EFFECTS OF CHEMICAL REACTION IN THERMAL AND MASS DIFFUSION OF MICROPOLAR FLUID SATURATED IN POROUS REGIME WITH RADIATION AND OHMIC HEATING

by

Hitesh KUMAR

Department of Information Technology, Ibri College
of Technology, Ibri, Oman

Original scientific paper
DOI: 10.2298/TSCI130415016K

The present paper analyzes the chemically reacting free convection magnetohydrodynamic micropolar flow, heat and mass transfer in porous medium past an infinite vertical plate with radiation, and viscous dissipation. The non-linear coupled partial differential equations are solved numerically using an implicit finite difference scheme known as Keller-Box method. The results for concentration, transverse velocity, angular velocity, and temperature are obtained and effects of various parameters on these functions are presented graphically. The numerical discussion with physical interpretations for the influence of various parameters also presented.

Key words: *thermal diffusion, mass diffusion, porous medium, micropolar fluid, radiation, ohmic heating, magnetic field, mixed convection, chemical reaction, viscous dissipation*

Introduction

Investigations of the flow streaming into a porous and permeable medium, assuming velocity of the flow not small (Reynolds number is moderately high) were obtained by Yamamoto and Iwamura [1], Yamamoto and Yoshida [2], Brinkman [3], Raptis *et al.* [4], and Raptis and Kafousias [5]. All previous authors used generalized Darcy's law. Flow equations used are boundary layer type equations in which convective acceleration was taken into account. But the generalized Darcy's law is derived without taking into account the angular velocity of the fluid particles. Raptis [6] in his research paper on a horizontal plate used flow equations with angular velocity. Such fluids are known as polar fluids in the literature. Raptis [7] in another research paper discussed magnetopolar fluid through a porous medium.

A micropolar continuum is a collection of interconnected particles in the form of small rigid bodies undergoing both translational and rotational motions. Typical examples of such materials are granular media and multimolecular bodies whose microstructures act as an evident part in their macroscopic responses. Rigid chopped fibers, elastic solids with rigid granular inclusions, and other industrial materials such as liquid crystals are examples of such materials. Theory of micro-elastic solids in which the micromotion of the material particles contained in a macrovolume element with respect to centroid presented by Eringen and Suhubi [8] and Suhubi and Eringen [9]. Eringen [10] developed a theory for a subclass of micromorphic materials which are called micropolar media and these materials show microrota-

Author's e-mail: hiteshrsharma@gmail.com

tional effect and microrotational interia. Chen [11] studied constitutive equations of micropolar electromagnetic fluids and Borrelli *et al.* [12] investigated numerical simulations of 3-D MHD stagnation-point flow of a micropolar fluid. Nowacki [13], Eringen [14, 15], Tauchert *et al.* [16], Tauchert [17] studied the micropolar theory with thermal effects and Kim [18] presented the phenomenon of heat and mass transfer in MHD micropolar flow over a vertical moving porous plate in a porous medium.

We know that fluids in geothermal region are electrically conducting. Flows arising from temperature difference have great significance not only for their own, but also for the applications to the geophysics and engineering. There are many interesting aspects of such flows and analytical solutions of such problem are presented by Gebhart and Pera [19], Sparrow *et al.* [20], Soundalgekar [21], Acharya *et al.* [22], and Singh and Chand [23].

