MIXED CONVECTION HEAT TRANSFER FROM A PARTICLE IN SUPERCRITICAL WATER

by

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This paper aims to study a steady laminar convection flow over a spherical particle in supercritical water in pseudo-critical zone. Forced convection, free convection, assisting convection, and opposing convection from a spherical particle in supercritical water were studied based on a numerical model fully accounting for variations of thermo-physical properties in pseudo-critical zone. Variation of specific heat plays a main role in determination of heat transfer coefficient. The gravity direction has a remarkable effect on heat transfer. Quantitative relationship between variable properties and Nusselt number has been established based on the simulation results.

Key words: *heat transfer, spherical particle, mixed convection, supercritical water*

Introduction

Supercritical water (SCW) fluidized bed reactor (FBR), which uses SCW as fluidization medium, quartz sand as bed materials, is a promising reactor for effectively gasify wet biomass and efficiently produce hydrogen [1, 2]. Forced convection, natural convection, and mixed convection from a particle in SCW flow are the basic particle-scale heat transfer modes in SCW-FBR. Both the complicated SCW-solid two-phase flow and the huge variations in property of SCW in pseudo-critical zone make these processes more complex [3, 4].

Mixed convection heat transfer from a sphere in infinite fluid has been studied by many researchers over the years. However, little attention has been paid to the flow of SCW. Most of the previous work on natural or mixed convection focused on the heat transfer from a spherical sphere particle in the *Boussinesq approximation* flow. Boussinesq approximation has been successfully applied to the studies on free convection and mixed convection heat transfer [5-8]. Chen and Mucoglu [9] found that as the buoyancy force increases, both the local friction factor and the local Nusselt number increase for assisting flow while decrease for opposing flow. Similar results were obtained by Nazar and Amin [6], and Bhattacharyya and Singh [10]. However, our previous work has shown that the effects of variable specific heat and conductivity on the heat transfer process are obvious [3, 4]. Boussinesq approximation is not suitable for free convection from a spherical particle to SCW flow in pseudo-critical zone. The huge variations of properties (density, viscosity, conductivity, and specific heat) in pseudo-critical zone result in more complicated problems.

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It is easy to obtain qualitative conclusions for mixed convection. However, it is much difficult to determine the quantitative relationship between operating parameters (Reynolds number, Grashof number, Rayleigh number, Richardson number, *etc.*) and flow or heat transfer parameters (drag coefficient, or Nusselt number). By using an analytical method, Hieber and Gebhart [11] derived an equation to calculate the drag coefficient induced by free convection in assisting convection by analytical method. The method suggested by Hieber and Gebhart [11], has been validated by Mograbi and Bar-Ziv [7] in a small Grashof number zone through experimental study, and by Kotouc *et al.* [8] through simulation work. Churchill [12] found the equation $\mathrm{Nu} = (\mathrm{Nu}_f^m + \mathrm{Nu}_n^m)^{1/m}$ with $m = 3$ was suitable for the results of assisting laminar boundary layer for isothermal and uniform heat flux plates, spheres, and cylinders. Oosthuizen and Bassey [13] correlated their data with $m = 4$. For the SCW flow over a particle, the effect of variable properties on heat transfer cannot be ignored. Therefore, it is difficult but necessary to determine the quantitative relationship between variable property properties and Nusselt number.

This paper aims to study the mixed convection over a sphere spherical particle in SCW in pseudo-critical zone with a moderate range of Reynolds number ($5 \le \mathrm{Re} \le 200$). The effects of variations of properties, Grashot and Reynolds numbers on the heat transfer coefficient have been investigated. Quantitative relationship between variable properties and Nusselt number has been established for forced convection, free convection, assisting convection, and opposing convection.

