margin below 5%, and separate cycles were created for different times and seasons.

The comparison results show that all three parameters significantly affect the fuel consumption of public transport vehicles.

*Effect of road gradient*: Analysis using both roads with slope and without slope profiles for the full day driving cycle revealed that fuel consumption increased by 38.46% due to the effect of the gradient.

*Comparison with SORT 2 profile*: The average speed of the full day driving cycle was found to be close to the SORT 2 profile. However, when comparing the full day driving cycle with the SORT 2 cycle that includes gradient effects, the values were found to be 21,99% higher.

*Seasonal impact*: The highest fuel consumption was observed during the summer period. Public transport vehicles complete the same route in different durations across seasons. Despite the distance being the same in summer, the route is completed in a shorter time. This is because the vehicle reaches higher speeds and achieves greater acceleration values compared to other seasons. The increased acceleration and higher speeds in a shorter time result in higher fuel consumption, fig. 7.

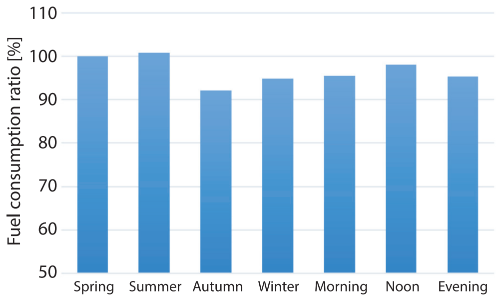


**Figure 7. Fuel consumption percentage comparison**

*Daily driving cycle comparison*: Among the daily driving cycles, the highest fuel consumption was observed during the noon period. Despite the distance being the same at noon, the route is completed in a shorter time. This is because the vehicle reaches higher speeds and achieves greater acceleration values compared to other times of the day. The increased acceleration and higher speeds in a shorter time result in higher fuel consumption, fig. 7.

## Conclusion

In this study, driving cycles were developed for public transport vehicles operating in Istanbul under various conditions. Ten parameters were considered while creating these driving cycles. For all cycles, the error rates concerning target values for the ten parameters were kept below 5%. A new method was used to link short term (1 month) data from 15 different bus routes with long term (1 year) data from the 500T bus route. This resulted in driving cycles that reflect seasonal effects for public transport vehicles in Istanbul. To accurately represent real life conditions, a different approach was taken by creating driving cycles based on three classifications. The first classification divided a day into three parts: morning, noon, and evening, resulting in three separate cycles. It was observed that the shortest completion time for the bus route was during noon, while the longest was in the evening. The second classification involved creating separate driving cycles for each season. Seasonal effects showed that the bus completed the 500T route in the shortest time during summer and in the longest time during autumn. The third classification compared driving cycles on roads with slope and without slope. Fuel consumption was calculated for each of the three classifications using a simulation program. It was found that the road gradient was the parameter that most significantly increased fuel consumption.

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## Nomenclature

ECE – European cycle
EUDC – extra-urban driving cycle
IETT – Istanbul electric tramway and tunnel operations general directorate

SORT – standardised on-road test cycles
UDC – urban driving cycle
WLTP – worldwide harmonised light vehicles test procedure

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