ANALYTIC APPROXIMATE SOLUTIONS FOR FLUID-FLOW IN THE PRESENCE OF HEAT AND MASS TRANSFER

by

Adem KILICMAN[a*], Yasir KHAN[b], Ali AKGUL[c], Naeem FARAZ[d], Esra Karatas AKGUL[e], and Mustafa INC[f]

[a] Department of Mathematics and Institute for Mathematical Research, University Putra Malaysia, Serdang, Selangor, Malaysia
[b] Department of Mathematics, Zhejiang University, Hangzhou, China
[c] Department of Mathematics, Art and Science Faculty, Siirt University, Siirt, Turkey
[d] Department of Mathematics, Shanghai University, Shanghai, China
[e] Department of Mathematics, Faculty of Education, Siirt University, Siirt, Turkey
[f] Department of Mathematics, Science Faculty, Firat University, Elazig, Turkey

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This paper outlines a comprehensive study of the fluid-flow in the presence of heat and mass transfer. The governing non-linear ODE are solved by means of the homotopy perturbation method. A comparison of the present solution is also made with the existing solution and excellent agreement is observed. The implementation of homotopy perturbation method proved to be extremely effective and highly suitable. The solution procedure explicitly elucidates the remarkable accuracy of the proposed algorithm.

Key words: *homotopy perturbation method, heat and mass transfer, fluid-flow*

Introduction

Most of the fluid-flows problems particularly heat and mass transfers are modeled for the non-linear PDE. There are several non-linear PDE in the literature and which can not be solved by analytical methods and hence need to be solved by using numerical methods which lead the researchers to observe the behavior of the system. There are several methods to approximate the solutions and the most commonly exercised methods are finite difference methods, Runge-Kutta methods, and finite element methods. In practical use, some of these methods are not easy and also require complex calculation [1]. Among these methods, the finite-difference methods are known as effective tools to solve several types of PDE [2]. Further, in the conditional stability analysis of explicit finite-difference schemes it is also necessary to put a severe constraint on the time parameter, while the implicit finite-difference schemes are observed that computationally expensive [3]. On the other side, these methods can be made highly effective and accurate, but require a structured grids.

In this study, we employ the homotopy perturbation method (HPM) to solve the non-linear ODE which arise in the heat generation and chemical reactions. The HPM was first introduced by He [4]. Well known remarkable feature of the HPM is that a few perturbation terms will be sufficient to obtain reasonable accurate solutions. The technique has been em-

* Corresponding author, e-mail: akilic@upm.edu.my

ployed by several researchers in order to solve a large variety of linear and non-linear problems [5-11].

Problem formulation

Consider basic governing equations of the problem with boundary conditions is:

$$f''' + \mathrm{Re}\left(f'^2 - ff''\right) - \mathrm{Gr}\,\theta - \mathrm{Gc}\,\phi = 0 \tag{1}$$

$$\frac{1}{\mathrm{Pr}}\theta'' + \mathrm{Ec}\,f'^2 + \delta\theta - \mathrm{Re}\,f\theta' = 0 \tag{2}$$

$$\frac{1}{S}\phi'' - \gamma\phi - \mathrm{Re}\,f\phi' = 0 \tag{3}$$

with boundary conditions:

$$\begin{aligned} &f = 0,\ \ f'' = 0,\ \ \theta = 1,\ \ \phi = 1 \ \text{ at } \ y = 0 \\ &f = 0,\ \ f' = 0,\ \ \theta = 0,\ \phi = 0 \ \text{at} \ \ y = \frac{1}{2} \end{aligned} \tag{4}$$

According to HPM [4], the homotopy construction of eqs. (1)-(3) can be expressed in the form:

$$\begin{aligned} &(1-p)L_1(f - f_0) + p\left[f''' + \mathrm{Re}\left(f'\,f'' - f\,f'''\right) - \mathrm{Gr}\,\theta' - \mathrm{Gc}\,\phi'\right] = 0 \\ &(1-p)L_2(\theta - \theta_0) + p\left(\theta'' + \mathrm{Pr}\,\mathrm{Ec}\,f'^2 + \mathrm{Pr}\,\delta\theta - \mathrm{Pr}\,\mathrm{Re}\,f\theta'\right) = 0 \\ &(1-p)L_3(\phi - \phi_0) + p\left(\phi'' - S\gamma\phi - S\,\mathrm{Re}\,f\phi'\right) = 0 \end{aligned} \tag{5}$$

$$f = f_0 + pf_1 + p^2 f_2 + \ldots,\quad \theta = \theta_0 + p\theta_1 + p^2\theta_2 + \ldots,\quad \phi = \phi_0 + p\phi_1 + p^2\phi_2 + \ldots \tag{6}$$