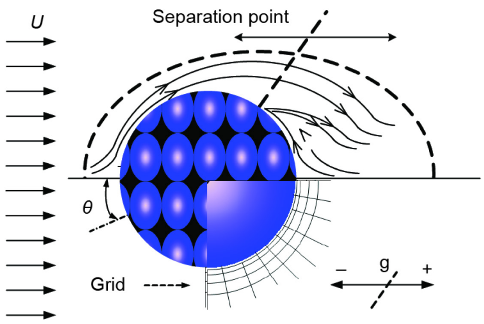

Figure 1. Schematics of a spherical particle in SCW under gravity

Problem formulation

Physical problem

Figure 1 shows the schematic of SCW flowing over a sphere under gravity and the non-uniform grid structure used in this work. The assisting convection is formed when gravity is in the direction opposite to flow velocity and marked with negative sign, and the opposing convection is formed when gravity is in the direction same with flow velocity and marked with positive sign. In the present work, it is considered that steady laminar flow and heat transfer from spherical particle to SCW flow occurs for particles with constant surface temperature. For laminar flow in SCW flow with variable fluid properties, the governing equations of the conservation of mass, momentum and energy, can be expressed:

$$\nabla \rho u = 0 \tag{1}$$

$$(u\nabla)\rho u = -\nabla P + \nabla(\mu \nabla u) + \mathrm{g} \tag{2}$$

$$(u\nabla)\rho h = \nabla(k\nabla T) \tag{3}$$

Grashof number is defined:

$$\mathrm{Gr} = \mathrm{g}\beta\rho^2 d^3 \frac{\Delta T}{\mu^2} \tag{4}$$

Richardson number is defined:

$$\mathrm{Ri} = \frac{\mathrm{Gr}}{\mathrm{Re}^2} \tag{5}$$

Thermal expansion coefficient of the fluid is defined:

$$\beta = -\frac{1}{\rho_\infty}\frac{\partial \rho}{\partial T} \tag{6}$$

Nusselt number is defined:

$$\mathrm{Nu} = \frac{d}{2k_\infty (T_\mathrm{w} - T_\infty)} \int_0^\pi k_\theta \left(\frac{\mathrm{d}T}{\mathrm{d}r}\right)_\theta \mathrm{d}\theta \tag{7}$$

The properties of SCW flow are calculated by ISAWP-IF97 equations. For model fluid of constant property flow, the properties are calculated at temperature and pressure in far field. The dimensionless parameters (Pr, Re, Ri, Gr, *etc.*) are based on thermal property evaluated at the ambient temperature.

Numerical method

The schematic of the non-uniform grid structure is shown in fig. 1. Grid independent tests have been conducted by varying the ratio of height of domain zone to the particle diameter for domain grid zone (H/d = 25, 50, and 70) and grid number (60 × 500, 80 × 1000, and 100 × 2000) at the Re = 5. The diameter of particle is 0.2 mm, which represents a typical Geldart B particle. With the domain size ratio (H/D) varying from 50 (grid number = 100 × 1000) to 75 (grid number = 100 × 2000), the values of C_d and Nusselt number are reduced by 0.015%, and 0.029%, respectively. The results show that the domain size ratio of 75 (grid number = 100 × 2000) should be chosen for numerical simulation. With an increase in Reynolds number, a smaller computed zone is needed due to a receding viscous interaction. Therefore, this grid is safer for a higher Reynolds number. Uniform temperature and velocity are set for upper stream. The boundary conditions of fixed temperature and no slip-velocity are set for spherical particle surface. The infinite flow field is achieved by keeping slip-velocity of the outside boundary the same with free stream. Finite volume method is used to solve eqs. (1)-(3). The discretization scheme of convective term is second-order upwind scheme. The SIMPLEC algorithm are applied for the pressure-velocity coupling. Under-relaxation factors of 0.1-0.5 are chosen for a steady convergence. A convergence criterion of 10^{-6} for each scaled residual component is specified for the relative error between two successive iterations. The validation of the present model have been conducted by comparing the total drag coefficient and average Nusselt number with empirical correlations, which can be found in our previous work [3, 4].

