

MHD TWO-PHASE FLUID FLOW AND HEAT TRANSFER WITH PARTIAL SLIP IN AN INCLINED CHANNEL

by

Zaheer ABBAS^{a*}, Jafar HASNAIN^a, and Muhammad SAJID^b

^a Department of Mathematics, The Islamia University of Bahawalpur,
Bahawalpur, Pakistan

^b Theoretical Physics Division, PINSTECH, Nilore, Islamabad, Pakistan

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The aim of this paper is to investigate the velocity and thermal slip effects in MHD flow and heat transfer of two-phase viscous fluid. It is assumed that both the phases have different densities, viscosities and electrical conductivities. The fully developed flow governed by a constant pressure gradient is passing through an inclined channel having inclination ϕ with horizontal axis. The electrical conductivity in phase I is assumed to be zero so that the constant applied magnetic field of strength B_0 in the transverse direction only effect the fluid in phase II. The method of successive approximation is used to develop the analytic solution of order 1 for the developed dimensionless coupled ordinary differential equations. The main focus is to discuss the influence of velocity and thermal slip parameters and Hartmann number on the velocity and temperature profiles.

Key words: MHD, two-phase flow, inclined channel, slip boundary conditions, heat transfer.

Introduction

Flow and heat transfer analysis of fluids is governed by the conservation laws of mass, momentum and energy. In the simple case when the fluid obeys the Newtonian law of viscosity the conservation of momentum give the Navier-Stokes equations. In many flow situations Navier-Stokes equations are nonlinear and do not have an exact solution. In general these equations are solved by assuming a no-slip at the boundary which states that at the solid boundary the fluid has the same velocity as that of the solid boundary. In situations where the surface is sufficiently smooth the no-slip boundary condition is replaced by Navier's slip boundary condition. In the Navier's slip condition the fluid velocity at the solid surface is assumed proportional to the shear stress at the solid surface. The above stated conditions are applicable at the fluid-solid interface.

However, in industry and technology and in some naturally existing fluids the fluid has two or more phases. These multi-phase fluids have applications in petroleum industry. The two-phase flow phenomena occur in pumping oil from a reservoir by adding a suitable amount of water into it. In the case of two phase fluids there exist a separating layer known as fluid-fluid interface. For these types of flow studies in addition to boundary condition at fluid-solid interface one needs to incorporate boundary conditions at fluid-fluid interface. Usually at the interface layer the continuity of velocity and the shear stress is ensured which provides us additional boundary conditions necessary to solve such flow problem having multi-phases.

* Corresponding author; e-mail: za_qau@yahoo.com

The flow of electrically conducting fluid through a pipe with suspended particles in the presence of a transverse magnetic field is encountered in MHD generators, pumps, accelerators and flowmeters. In these type of flows when fluid have more concentrated particles, their mutual interaction results in higher particle-phase viscous stresses. The laminar flow through a pipe for two-phase fluid is discussed by Packham and Shail [1]. The effects of an applied magnetic field in for flow through a rectangular channel was analyzed by Shail [2]. Saffman [3] investigated the stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. Datta and Mishra [4] have analyzed the boundary layer flow of a dusty fluid on a semi-infinite flat plate. In the literature one can find articles on the steady flow of two-phase fluid over a flat plate. These studies include Marble [5], Soo [6], Ottermann and Lee [7], Tabakoff and Hameed [8], Peddieson [9] and Fernandez [10] and references there in. The steady two-phase fluid flow of fluid-particle suspension past a stretching sheet is discussed by Kumar and Sharma [11]. In another paper Chakrabarti and Gupta [12] studied the hydromagnetic flow and heat transfer in a fluid initially at rest and at uniform temperature over a stretching surface. Vajravelu and Nayfeh [13] analyzed the MHD flow of a dusty fluid past a stretching sheet by considering the effect of uniform suction. Ezzat *et al.* [14] studied the hydromagnetic flow of dusty fluid through a porous medium using the state space approach and inversion of the Laplace transformation method. Gireesha *et al.* [15] presented the boundary layer flow and heat transfer of a dusty fluid over a stretching sheet with the effect of non-uniform heat source/sink and obtain numerical solution. Recently, Gireesha *et al.* [16] discussed three-dimensional Couette flow of a dusty fluid with heat transfer analytically using the classical perturbation method.