All industrial chemical processes are designed to transform cheaper raw materials to high value products (usually via chemical reaction). A reactor, in which such chemical transformations take place, has to carry out several functions like bringing reactants into intimate contact, providing an appropriate environment (temperature and concentration fields) for adequate time and allowing for removal of products. Fluid dynamics play a pivotal role in establishing the relationship between reactor hardware and reactor performance. For a specific chemical catalyst, the reactor performance is a complex function of the underlying transport processes. The first step in any reaction engineering analysis is formulating a mathematical framework to describe the rate (and mechanisms) by which one chemical species are converted into another in the absence of any transport limitations (chemical kinetics). Once the intrinsic kinetics are available, the production rate and composition of the products can be related, in principle, to reactor volume, reactor configuration, and mode of operation by solving mass, momentum and energy balances over the reactor. This is the central task of a reaction and reactor engineering activity. Analysis of the transport processes and their interaction with chemical reactions can be quite difficult and is intimately connected to the underlying fluid dynamics. Such a combined analysis of chemical and physical processes constitutes the core of chemical reaction engineering. Recent advances in understanding the physics of flows and computational flow modeling (CFM) can make tremendous contributions in chemical engineering.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Natural convection processes involving the combined mechanisms are also encountered in many natural processes, such as evaporation, condensation and agricultural drying, and in many industrial applications, such as the curing of plastics, cleaning and chemical processing of materials relevant to the manufacture of printed circuitry, manufacture of pulp-insulated cables, *etc*. Diffusion rates can be altered tremendously by chemical reactions. The Effect of a chemical reaction depends whether the reaction is homogeneous or heterogeneous. This depends on whether they occur in an interface or as a single phase volume reaction. In a well-mixed system, the reaction is heterogeneous if the reactants are in multiple phase, and homogeneous if the reactants are in the same phase. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. Cooling towers are the cheapest way to cool large quantities of water. For example, the formation of smog is a first-order homogeneous chemical reaction. Consider the emission of NO_2 from automobiles and other smoke stacks. This NO_2 reacts chemically in the atmosphere with unburned hydrocarbons (aided by sunlight) and produces peroxyacetylnitrate, which forms an envelope termed as the photochemical smog. Kandasamy et al. [24] studied thermophoresis and variable viscosity effects on MHD mixed convective heat and mass transfer past a porous wedge in the presence of chemical reaction. Kandasamy and Devi [25] studied the effects of chemical reaction, heat and mass transfer on non-linear laminar boundary-layer flow over a wedge with suction or injection.

Recently, considerable attention has also been focused on new applications of MHD and heat transfer for example metallurgical processing, melt refining involves magnetic field application to control excessive heat transfer rates. Both laminar and turbulent flows are of interest. Many studies in MHD thermo-convection flows have been conducted. Chamkha [26] studied the free convection boundary-layer flow over an isothermal plate in the presence of a non-uniform transverse magnetic field. Recently, Asghar *et. al.* [27] investigated the MHD flow due to non-coaxial rotations of a porous disk, moving with uniform acceleration in its own plane and a second grade fluid at infinity.

When the temperature of surrounding fluid is high, the radiation effects play an important role that cannot be ignored, Modest [28] and Siegel and Howell [29]. The effects of radiation on temperature have become more important industrialized. Many processes in engineering areas occur at high temperature and acknowledge radiation heat transfer become very important for the design of pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. In such cases one has to take into account the effects of radiation and free convection. For an impulsively started infinite vertical isothermal plate, Ganesan *et al.* [30] studied the effects of radiation and free convection, by using Rosseland approximation, Brewster [31]. Problem of radiative heat transfer with hydromagnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux is solved analytically by Kumar [32]. Hossain and Takhar [33], Raptis and Massals [34], and Hossain *et al.* [35] studied the radiation effect on free and forced convection flows past a vertical plate, including various physical aspects. Aboeldahab Emad [36] studied the radiation effect on heat transfer in an electrically conducting fluid at the stretching surface. At high operating temperature, radiation effect can be quite significant, Ghaly and Elbarbary [37]. Heat and mass transfer effects on moving plate in the presence of thermal radiation have been studied by Muthucumaraswamy and Kumar [38] using Laplace technique. For the problem of coupled heat and mass transfer in MHD free convection, the effect of both viscous dissipation and *ohmic heating* are not studied in the previous investigations. However, it is more realistic to include these two effects to explore the impact of the magnetic field on the thermal transport in the boundary layer. With this awareness, the effect of *ohmic heating* on the MHD free convection heat transfer has been examined for a Newtonian fluid by Hossain [39]. Chen [40] studied the problem of combined heat and mass transfer of an electrically conducting fluid in MHD natural convection, adjacent to a vertical surface with *ohmic heating*.

The object of the paper is to study the steady mixed convection flow of a laminar, incompressible MHD micropolar fluid and thermal and mass diffusion in porous medium with the effects of radiation and *ohmic heating* in the presence of chemical reaction. The non-linear coupled partial differential equations that appear in the paper are solved numerically by Cebeci and Bradshaw [41, 42], using an implicit finite-difference scheme which is the Keller-Box method.