Results and discussion

Figures 2(a) and 2(b) show typical the flow streamlines and temperature contours for constant property flow and SCW flow at the Re = 30, respectively. The ambient pressure is 23 MPa, and the pseudo-critical temperature at 23 MPa is 650.125 K. The ambient temperature and particle surface temperature are 647 and 657 K, respectively. Flow separation of SCW flow occurs earlier than that of constant property flow. When variable property was incorporated in calculation, a larger recirculation zone and a higher temperature gradient were observed. Figure 2(c) shows the effects of assisting convection on the flow streamlines and temperature contours for SCW flow. Under the effect of assisting effect of buoyancy force, the wake vortex at the rear of sphere was vanished, and higher velocity gradient and temperature gradient near the sphere surface were obtained. The density of SCW decreases with an elevated surface temperature, which leads to a strong upward jet along the downstream side of the sphere under the effect of buoyancy force. Figure 2(d) shows flow streamlines and temperature contours of opposing convection. The wake vortex at the rear of sphere was vanished and the flow direction turn opposite under the opposing effect of buoyancy force. Therefore, the gravity direction has a remarkable effect on the flow separation.

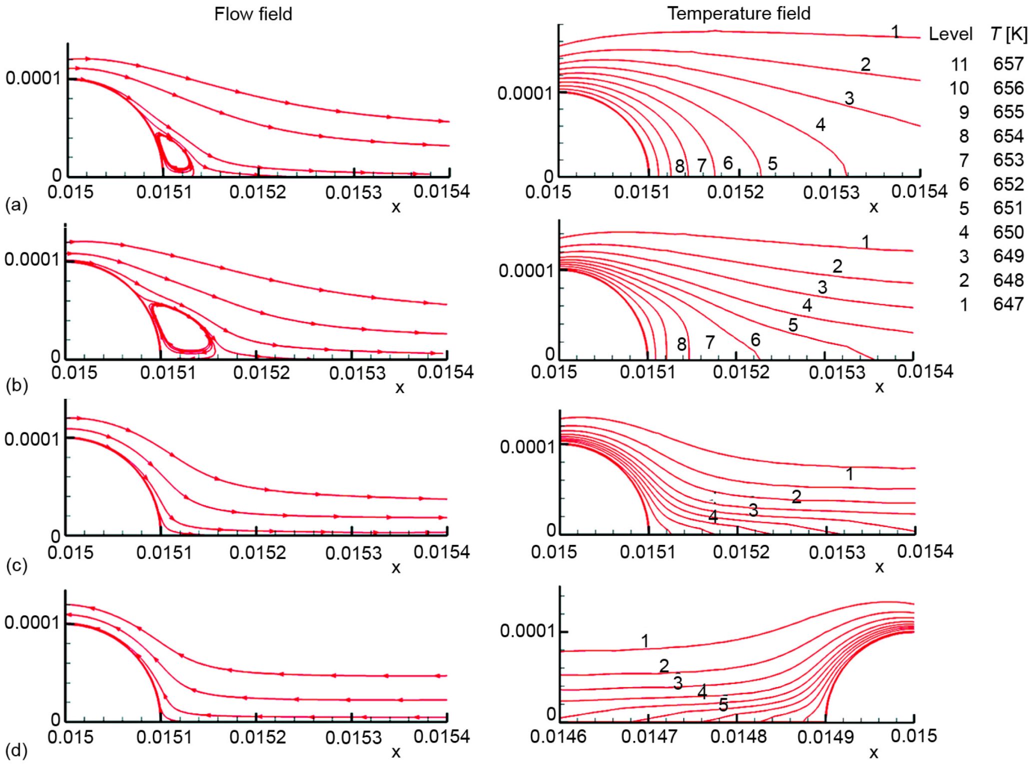

Figure 2. Flow streamlines and temperature contours around a spherical particle, P = 23 MPa, T_w = 657 K, T_∞ = 647 K, Re = 30; (a) forced convection for constant property flow, (b) forced convection for SCW flow, (c) assisting convection for SCW flow, (d) opposing convection for SCW flow