Assuming $L_1 f = 0$, $L_2\theta = 0$, and $L_3\phi = 0$, making substitution f, θ, and ϕ from eq. (6) into eq. (5) and by using simple algebraic simplification and arrangement on powers of p-terms, we obtain the following sets of equations:

$$\begin{aligned} &p^{(0)} : L_1 f_0 = 0,\quad L_2\theta_0 = 0,\quad L_3\phi_0 = 0 \\ &f_0(0) = 0,\quad f_0''(0) = 0,\quad \theta_0(0) = 1,\quad \phi_0(0) = 1 \\ &f_0(1/2) = 0,\quad f_0'(1/2) = 0,\quad \theta_0(1/2) = 0,\quad \phi_0(1/2) = 0 \end{aligned} \tag{7}$$

where L_1, L_2, and L_3 are defined:

$$L_1 = \frac{\partial^4}{\partial\eta^4},\quad L_2 = \frac{\partial^2}{\partial\eta^2},\ \text{ and }\ L_3 = \frac{\partial^2}{\partial\eta^2} \tag{8}$$

On solving eq. (7) we get initial guess:

$$f_0(\eta) = \frac{3\eta}{2} - 2\eta^3,\quad \theta_0(\eta) = 1 - 2\eta,\ \text{ and }\ \phi_0(\eta) = 1 - 2\eta \tag{9}$$

$$p^{(1)}: L_1 f_1 + \mathrm{Re}\left(f_0' f_0'' - f_0 f_0'''\right) - \mathrm{Gr}\,\theta_0'' - \mathrm{Gc}\,\phi_0' = 0$$

$$f_1(0) = 0, \quad f_1''(0) = 0, \quad f_1(1/2) = 0, \quad f_1'(1/2) = 0$$

$$p^{(1)}: L_2\theta_1 + \mathrm{Pr}\,\mathrm{Ec}\, f_0'^2 + \mathrm{Pr}\,\delta\theta_0 - \mathrm{Pr}\,\mathrm{Re}\, f_0\theta_0' = 0, \quad \theta_1(0) = \theta_1(1/2) = 0 \tag{10}$$

$$p^{(1)}: L_2\phi_1 - S\gamma\phi_0 - S\,\mathrm{Re}\, f_0\phi_0' = 0, \quad \phi_1(0) = \phi_1(1/2) = 0$$

$$\vdots$$

$$p^{(j)}: L_1 f_j + \mathrm{Re}\left(\sum_{k=0}^{j-1} f_k' f_{j-1-k}'' - \sum_{k=0}^{j-1} f_k f_{j-1-k}'''\right) - \mathrm{Gr}\,\theta_j' - \mathrm{Gc}\,\phi_j' = 0$$

$$f_j(0) = 0, \quad f_j''(0) = 0, \quad f_j(1/2) = 0, \quad f_j'(1/2) = 0$$

$$p^{(j)}: L_2\theta_j + \mathrm{Pr}\,\mathrm{Ec}\sum_{k=0}^{j-1} f_k' f_{j-1-k}' + \mathrm{Pr}\,\delta\theta_j - \mathrm{Pr}\,\mathrm{Re}\sum_{k=0}^{j-1} f_k\theta_{j-1-k}' = 0, \quad \theta_j(0) = \theta_j(1/2) = 0 \tag{11}$$

$$p^{(j)}: L_3\phi_j - S\gamma\phi_j - S\,\mathrm{Re}\sum_{k=0}^{j-1} f_k\phi_{j-1-k}' = 0, \quad \phi_j(0) = \phi_j(1/2) = 0$$

$$\vdots$$

On solving eqs. (10) and (11) One can use one of the software such as MATHEMATICA, MAPLE or MATLAB. Then we write first order approximations:

$$f = \frac{3}{2}\eta - \frac{\mathrm{Gr}}{192}\eta - \frac{\mathrm{Gc}}{192}\eta - \frac{\mathrm{Re}}{560}\eta - 2\eta^3 + \frac{\mathrm{Gr}}{16}\eta^3 + \frac{\mathrm{Gc}}{16}\eta^3 + \frac{3\,\mathrm{Re}}{280}\eta^3 - \frac{\mathrm{Gr}}{12}\eta^4 - \frac{\mathrm{Gc}}{12}\eta^4 - \frac{2\,\mathrm{Re}}{35}\eta^7$$