Governing equations and analysis

Consider the mixed convection flow of an incompressible and electrically conducting viscous thermo-micropolar fluid past an infinite porous vertical plate (fig. 1). A magnetic

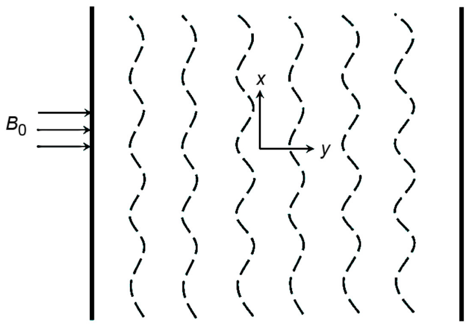

Figure 1. Physical model and co-ordinate system

field, B_0, of uniform strength is applied transversely to the direction of the flow that is y-axis and the induced magnetic field is neglected. Taking the x-axis along the vertical porous plate in upward direction and y-axis normal to it. Since the length of the plate is large and fluid flow extends to infinity, therefore all physical variables are independent of x and hence the functions of y only, the governing equations of continuity, momentum, concentration, angular velocity, and energy for the flow in the presence of radiation, chemical reaction and viscous dissipation are:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$v^* = -V_0 \text{ (constant)} \tag{2}$$

$$\frac{\mathrm{d}p^*}{\partial y^*} = 0 \Rightarrow p^* \text{ is independent of } y^* \tag{3}$$

$$\rho\, v^* \frac{\partial u^*}{\partial y^*} = (\kappa + \mu)\frac{\partial^2 u^*}{\partial y^{*2}} + \rho g \beta_\mathrm{T}(T^* - T_\infty) + \rho g \beta_\mathrm{c}(C^* - C_\infty) + \kappa \frac{\partial \omega_a^*}{\partial y^*} - \frac{\mu}{K^*}u^* - \sigma B_0^2 u^* \tag{4}$$

$$\rho j\left(v^* \frac{\partial \omega_\mathrm{a}^*}{\partial y^*}\right) = \gamma \frac{\partial^2 \omega_\mathrm{a}^*}{\partial y^{*2}} - 2\kappa\omega_\mathrm{a}^* \tag{5}$$

$$\rho c_p v^* \frac{\partial T^*}{\partial y^*} = k\frac{\partial^2 T^*}{\partial y^{*2}} + \mu\left(\frac{\partial u^*}{\partial y^*}\right)^2 - \frac{\partial q_\mathrm{r}^*}{\partial y^*} - \sigma B_0^2 u^{*2} \tag{6}$$

$$v^* \frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - k_l(C^* - C_\infty^*) \tag{7}$$

with the boundary conditions:

$$\begin{gathered} u^* = V_0, \quad \frac{\partial \omega_\mathrm{a}^*}{\partial y^*} = -\frac{\partial^2 u^*}{\partial y^{*2}}, \quad T^* = T_\mathrm{w}, \quad -D\frac{\partial C^*}{\partial y^*} = m_\mathrm{w}, \quad \text{at } y = 0 \\ u^* \to 0, \quad \omega_\mathrm{a}^* \to 0, \quad T^* \to T_\infty, \quad C^* \to C_\infty, \quad \text{as } y \to \infty \end{gathered} \tag{8}$$

where $V_0 > 0$, $\gamma = (\mu + \kappa/2)j = \mu(1 + a/2)j$ and $j = \vartheta^2/V_0^2$.

The radiative heat flux, may be written [43]:

$$\frac{\partial q_\mathrm{r}^*}{\partial y^*} = 4(T^* - T_\infty)I' \tag{9}$$

where $I' = \int_0^\infty K_{\lambda \mathrm{w}}(\partial e_{b\lambda}/\partial T)\mathrm{d}\lambda$, $K_{\lambda w}$ is the absorption coefficient at the wall and $e_{b\lambda}$ – the Plank's function.