Forced convection

In order to study the effect of thermal properties on heat transfer, simulations were conducted by varying only one property and keeping others same or unchanged with free stream. The model fluid with only one property variation can be called variable *property* flow, *e. g.* the model fluid with only density variation can be called variable density fluid. Figure 3 shows the effect of each property on the local Nusselt number for forced convection. It is clear that the model fluid with variable specific heat shows a very good agreement with that of SCW flow. The deviation is within 15% at Re = 15, and 5% at Re = 30. At the same time, model fluid with variable thermal conductivity predicts a larger Nusselt number than that of constant property flow, but much smaller Nusselt number than that of SCW flow. On the contrary, the variable density and viscosity have little effect on the heat transfer process. Those are attributed to the peak of specific heat and thermal conductivity at pseudo-critical point. High conductivity results in high heat transfer rate and high heat diffusion, while high specific heat provides fluid with heat storage capacity. Therefore, fluid with a higher specific heat hinders the heat diffusion and stores the energy in a smaller regime, which leads to high temperature gradient.

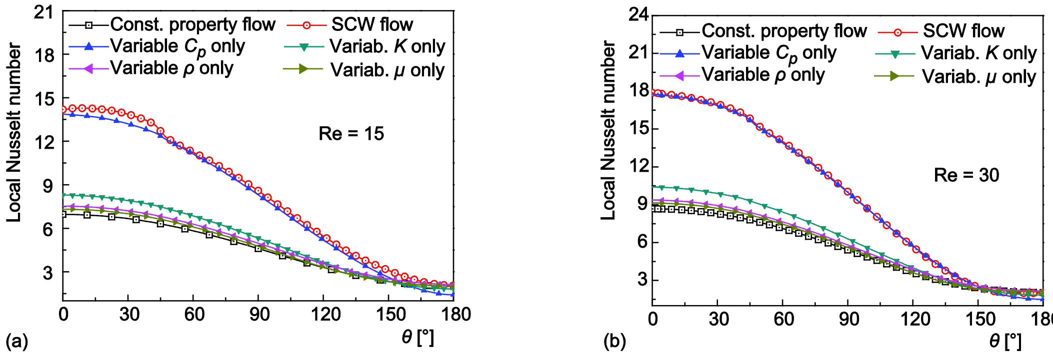

Figure 3. Local Nusselt number for variable properties, P = 23 MPa, T_∞ = 647 K, T_w = 657 K: (a) Re = 15, (b) Re = 30

Figure 3 shows that variable specific heat plays a primary role in determination of the heat transfer process in SCW flow. A correlation based on the revised specific heat ratio can be used to predict the heat transfer in SCW flow, as shown in eq. (8).

$$\frac{\mathrm{Nu_f}}{\mathrm{Nu_{fc}}} = 1.044\left(\frac{C_{pm}}{C_{p\infty}}\right)^{-0.6225\,\mathrm{Re}^{0.1095}} \tag{8}$$

where C_{pm} is determined by the temperature of T_m [$T_m = T_\infty - 0.6(T_\infty - T_w)$], which is an empirically calculated temperature value, and the value 0.6 is revised for present situation [14]. The $\mathrm{Nu_{fc}}$, represents the Nusselt number for constant property flow over a particle, which is calculated by Whitaker's [15] correlation:

$$\mathrm{Nu_{fc}} = 2 + (0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3})\,\mathrm{Pr}^{0.4} \tag{9}$$

In order to a good agreement between situation results and correlation, eq. (8), are observed in fig. 4, and the deviation is within ± 12.5%.