$$\theta = 1 - 2\eta + \frac{33\,\mathrm{Ec}\,\mathrm{Pr}}{80}\eta + \frac{9\,\mathrm{Pr}\,\mathrm{Re}}{80}\eta + \frac{\mathrm{Pr}\,\delta}{6}\eta - \frac{9\,\mathrm{Ec}\,\mathrm{Pr}}{8}\eta^2 - \frac{\delta\,\mathrm{Pr}}{2}\eta^2 - \frac{\mathrm{Re}\,\mathrm{Pr}}{2}\eta^3 + \frac{\delta\,\mathrm{Pr}}{3}\eta^3 + \\ + \frac{3\,\mathrm{Ec}\,\mathrm{Pr}}{2}\eta^4 + \frac{\mathrm{Re}\,\mathrm{Pr}}{2}\eta^5 - \frac{6\,\mathrm{Ec}\,\mathrm{Pr}}{5}\eta^6$$

$$\phi = 1 - 2\eta + \frac{9\,\mathrm{Re}\,S}{80}\eta - \frac{\mathrm{Re}\,S}{2}\eta^3 + \frac{\mathrm{Re}\,S}{5}\eta^5 - \frac{S\lambda}{6}\eta + \frac{S\lambda}{2}\eta^2 - \frac{S\lambda}{3}\eta^3$$

Results and discussion

By looking at the graphical representation of the results we notice that a very useful demonstration of the efficiency and accuracy of the method HPM for considered problems. In order to verify the accuracy of the present method, we have compared HPM results with the numerical and HAM results. The tabs. 1-4 clearly reveal that present solution method namely HPM shows excellent agreement with the HAM and numerical solution. This analysis shows that HPM suits for boundary-layer flow problem in the presence of heat and mass transfer.

In figs. 1-4 we show the velocity, $f'(\eta)$, temperature, $\theta(\eta)$, and concentration profiles $\phi(\eta)$ obtained by the HPM. The effect of Grashof number, Gc, (is also known as the local solutal) on the velocity is shown in the fig. 1. It is noted from fig.1 that initially f' increases but after the center of the channel it decreases as Grashof number increases. Figures 2 and 3 illustrate the effect of δ and Eckert number on temperature θ. Figure 2 shows that those positive values of δ increases temperature θ and the negative values of δ decreases temperature θ. From fig. 3 it is found that θ is an increasing function of Eckert number. Figure 4 depicts the influence of chemical reaction parameter γ on the concentration profiles $\phi(\eta)$. It is noticed that $\phi(\eta)$ decreases when γ increases.

Table 1. Comparison between the HPM, HAM, and numerical solutions of $f'(0)$, $\theta'(0)$, $\phi'(0)$ for different values of Reynolds numbers

$f'(0)$	HPM	HAM	Numerical
0.1	1.48951	1.4895	1.4895
1.0	1.48782	1.4878	1.4878
5.0	1.47979	1.4798	1.4798
$\theta'(0)$	HPM	HAM	Numerical
0.1	–1.40213	–1.4021	–1.4021
1.0	–1.29903	–1.2990	–1.2990
5.0	–0.868941	–0.8690	–0.8690
$\phi'(0)$	HPM	HAM	Numerical
0.1	–2.15309	–2.1531	–2.1531
1.0	–2.05614	–2.0561	–2.0561
5.0	–1.64838	–1.6484	–1.6484

Table 2. Comparison between the HPM, HAM, and numerical solutions of $\theta'(0)$ for different values of Eckert number

$\theta'(0)$	HPM	HAM	Numerical
0	–1.71593	–1.71593	–1.71593
1	–1.29903	–1.29903	–1.29903
2	–0.882127	–0.88213	–0.88213
3	–0.465201	–0.46520	–0.46520
4	–0.048258	–0.04826	–0.04826
5	0.368703	0.36871	0.36871
10	2.45377	2.45377	2.45377
20	6.62523	6.62523	6.62523

Table 3. Comparison between the HPM, HAM, and numerical solutions of $\theta'(0)$ for different values of δ

$\theta'(0)$	HPM	HAM	Numerical
0	–1.47915	–1.47915	–1.47915
0.5	–1.38989	–1.38989	–1.38989
1.0	–1.29903	–1.29903	–1.29903
1.5	–1.20654	–1.20654	–1.20654
2.0	–1.11233	–1.11233	–1.11233
2.5	–1.01634	–1.01634	–1.01634
3.0	–0.918509	–0.91851	–0.91851
3.5	–0.818754	–0.81875	–0.81875
4.0	–0.716997	–0.71700	–0.71699