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced:

$$y=\frac{V_0\,y^*}{\vartheta},\quad u=\frac{u^*}{V_0},\quad M=\frac{\sigma B_0^2\vartheta}{\rho V_0^2},\quad \mathrm{Pr}=\frac{\mu c_p}{k},\quad \theta=\frac{T^*-T_\infty}{T_w-T_\infty},\quad C=\frac{C^*-C_\infty}{\dfrac{m_\mathrm{w}\vartheta}{V_0 D}},$$

$$\mathrm{Ec}=\frac{V_0^2}{c_p(T_\mathrm{w}-T_\infty)},\quad \mathrm{Gr}=\frac{\mathrm{g}\beta_T\vartheta(T_\mathrm{w}-T_\infty)}{V_0^3},\quad \mathrm{Gc}=\frac{\mathrm{g}\beta_\mathrm{c}m_\mathrm{w}\vartheta^2}{V_0^4 D},\quad \omega_\mathrm{a}=\frac{\vartheta\omega_\mathrm{a}^*}{V_0^2},$$

$$\mathrm{Sc}=\frac{\vartheta}{D},\quad F=\frac{4\vartheta I'}{\rho c_p V_0^2},\quad K_c=\frac{\vartheta k_l}{v_\mathrm{w}^2},\quad K=\frac{K^* V_0^2}{\vartheta^2},\quad R=\frac{\kappa}{\mu}$$

The equations (4)-(7) change to:

$$(1+R)\frac{\mathrm{d}^2u}{\mathrm{d}y^2}+\frac{\mathrm{d}u}{\mathrm{d}y}-\left(M+\frac{1}{K}\right)u+R\frac{\mathrm{d}\omega_a}{\mathrm{d}y}+\mathrm{Gr}\theta+\mathrm{Gc}C=0 \tag{10}$$

$$\left(1+\frac{R}{2}\right)\frac{\mathrm{d}^2\omega_\mathrm{a}}{\mathrm{d}y^2}+\frac{\mathrm{d}\omega_\mathrm{a}}{\mathrm{d}y}-2R\omega_\mathrm{a}=0 \tag{11}$$

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}y^2}+\mathrm{Pr}\frac{\mathrm{d}\theta}{\mathrm{d}y}-F\mathrm{Pr}\theta+\mathrm{Pr\,Ec}\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2+\mathrm{Pr\,Ec}Mu^2=0 \tag{12}$$

$$\frac{\mathrm{d}^2C}{\mathrm{d}y^2}+\mathrm{Sc}\frac{\mathrm{d}C}{\mathrm{d}y}-K_c\,\mathrm{Sc}\,C=0 \tag{13}$$

Boundary conditions changes to:

$$\text{at}\quad y=0,\quad u=1,\quad \theta=1,\quad \frac{\mathrm{d}\omega_\mathrm{a}}{\mathrm{d}y}=-\frac{\mathrm{d}^2u}{\mathrm{d}y^2},\quad \frac{\mathrm{d}C}{\mathrm{d}y}=-1$$

$$\text{as}\quad y\to\infty,\quad u\to 0,\quad \theta\to 0,\quad \omega_\mathrm{a}\to 0,\quad C\to 0 \tag{14}$$

Discussions and results

System of eqs. (10)-(13) subject to the boundary conditions (14) are highly coupled and solved numerically using the Keller-Box method as described by Cebeci and Bradshaw [41, 42] which is essentially a matrix method that solves the discretized equations, by finding simple centered-difference derivatives and average of the midpoints of net rectangles. Then, a block-tridiagonal factorization scheme is applied on the coefficient matrix of the finite-difference equations. In order to understand the physical solution, the numerical values of concentration, transverse velocity, angular velocity, and temperature are presented in figs. 2-10.

Figure 2 represents the profile of concentration plotted for the different values of Schmidt number, it shows concentration decreases with Schmidt number. Physically, the increase of Schmidt number means decrease of molecular diffusivity, *D*. That results in decrease of concentration boundary layer. Hence, the concentration of the species is higher for small values of Schmidt number and lower for larger values of Schmidt number.