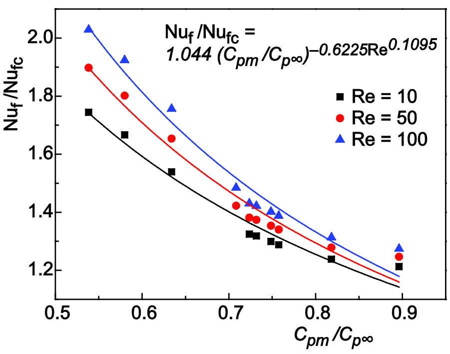

Figure 4. Nusselt number for forced convection from particle to SCW flow

Free convection

Based on our previous work [3], both variable specific heat and conductivity have a great effect on the Nusselt number. In order to obtain a quantitative relationship between Nusselt number and the specific heat, free convection over a spherical particle in flow with variable specific heat and density was conducted. This model fluid has the same specific heat and density with that of SCW flow, and the same viscosity and conductivity with that of far field. The temperatures in far field and sphere surface are set 637-697 K, which makes the variation of specific heat cover all pseudo-critical zone. Equation (10) was achieved to calculate the Nusselt number for flow with variable specific heat ($\mathrm{Nu_{nCp}}$).

$$\frac{\mathrm{Nu_{nCp}}}{\mathrm{Nu_{nc}}}=1.21\left(\frac{C_{pm}}{C_{p\infty}}\right)^{-0.82} \tag{10}$$

where $\mathrm{Nu_{nc}}$ is calculated by Churchill [12] correlation, as shown in eq. (11)

$$\mathrm{Nu_{nc}}=2+\frac{0.589\cdot\mathrm{Ra}^{0.25}}{\left[1+\left(\frac{0.43}{\mathrm{Pr}}\right)^{9/16}\right]^{4/9}} \tag{11}$$

Rayleigh number in eq. (11) is reversed by density ratio between free stream and particle wall:

$$\mathrm{Ra}=\mathrm{Gr}\ \mathrm{Pr}\ \frac{\rho_\infty}{\rho_w} \tag{12}$$

Figure 5(a) compares the simulation results with eq. (10), and the maximum deviation between simulation results and correlation is below 5%.

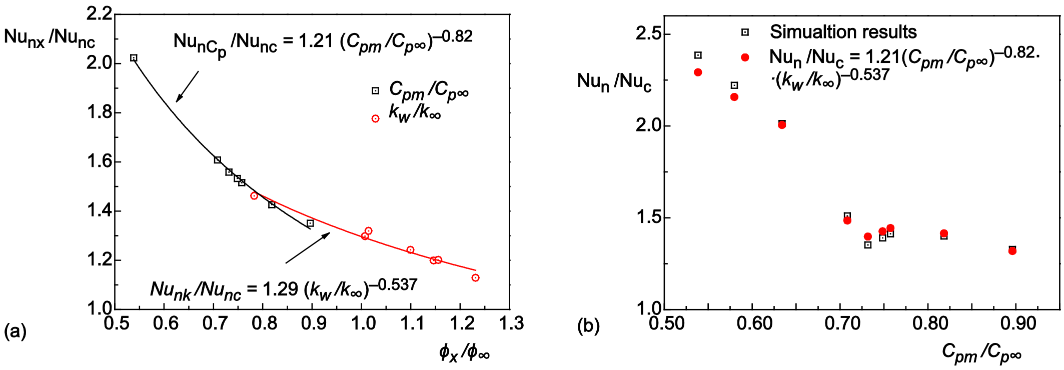

Figure 5. Nusselt number for free convection: (a) correlation for variable property flow, (b) correlation for SCW flow

In order to obtain a quantitative relationship between Nusselt number and conductivity, free convection over spherical particle in flow with variable conductivity and density was conducted. This model fluid has the same conductivity and density with that of SCW flow, and the same viscosity and specific heat with that of far field. The temperatures in far field and sphere surface are set 637-697 K, which makes the ratio of conductivity at sphere surface to free stream cover all pseudo-critical zone. Nusselt number for variable specific heat flow ($\mathrm{Nu_{nk}}$) can be predicted by eq. (10):

$$\frac{\mathrm{Nu_{nk}}}{\mathrm{Nu_{nc}}}=1.29\left(\frac{k_w}{k_\infty}\right)^{-0.537} \tag{13}$$

Figure 5(a) compares the simulation results with eq. (13), and the maximum deviation between simulation results and correlation is below 6.5%.