In above reference articles the considered geometry is horizontal. The applications in heat transfer particularly in solar collector technology encouraged researchers to look into the two-phase flow problem with inclined geometry. The review articles by Raithby and Hollands [17] and Catton [18] includes the basic details in this direction. The problem of two-phase MHD flow through an inclined channel was investigated by Malashetty and Umavathi [19]. They also analyzed heat transfer and presented approximate solutions for zero and first order using perturbation method. In another paper, Malashetty *et al.* [20] discussed perturbation solution for the fully developed free convection magnetohydrodynamic two fluid flow and heat transfer in an inclined channel. Recently, Umavathi *et al.* [21] studied the unsteady flow and heat transfer in a horizontal composite channel consisting of two parallel permeable plates with half of the distance between them filled by a fluid-saturated porous layer and the other half by a clear viscous fluid.

The objective of present investigation is to study the steady two-phase magnetohydrodynamic flow and heat transfer through an inclined channel in the presence of velocity and thermal slip effects at the upper and lower walls. The phase in lower channel is electrically conducting and in the upper channel the phase has zero electrical conductivity. Therefore, the applied constant magnetic field only effects the phase in the lower channel. The approximate analytical solutions for zero and first orders of the resultant coupled differential equations are obtained using method of successive approximation.

Formulation of the problem

Let us consider a channel inclined at an angle ϕ with the horizontal axis. We choose a Cartesian coordinate system in such a way that x -axis is along the channel and y -axis is normal to it as shown in fig. 1. Channel is divided into two regions such that the region $0 \leq y \leq h_1$ is filled with electrically conducting fluid having density ρ_1 , viscosity μ_1 and thermal conductivity K_1 . The other region $h_2 \leq y \leq 0$ is filled with another electrically conducting fluid having density ρ_2 , viscosity μ_2 , thermal conductivity K_2 and electrically conductivity σ_2 . A magnetic field is of constant strength

B_0 is imposed in the y -direction perpendicular to the channel. The flow of both phases is governed by a constant applied pressure gradient. Under these assumptions, the governing equations of momentum and energy for both phases are given as:

– for Phase I

$$\mu_1 \frac{d^2 u_1}{dy^2} + \rho_1 g \beta_1 \sin \phi (T_1 - T_{w_2}) = \frac{\partial p}{\partial x} \quad (1)$$

$$\frac{dT_1}{dy^2} + \frac{\mu_1}{K_1} \left(\frac{du_1}{dy} \right) = 0 \quad (2)$$

– for Phase II

$$\mu_2 \frac{d^2 u_2}{dy^2} + \rho_2 g \beta_2 \sin \phi (T_2 - T_{w_2}) + \sigma_2 B_0^2 u_2 = \frac{\partial p}{\partial x} \quad (3)$$

$$\frac{dT_2}{dy^2} + \frac{\mu_2}{K_2} \left(\frac{du_2}{dy} \right)^2 + \frac{\sigma_2 B_0^2 u_2^2}{K_2} = 0 \quad (4)$$

where u_1 and u_2 are the x -component of the velocities, T_1 and T_2 the temperatures, β_1 and β_2 the coefficient of thermal expansion for phase I and phase II, respectively, g is the acceleration due to gravity. Referring to fig. 1, the boundary and interface conditions on velocity are:

$$u_1 = -b_1 \frac{du_1}{dy} \quad \text{at} \quad y = h_1, \quad u_2 = b_2 \frac{du_2}{dy} \quad \text{at} \quad y = h_2 \quad (5)$$

$$u_1(0) = u_2(0), \quad \mu_1 \frac{du_1}{dy} = \mu_2 \frac{du_2}{dy} \quad \text{at} \quad y = 0$$

where b_1 and b_2 are the velocity slip lengths. Since the walls are maintained at constant different temperatures T_{w_1} and T_{w_2} at $y = h_1$ and $y = -h_2$, respectively, the boundary conditions on T_1 and T_2 are given by:

$$T_1(h_1) = T_{w_1} - d_1 \frac{dT_1}{dy}, \quad T_2(-h_2) = T_{w_2} + d_2 \frac{dT_2}{dy} \quad (6)$$

$$T_1(0) = T_2(0), \quad K_1 \frac{dT_1}{dy} = K_2 \frac{dT_2}{dy} \quad \text{at} \quad y = 0$$

where d_1 and d_2 are the thermal slip constants. Introducing the following dimensionless variables and parameters:

$$\begin{aligned} u_1^* &= u_1/\bar{u}_1, \quad u_2^* = u_2/\bar{u}_1, \quad y_1^* = y_1/h_1, \quad y_2^* = y_2/h_2, \quad \theta = (T - T_{w_2})/(T_{w_1} - T_{w_2}), \\ m &= \mu_1/\mu_2, \quad k = K_1/K_2, \quad h = h_2/h_1, \quad n = \rho_2/\rho_1, \quad b = \beta_2/\beta_1, \\ \text{Gr} &= g \beta_1 h_1^3 (T_{w_1} - T_{w_2})/\nu_1^2, \quad \text{M} = B_0 h_2 \sqrt{\sigma_2/\mu_2}, \quad \text{Pr} = \mu_1 C_p / K_1, \\ P &= (h_1^2/\mu_1 \bar{u}_1) (\partial p/\partial x), \quad \text{Re} = \bar{u}_1 h_1/\nu_1, \quad \text{Ec} = \bar{u}_1^2 / C_p (T_{w_1} - T_{w_2}) \end{aligned} \quad (7)$$

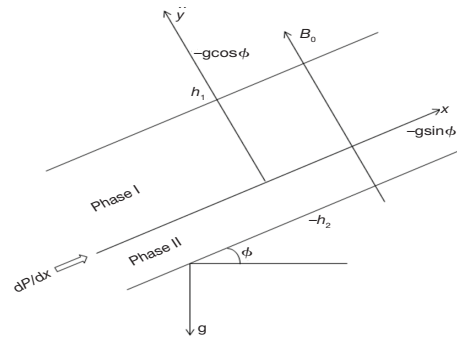


Figure 1. Physical configuration and co-ordinate system

Here Gr is the Grashof number, Ec the Eckert number, Pr the Prandtl number, Re the Reynolds number, M the Hartmann number, P the non-dimensional pressure gradient, and \bar{u}_1 the average velocity.

In new variables our problem takes the following form:

– Phase I

$$\frac{d^2 u_1}{dy^2} + \frac{\text{Gr} \sin \phi}{\text{Re}} \theta_1 = P \quad (8)$$

$$\frac{d^2 \theta_1}{dy^2} + \text{PrEc} \left(\frac{du_1}{dy} \right)^2 = 0 \quad (9)$$

– Phase II

$$\frac{d^2 u_2}{dy^2} + \frac{\text{Gr}}{\text{Re}} b m n h^2 \sin \phi \theta_2 - M^2 u_2 = m h^2 P \quad (10)$$

$$\frac{d^2 \theta_2}{dy^2} + \text{PrEc} \left(\frac{k}{m} \right) \left(\frac{du_2}{dy} \right)^2 + \left(\frac{M^2}{m} \right) \text{PrEc} k u_2^2 = 0 \quad (11)$$

The asterisks have been dropped with the understanding that all the quantities are now dimensionless. The velocity, temperature and interface boundary conditions (5) and (6) in non-dimensional form are:

$$u_1 = -\gamma_1 \frac{du_1}{dy}, \quad \text{at } y = 1, \quad u_2 = \gamma_2 \frac{du_2}{dy}, \quad \text{at } y = -1, \quad (12)$$

$$u_1(0) = u_2(0), \quad \frac{du_1}{dy} = \left(\frac{1}{mh} \right) \frac{du_2}{dy} \quad \text{at } y = 0$$

$$\theta_1(1) = 1 - \lambda_1 \frac{d\theta_1}{dy}, \quad \theta_2(-1) = \lambda_2 \frac{d\theta_1}{dy}, \quad (13)$$

$$\theta_1(0) = \theta_2(0), \quad \frac{d\theta_1}{dy} = \left(\frac{1}{kh} \right) \frac{d\theta_2}{dy} \quad \text{at } y = 0$$

where $\gamma_1 = b_1/h_1$, and $\gamma_2 = b_2/h_2$ are the velocity slip parameters and $\lambda_1 = d_1/h_1$, $\lambda_2 = d_2/h_2$ are the thermal slip parameters.