Different variations in chemical reaction parameter are plotted for concentration in fig. 3. It is seen that an increase in K_c leads to decrease *C*. It is considered here, to be a homogeneous first-order chemical reaction. The diffusing species either can be destroyed or generated in the homogeneous reaction. The chemical reaction parameter can be adjusted to meet these circumstances if one takes $K_c > 0$ for a destructive reaction, $K_c < 0$ for a generative reaction, and

K_c = 0 for no reaction. The destructive chemical reaction is taken into account here. Consequently, the concentration falls off for the increments of the chemical reaction parameter. This shows that the diffusion rates can be altered by the chemical reaction parameter. In mixed convection regime, concentration of the fluid decrease with increase of destructive reaction and thermophoresis particle deposition. Also, it can be seen that the thermophoretic deposition velocity becomes sensitive to the variation of these parameters, this is of particular benefit in processes, which require extreme cleanliness of the surfaces and therefore, the thermophoretic and destructive reaction are expected to transship the concentration boundary layer significantly.

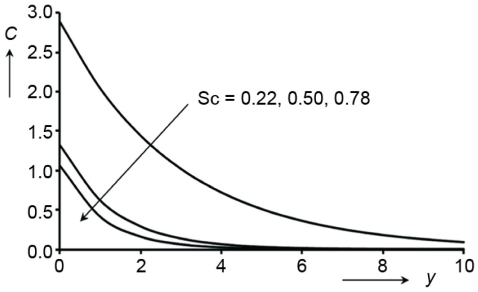

Figure 2. The dimensionless concentration *vs.* various values of Schmidt number when K_c = 0.2, R = 0.3, Pr = 2.0, F = 3.0, M = 2.0, K = 2.0, Gr = 1.0, Gc = 0.5, and Ec = 0.01

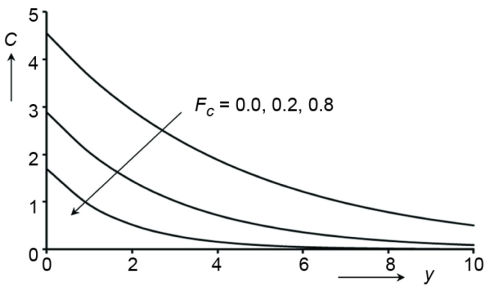

Figure 3. The dimensionless concentration *vs.* various values of K_c when Sc = 0.22, R = 0.3, Pr = 2.0, F = 3.0, M = 2.0, K = 2.0, Gr = 1.0, Gc = 0.5, and Ec = 0.01

Figure 4 represents the effect of material parameter R on angular velocity, ω_a. An increase in the material parameter increases angular velocity, also. Increase in R implies that spin gradient viscosity supersedes the vortex viscosity in value, as R increase it enhances the micro-rotation hence the spin gradient viscosity is boosted and micro elements rotates faster therefore the angular velocity rises.

Transverse velocity is drawn in fig. 5 for different values of radiation parameter, F. The velocity increase with F. Variation occurs in the vicinity of the plate, and as we move away from the plate the effects of the radiation become constant and streamlines reduces to zero.

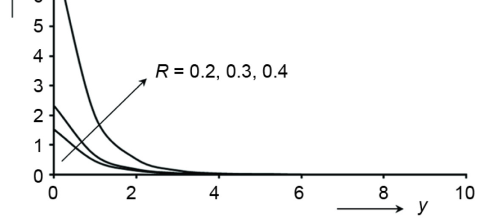

Figure 4. The dimensionless angular velocity *vs.* various values of R when Sc = 0.22, K_c = 0.2, Pr = 2.0, F = 3.0, M = 2.0, K = 2.0, Gr = 1.0, Gc = 0.5, and Ec = 0.01

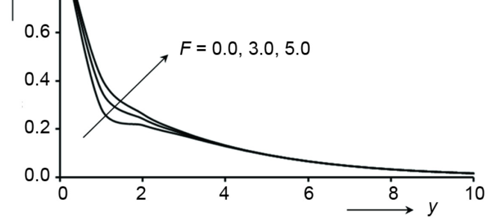

Figure 5. The dimensionless velocity *vs.* various values of F when Sc = 0.22, K_c = 0.2, R = 0.3, Pr = 2.0, M = 2.0, K = 2.0, Gr = 1.0, Gc = 0.5, and Ec = 0.01

The effects of the magnetic parameter, M, on velocity, u, are presented in fig. 6. The velocity decreases with an increase in M. Physically, the effect of increasing magnetic field strength is to increase the retarding force and hence reduces the velocity. This is the classical Hartmann result.