Then a heat transfer correlation of free convection in SCW flow over a particle can be expressed:

$$\frac{\mathrm{Nu_n}}{\mathrm{Nu_{nc}}} = 1.21\left(\frac{C_{pm}}{C_{p\infty}}\right)^{-0.82}\left(\frac{k_w}{k_\infty}\right)^{-0.537} \tag{14}$$

Obviously, this correlation is a combination of eqs. (10) and (13). Figure 5(b) compares the simulation results with eq. (14), and the deviation between simulation results and correlation is within ±8%.

Assisting convection

The local Nusselt number around sphere for assisting convection is shown in fig. 6. From the figure it can be observed that gravity hinders the heat transfer from sphere in recirculation zone, but enhances the heat transfer before the separation point. With an increase in Grashof number, the local Nusselt number before the separation point increases, but the local Nusselt number after the separation point decreases. Those are attributed to the assistance of free convection to assists the forced convection. The thinner or thicker thermal boundary layer results from a higher or lower fluid velocity driven by boundary force.

Figure 7 shows the effect of Grashof number on the average Nusselt number. The average Nusselt number increases with an increase in Grashof number. These results are consistent with [5]. The Nusselt number for assisting flow can be obtained by introducing eq. (8) and (14) into eq. (15). The average error when m = 3 and 4 is 13.2%, and 11.7%, respectively. Here, m = 4 is suggested for a smaller divergence.

$$\mathrm{Nu} = (\mathrm{Nu}_f^{\,m} + \mathrm{Nu}_n^{\,m})^{1/m} \tag{15}$$

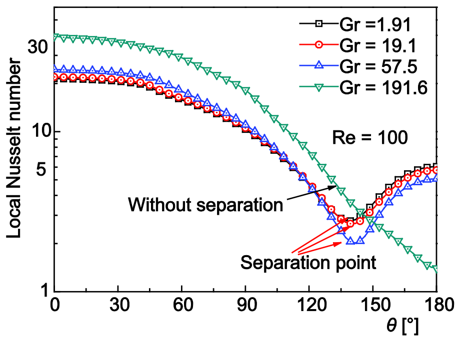

Figure 6. Local Nusselt number around sphere for assisting convection, *P* = 23 MPa, T_∞ = 647 K

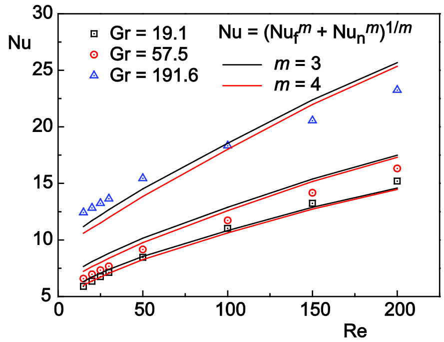

Figure 7. Average Nusselt number around sphere for assisting convection, *P* = 23 MPa, T_∞ = 647 K

Opposing convection

Figure 8 shows the effect of Grashof number on the local Nusselt number around sphere for opposing convection. When the flow separation occurs in the rear end of spherical particle, the local Nusselt number decreases around the sphere before the separation point, and increases after the separation point. An increase in Grashof number induces the decrease or increase in local Nusselt number before or after the separation point until flow separation occurs before the head of sphere. While the flow direction turns in reverse, the local Nusselt number increases around the sphere surface. These are due to that the flow velocity before the separation point is decelerated by buoyancy force together with the viscous interaction, but the flow velocity after the separation point is accelerated, which results in a strong convection interaction and a high heat transfer from the sphere to fluid.