Perturbation solution

The governing equations of momentum (8) and (10), along with the energy equations (9) and (11) are solved subject to the boundary and interface conditions (12) and (13) for the velocity and temperature distributions. The equations are coupled and nonlinear because of the inclusion of the dissipation terms in energy equation. Following Malashetty and Umavathi [19], we can approximate the solution of (8)-(11) subject to the boundary conditions (12) and (13) valid for small values of $\varepsilon \ll 1$ ($= \text{PrEc}$) because in most of the practical problems the Eckert number is very small and is of order 10^{-5} (see [19]). The solutions are assumed in the form:

$$(u_i, \theta_i) = \sum_{j=0}^{\infty} (u_{ij}, \theta_{ij}) \varepsilon^j \quad (14)$$

where u_j, θ_j are the perturbations in u and θ , respectively.

Using eqs. (14) in eqs. (8-13) after comparing the like powers of ε and neglecting the terms of $O(\varepsilon^2)$, we obtain the following equations.

For phase I

– *Zeroth-order equations*

$$\frac{d^2 u_{10}}{dy^2} + \frac{\text{Gr} \sin \phi}{\text{Re}} \theta_{10} = P \quad (15)$$

$$\frac{d^2 \theta_{10}}{dy^2} = 0 \quad (16)$$

– *First-order equations*

$$\frac{d^2 u_{11}}{dy^2} + \frac{\text{Gr} \sin \phi}{\text{Re}} \theta_{11} = 0 \quad (17)$$

$$\frac{d^2 \theta_{11}}{dy^2} + \left(\frac{du_{10}}{dy} \right)^2 = 0 \quad (18)$$

For phase II

– *Zeroth-order equations*

$$\frac{d^2 u_{20}}{dy^2} + \frac{\text{Gr}}{\text{Re}} \sin \phi b m n h^2 \theta_{20} - M^2 u_{20} = m h^2 P \quad (19)$$

$$\frac{d^2 \theta_{20}}{dy^2} = 0 \quad (20)$$

– *First-order equations*

$$\frac{d^2 u_{21}}{dy^2} + \frac{\text{Gr}}{\text{Re}} \sin \phi b m n h^2 \theta_{21} - M^2 u_{21} = 0 \quad (21)$$

$$\frac{d^2 \theta_{21}}{dy^2} + \left(\frac{k}{m} \right) \left(\frac{du_{20}}{dy} \right)^2 + M^2 \left(\frac{k}{m} \right) u_{20}^2 = 0 \quad (22)$$

The corresponding boundary conditions (12) and (13) reduces to:

$$u_{10} = -\gamma_1 \frac{du_{10}}{dy} \quad \text{at } y=1 \quad u_{20} = \gamma_2 \frac{du_{20}}{dy}, \quad \text{at } y=-1 \quad (23)$$

$$u_{10}(0) = u_{20}(0), \quad \frac{du_{10}}{dy} = \left(\frac{1}{mh} \right) \frac{du_{20}}{dy} \quad \text{at } y=0$$

$$\theta_{10}(1) = 1 - \lambda_1 \frac{d\theta_{10}}{dy}, \quad \theta_{20}(-1) = \lambda_2 \frac{d\theta_{20}}{dy} \quad (24)$$

$$\theta_{10}(0) = \theta_{20}(0), \quad \frac{d\theta_{10}}{dy} = \left(\frac{1}{kh}\right) \frac{d\theta_{20}}{dy} \quad \text{at } y = 0$$

$$u_{11} = -\gamma_1 \frac{du_{11}}{dy}, \quad \text{at } y = 1, \quad u_{21} = \gamma_2 \frac{du_{21}}{dy}, \quad \text{at } y = -1 \quad (25)$$

$$u_{11}(0) = u_{21}(0), \quad \frac{du_{11}}{dy} = \left(\frac{1}{mh}\right) \frac{du_{21}}{dy} \quad \text{at } y = 0$$

and

$$\theta_{11}(1) = -\lambda_1 \frac{d\theta_{11}}{dy}, \quad \theta_{21}(-1) = \lambda_2 \frac{d\theta_{21}}{dy} \quad (26)$$

$$\theta_{11}(0) = \theta_{21}(0), \quad \frac{d\theta_{11}}{dy} = \left(\frac{1}{kh}\right) \frac{d\theta_{21}}{dy} \quad \text{at } y = 0$$