Figures 7 and 8 are plotted for u and ω_a for different values of permeability parameter K. It is seen here that increasing K increases u. An increase in the porosity parameter physically means reduce the drag force and hence causes the flow velocity to increase. An increase in K will reduce the resistance of the porous medium which leads to increase the velocity. Increasing in K also leads to decreasing of ω_a. The maximum micro-rotation occurs for the lowest permeability. As K rises ω_a falls, this implies a reversal in spin. It would appear, therefore, that larger permeability materials can be used to reduce micro-rotational effects in suspension fluids.

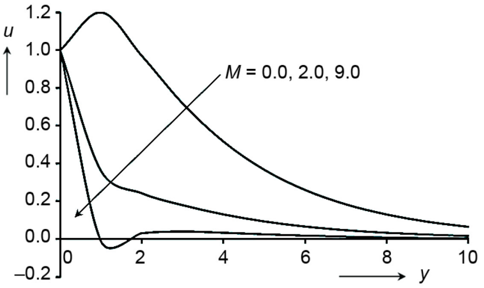

Figure 6. The dimensionless velocity *vs.* various values of M when Sc = 0.22, K_c = 0.2, R = 0.3, Pr = 2.0, F = 3.0, K = 2.0, Gr = 1.0, Gc = 0.5, and Ec = 0.01

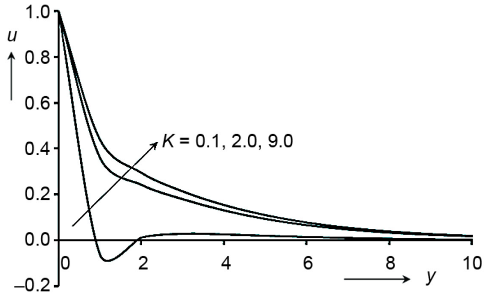

Figure 7. The dimensionless velocity *vs.* various values of K when Sc = 0.22, K_c = 0.2, R = 0.3, Pr = 2.0, F = 3.0, M = 2.0, Gr = 1.0, Gc = 0.5, and Ec = 0.01

Figure 9 is drawn for the temperature of the fluid, θ, for varying Prandtl number. It is noted here that θ decreases as Prandtl number increase, and observed that the thermal boundary layer thickness becomes shorter for larger values of Prandtl number. This phenomenon occurs because when Prandtl number increases, that implies lower effective thermal diffusivity for a fixed kinematic viscosity and this leads to the decrease of the thermal boundary layer. Figure 10 shows the effects of radiation parameter F on θ. The figure depicts that θ decreases as F increase. Also for the higher values of radiation parameter correspond to an increased dominance of conduction over radiation thereby decreasing buoyancy force, hence reducing the thickness of the thermal boundary layers.

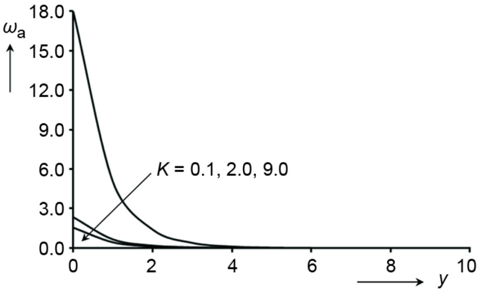

Figure 8. The dimensionless angular velocity *vs.* various values of K when Sc = 0.22, K_c = 0.2, R = 0.3, Pr = 2.0, F = 3.0, M = 2.0, Gr = 1.0, Gc = 0.5, and Ec = 0.01

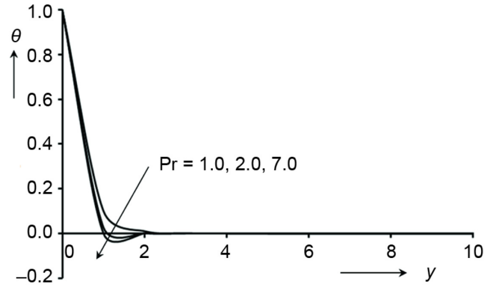

Figure 9. The dimensionless temperature *vs.* various values of Prandtl number when Sc = 0.22, K_c = 0.2, R = 0.3, F = 3.0, M = 2.0, K = 2.0, Gr = 1.0, Gc = 0.5, and Ec = 0.01