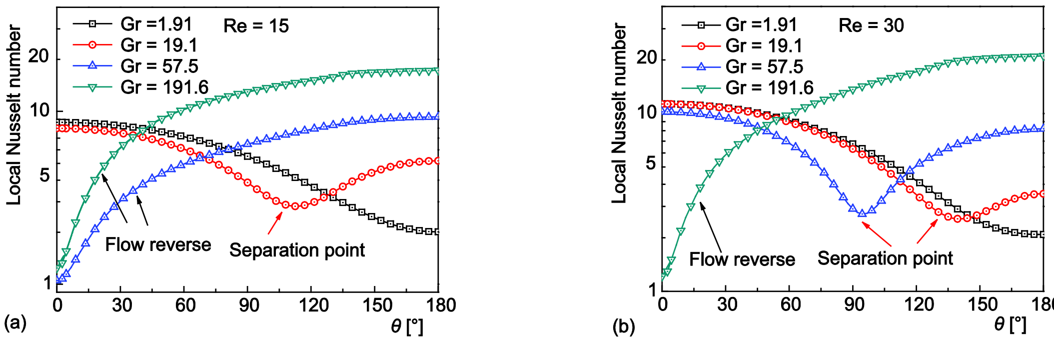

Figure 8. Local Nusselt number around sphere for opposing convection, P = 23 MPa, T_∞ = 647 K; (a) Re = 15, (b) Re = 30

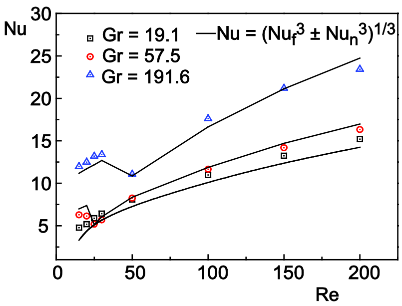

Figure 9. Average Nusselt number around sphere for opposing convection, P = 23 MPa, T_∞ = 647 K

The average Nusselt number for opposing flow is dependent on two parts: one part from forced convection and the other part from free convection. Traditionally, the effects of these two parts are opposite [16]. Figure 9 shows that the average Nusselt number for opposing convection varies with Reynolds and Grashof numbers. For low Reynolds and high Grashof number, the average Nusselt number is dominated by the free convection. For high Reynolds and low Grashof numbers, the average Nusselt number is close to the forced convection. The Nusselt number for opposing flow can be predicted by eq. (15) with minus sign instead of the plus sign while flow direction is consistent with the free stream. However, when flow direction is opposed to the free stream, plus sign shall be used. Figure 9 compares the simulation results with the correlation (m = 3). A correctly predicted trend is observed for the correlation, and the deviation is within ±7.8%.

Conclusions

In the present work, forced convection, free convection, assisting convection, and opposing convection heat transfer from a spherical particle in SCW were studied based on a numerical model fully accounting for variations of thermo-physical properties in pseudo-critical zone. It is found that variation of specific heat plays a main role in determination of heat transfer coefficient for forced convection in SCW flow. For assisting convection, increasing Grashof number hinders the heat transfer from a sphere in recirculation zone, but enhances the heat transfer before the separation point. The situation is on the contrary to the opposing convection. Quantitative relationship between variable property ratios and Nusselt number of forced convection, free convection, and mixed convection has been established in this work. For opposing convection, the eq. (15) with a plus sign is validated while the flow direction is opposed to the free stream.

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Nomenclature

C_d – drag coefficient
C_p – special heat, [J/(kg · K)]
D – diameter of sphere particle, [mm]
Gr – Grashof number
H – height of domain zone
h – enthalpy, [Jkg^{-1}]
k – conductive coefficient, [$Wm^{-1}K^{-1}$)]
Nu – Nusselt number
P – pressure, [MPa]
Pr – Prandtl number (= $C_{p\infty}\mu_\infty/k_\infty$)
Ra – reversed Rayleigh number [= Gr Pr (ρ_∞/ρ_w)]
Re – Reynolds number (= $\rho_\infty u_\infty d/\mu_\infty$)
Ri – Richardson number (= Gr/Re^2)
T – temperature, [K]
U – characteristic velocity, [ms^{-1}]
u – velocity, [ms^{-1}]