Solution of eqs. (16) and (20), and (15) and (19) using boundary conditions (23) and (24) are:

$$\theta_{10} = \frac{y + kh(1 + \lambda_2)}{1 + \lambda_1 + kh(1 + \lambda_2)} \quad (27)$$

$$\theta_{20} = \frac{kh(1 + \lambda_2 + y)}{1 + \lambda_1 + kh(1 + \lambda_2)} \quad (28)$$

$$u_{10} = a_1 y^3 + a_2 y^2 + c_1 y + c_2 \quad (29)$$

$$u_{20} = d_1 \cosh My + d_2 \sinh My + f_1 + f_2 y \quad (30)$$

where

$$G = \frac{Gr}{Re} \sin \phi, \quad a_1 = -\left\{ \frac{G}{6[1 + \lambda_1 + kh(1 + \lambda_2)]} \right\}, \quad a_2 = \frac{1}{2} \left\{ P - \frac{khG(1 + \lambda_2)}{[1 + \lambda_1 + kh(1 + \lambda_2)]} \right\},$$

$$f_1 = -\frac{mh^2 p}{M^2} + f_2(1 + \lambda_2), \quad f_2 = \frac{kGbmnh^3}{M^2 [1 + \lambda_1 + kh(1 + \lambda_2)]},$$

$$d_2 = \frac{mh}{\left[\frac{mh \left(\frac{\sinh M +}{+\gamma_2 M \cosh M} \right) +}{+M(1 + \gamma_1) \left(\frac{\cosh M +}{+\gamma_2 M \sinh M} \right)} \right]} \left\{ \frac{f_1 - f_2(1 + \gamma_2) -}{\left[\frac{f_1 + a_1(1 + 3\gamma_1) +}{+\frac{f_2(1 + \gamma_1)}{mh} + a_2(1 + 2\gamma_1)} \right]} \left(\frac{\cosh M +}{+\gamma_2 M \sinh M} \right) \right\}$$

$$d_1 = \frac{(\sinh M + \gamma_2 M \cosh M)d_2 - f_1 + f_2(1 + \gamma_2)}{\cosh M + \gamma_2 M \sinh M}, \quad c_1 = \frac{d_2 M + f}{mh}, \quad c_2 = d_1 + f_1$$

Similarly, solution of eqs. (18), (22) and (17), (21) using boundary conditions (25) and (26) are:

$$\theta_{11} = g_1 y^6 + g_2 y^5 + g_3 y^4 + g_4 y^3 + g_5 y^2 + e_1 y + e_2 \quad (31)$$

$$\theta_{21} = g_6 \cosh 2My + g_7 \sinh 2My + g_8 y \cosh 2My + g_9 y \sinh 2My + g_{10} \cosh My + g_{11} \sinh My + g_{12} y^4 + g_{13} y^3 + g_{14} y^2 + e_3 y + e_4 \quad (32)$$

$$u_{11} = r_1 y^8 + r_2 y^7 + r_3 y^6 + r_4 y^5 + r_5 y^4 + r_6 y^3 + r_7 y^2 + j_1 y + j_2 \quad (33)$$

$$u_{21} = j_3 \cosh My + j_4 \sinh My + p_1 \cosh 2My + p_2 \sinh 2My + p_3 y^2 \cosh My + p_4 y^2 \sinh My + p_5 y \cosh My + p_6 y \sinh My + p_7 \cosh My + p_8 \sinh My + p_9 y^4 + p_{10} y^3 + p_{11} y^2 + p_{12} y + p_{13} \quad (34)$$

where

$$g_1 = -\frac{3a_1^2}{10}, \quad g_2 = -\frac{3a_1 a_2}{5}, \quad g_3 = -\frac{2a_2^2 + 3a_1 c_1}{6}, \quad g_4 = -\frac{2a_2 c_1}{3}, \quad g_5 = -\frac{c_1^2}{2},$$

$$g_6 = \frac{t_1}{4M^2}, \quad g_7 = \frac{t_2}{4M^2}, \quad g_8 = \frac{t_3}{M^2}, \quad g_9 = \frac{t_4}{M^2}, \quad g_{10} = \frac{t_5}{M^2} - \frac{2t_4}{M^3},$$

$$g_{11} = \frac{t_6}{M^2} - \frac{2t_3}{M^3}, \quad g_{12} = \frac{t_7}{12}, \quad g_{13} = \frac{t_8}{6}, \quad g_{14} = \frac{t_9}{2},$$

$$t_1 = -\frac{kM^2}{m}(d_1^2 + d_2^2), \quad t_2 = -\frac{2kd_1 d_2 M^2}{m}, \quad t_3 = -\frac{2kd_1 f_2 M^2}{m}, \quad t_4 = -\frac{2kd_2 f_2 M^2}{m},$$