Table 4. Comparison between the HPM, HAM, and numerical solutions of $\phi(0)$ for different values of chemical reaction parameter

$\phi(0)$	HPM	HAM	Numerical
0	–1.88893	–1.88893	–1.88893
0.5	–1.97323	–1.97323	–1.97323
1.0	–2.05614	–2.05614	–2.05614
1.5	–2.13770	–2.13770	–2.13770
2.0	–2.21796	–2.21796	–2.21796
2.5	–2.29697	–2.29697	–2.29697
3.0	–2.37476	–2.37476	–2.37476
3.5	–2.45137	–2.45137	–2.45137
4.0	–2.52685	–2.52685	–2.52685

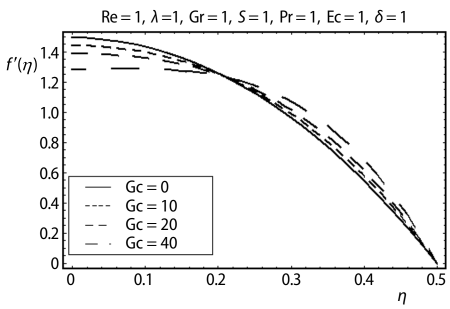

Figure 1. Variation of local solutal Grashof number on the velocity f'

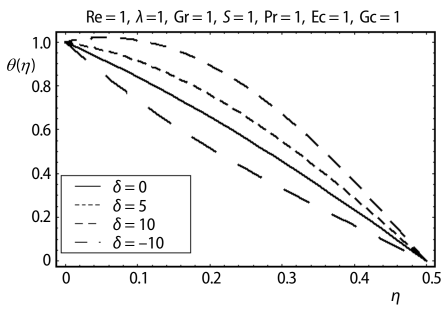

Figure 2. Variation of δ on the temperature θ

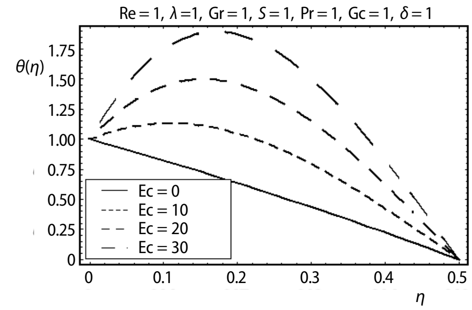

Figure 3. Variation of Eckert number on the temperature θ

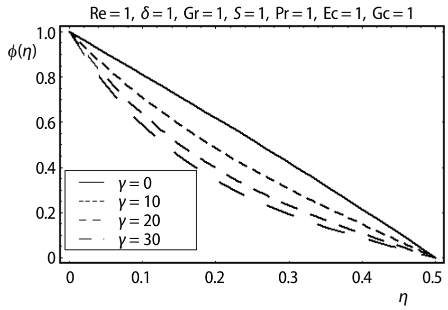

Figure 4. Variation of γ on the temperature ϕ

Conclusions

In this paper, the non-linear ODE which result from the similarity solutions of a steady viscous fluid with heat generation and chemical reaction were solved by using an analytical solution method as known the HPM. Comparison of the results obtained using the developed HPM with numerical and HAM results. The variations of various emerging parameters on the velocity, temperature as well as concentration profiles are also discussed through the graphs and tables, respectively. Then we can easily make the following observations.

- The HPM is an effective and easy to use if one compares with HAM and numerical solution method.
- The tangential velocity at the wall is an increasing function of Reynolds number.
- Behaviors of δ and Econ temperature $\theta(\eta)$ are similar.
- Concentration profile decreases by increasing chemical reaction parameter.

The proposed analytical approach for this problem might have many more applications and thus possible to apply in similar ways to the other boundary-layer flows to get accurate series solutions.

Nomenclature

Ec – Eckert number, [–]
f – dimensionless velocity profile, [–]
Gc – the local solutal Grashof number, [–]
Gr – the local thermal Grashof number, [–]
Pr – Prandtl number, [–]
Re – Reynolds numbers, [–]
S – Schmidt number, [–]

Greek symbols

γ – chemical reaction parameter, [–]
θ – dimensionless temperature profile, [–]
ϕ – dimensionless concentration, [–]

Acronym

HAM – homotopy analysis method

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