Greek symbols

θ – streamwise angle, [°]
μ – viscosity, [Pa·s]
ρ – density, [kgm^{-3}]
ϕ – variables of properties

Subscripts

c – constant property flow
f – forced convection
m – film temperature
n – nature or free convection
w – wall surface
χ – reference temperature
∞ – far field

Superscript

m – index number

Acronyms

SCW – supercritical water
FBR – fluidized bed reactor

References

[1] Lu, Y. J., *et al.*, Hydrogen Production by Biomass Gasification in Supercritical Water with a Fluidized Bed Reactor, *Int. J. Hydrogen Energ., 33* (2008), 21, pp. 6066-6075
[2] Lu, Y., *et al.*, Minimum Fluidization Velocities for Supercritical Water Fluidized Bed within the Range of 633–693 K and 23–27 MPa, *Int. J. Multiphas. Flow*, *49* (2013), pp. 78-82
[3] Wei, L., *et al.*, Numerical Study on Laminar Free Convection Heat Transfer between Sphere Particle and High Pressure Water in Pseudo-Critical Zone, *Thermal Science, 18* (2014), 4, pp. 1293-1303
[4] Wei, L., *et al.*, Numerical Study on the Mixed Convection Heat Transfer between a Sphere Particle and High Pressure Water in Pseudo-Critical Zone, *Advances in Mechanical Engineering, 2013* (2013), ID 527182
[5] Antar, M., El-Shaarawi, M., Mixed Convection Around a Liquid Sphere in an Air Stream, *Heat Mass Transfer*, *38* (2002), 4, pp. 419-424
[6] Nazar, R., Amin, N., Free Convection Boundary Layer on an Isothermal Sphere in a Micropolar Fluid, *Int. Commun. Heat Mass, 29* (2002), 3, pp. 377-386
[7] Mograbi, E., Bar-Ziv, E., Dynamics of a Spherical Particle in Mixed Convection Flow Field, *J. Aerosol Sci.*, 36 (2005), 3, pp. 387-409
[8] Kotouc, M., *et al.*, Loss of Axisymmetry in the Mixed Convection, Assisting Flow Past a Heated Sphere, *Int. J. Heat Mass Transfer*, *51* (2008), 11-12, pp. 2686-2700
[9] Chen, T., Mucoglu, A., Analysis of Mixed Forced and Free Convection about a Sphere, *Int. J. Heat Mass Transfer*, *20* (1977), 8, pp. 867-875

[10] Bhattacharyya, S., Singh, A., Mixed Convection from an Isolated Spherical Particle, *Int. J. Heat Mass Transfer*, *51* (2008), 5, pp. 1034-1048
[11] Hieber, C., Gebhart, B., Mixed Convection from a Sphere at Small Reynolds and Grashof Numbers, *J. Fluid Mech.*, *38* (1969), 1, pp. 137-159
[12] Churchill, S. W., Comprehensive, Theoretically Based, Correlating Equations for Free Convection from Isothermal Spheres, *Chem. Eng. Commun.*, *24* (1983), 4-6, pp. 339-352
[13] Oosthuizen, P., Bassey, M., An Experimental Study of Combined Forced– and Free-Convective Heat Transfer from Flat Plates to Air at Low Reynolds Numbers, *J. Heat Transfer*, *95* (1973), 1, pp. 120-121
[14] Dang, C., Hihara, E., Numerical Study on In-Tube Laminar Heat Transfer of Supercritical Fluids, *Appl. Therm. Eng.*, *30* 2010), 13, pp. 1567-1573
[15] Whitaker, S., Forced Convection Heat Transfer Correlations for Flow in Pipes, Past Flat Plates, Single Cylinders, Single Spheres, and for Flow in Packed Beds and Tube Bundles, *AIChE J.*, *18* (1972), 2, pp. 361-371
[16] Rohsenow, W. M., *et al.*, *Handbook of Heat Transfer*, McGraw-Hill, New York, USA, 1998

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