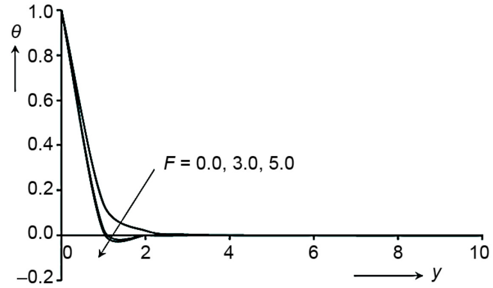

Figure 10. The dimensionless temperature *vs.* various values of *F* when Sc = 0.22, K_c = 0.2, *R* = 0.3, Pr = 2.0, *M* = 2.0, *K* = 2.0, Gr = 1.0, Gc = 0.5, and Ec = 0.01

Conclusions

In the present research, the effects of Schmidt number, porous medium, magnetic field, Prandtl number, radiation (when $T_w > T_\infty$), material parameters, and chemical reaction over various field functions are obtained. It is concluded the following.

- Increase in Schmidt number or chemical reaction results to reduce concentration boundary layer.
- Porosity increases the velocity boundary layer; whereas it decrease the rotational velocity. Therefore, it is also concluded that larger permeability materials can be used to reduce micro-rotational effects.
- Increasing magnetic field force weakens the velocity boundary layer, as a retarding force.
- The effect of Prandtl number is to reduce the thickness of the thermal boundary layer.
- The presence of radiation increases the velocity boundary layer, near the wall.
- Increasing radiation reduces buoyancy force and hence the thickness of the thermal boundary layer.
- An increase in the material parameter enhances the spin gradient viscosity and the angular velocity also increases.

Acknowledgments

The author is very much thankful to the learned referee and Prof. S. S. Tak, Jai Narain Vyas University, Jodhpur, India, for offering their valuable suggestions and assistance to improve the paper.

Nomenclature

B_0 – magnetic field coefficient, [T]
C – dimensionless species concentration
C^* – species concentration, [molm^{-3}]
C_∞ – far field concentration, [molm^{-3}]
c_p – specific heat, [Jkg^{-1}K^{-1}]
D – mass diffusion coefficient, [m^{2}s^{-1}]
Ec – Eckert number
F – radiation parameter
Gc – solutal Grashof number
Gr – thermal Grashof number
g – acceleration due to gravity, [ms^{-2}]
j – micro inertia density, [m^2]
K – dimensionless permeability parameter
K^* – permeability of porous medium, [Hm^{-1}]
K_c – chemical reaction parameter
k – thermal conductivity, [Wm^{-1}K^{-1}]
k_l – rate of chemical reaction, [s^{-1}]
M – magnetic field parameter
m_w – wall mass flux, [molm^{-2}s^{-1}]
Pr – Prandtl number
p^* – pressure, [Pa]
q_r^* – radiative heat flux in y-direction, [Wm^{-2}]
R – material parameter
Sc – Schmidt number
T^* – temperature of the fluid, [K]
T_∞ – far field temperature, [K]
T_w – wall temperature, [K]
u – dimensionless axial velocity
u^* – axial velocity, [ms^{-1}]
V_0 – suction velocity, [ms^{-1}]
v^* – transverse velocity, [ms^{-1}]
y – dimensionless horizontal co-ordinate
y^* – horizontal co-ordinate, [m]

Greek symbols

β_c – coefficient of concentration expansion, [m^{3}mol^{-1}]
β_T – coefficient of thermal expansion, [K^{-1}]
γ – spin gradient viscosity, [kgms^{-1}]
θ – dimensionless temperature
ϑ – kinematic viscosity, [m^{2}s^{-1}]
κ – vortex viscosity, [Pa·s]

λ – wave length, [m]
μ – dynamic coefficient of viscosity, [Pa·s]
ρ – density, [kgm^{-3}]
σ – electrical conductivity, [Sm^{-1}]
ω_{a_*} – dimensionless angular velocity
$\omega_a{}^*$ – angular velocity vector normal to the x-y plane, [rads^{-1}]

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Paper submitted: April 15, 2013
Paper revised: February 10, 2014
Paper accepted: February 19, 2014