$$t_5 = -\frac{2kM}{m}(d_1 f_1 M + d_2 f_2), \quad t_6 = -\frac{2kM^2}{m}\left(\frac{d_1 f_2}{M} + d_2 f_1\right), \quad t_7 = -\frac{kM^2 f_2^2}{m},$$

$$t_8 = -\frac{2kf_1 f_2 M^2}{m}, \quad t_9 = -\frac{k}{m}(f_1^2 M^2 + f_2^2),$$

$$e_1 = \frac{1}{\begin{bmatrix} 1 + \lambda_1 + \\ +kh(1 + \lambda_2) \end{bmatrix}} \left\{ \begin{aligned} & \left[(g_6 - 2\lambda_2 g_7 M) \cosh 2M - (g_7 - 2\lambda_2 g_6 M) \sinh 2M + 2M g_7 (1 + \lambda_2) + \right. \\ & \quad \left. + g_8 (1 + \lambda_2) + M g_{11} (1 + \lambda_2) + g_{12} (1 + 4\lambda_2) - g_{13} (1 + 3\lambda_2) + \right. \\ & \quad \left. + g_{14} (1 + 2\lambda_2) - \left[(1 + 6\lambda_1) g_1 + (1 + 5\lambda_1) g_2 + (1 + 4\lambda_1) g_3 + \right. \right. \\ & \quad \left. \left. + (1 + 3\lambda_1) g_4 + (1 + 2\lambda_1) g_5 + g_6 + g_{10} \right] + \right. \\ & \quad \left. + \left[g_{10} - g_8 (1 + \lambda_2) + \right] \cosh M + \left[g_9 (1 + \lambda_2) - g_{11} + \right. \right. \\ & \quad \left. \left. + \lambda_2 M (g_9 - g_{11}) \right] \sinh M \right\}, \end{aligned}$$

$$e_2 = -\left((1 + 6\lambda_1) g_1 + (1 + 5\lambda_1) g_2 + (1 + 4\lambda_1) g_3 + (1 + 3\lambda_1) g_4 + (1 + 2\lambda_1) g_5 + (1 + \lambda_1) e_1 \right),$$

$$e_3 = k h e_1 - 2M g_7 - g_8 - M g_{11}, \quad e_4 = e_2 - g_6 - g_{10},$$

$$r_1 = -\frac{Gg_1}{56}, r_2 = -\frac{Gg_2}{42}, r_3 = -\frac{Gg_3}{30}, r_4 = -\frac{Gg_4}{20}, r_5 = -\frac{Gg_5}{12}, r_6 = -\frac{Ge_1}{6}, r_7 = -\frac{Ge_2}{2},$$

$$\tilde{p} = bmnGh^2, p_1 = -\frac{\tilde{p}g_6}{3M^2}, p_2 = -\frac{\tilde{p}g_7}{3M^2}, p_3 = -\frac{\tilde{p}g_9}{4M}, p_4 = -\frac{\tilde{p}g_8}{4M},$$

$$p_5 = -\tilde{p}\left(\frac{g_{11}}{2M} - \frac{g_8}{4M^2}\right), p_6 = -\tilde{p}\left(\frac{g_{10}}{2M} - \frac{g_9}{4M^2}\right), p_7 = -\tilde{p}\left(\frac{g_9}{8M^3} - \frac{g_{10}}{4M^2}\right),$$

$$p_8 = -\tilde{p}\left(\frac{g_8}{8M^3} - \frac{g_{11}}{4M^2}\right), p_9 = \frac{\tilde{p}g_{12}}{M^2}, p_{10} = \frac{\tilde{p}g_{13}}{M^2}, p_{11} = \frac{\tilde{p}}{M^2}\left(g_{14} + \frac{12g_{12}}{M^2}\right),$$

$$p_{12} = \frac{\tilde{p}}{M^2}\left(e_3 + \frac{6g_{13}}{M^2}\right), p_{13} = \frac{\tilde{p}}{M^2}\left(e_4 + \frac{2g_{14}}{M^2} + \frac{24g_{12}}{M^4}\right),$$

$$j_1 = \frac{M}{\left[\begin{array}{l} M \cosh M \left(\frac{1+\gamma_1}{+\gamma_2 mh} \right) + \\ + \left[\frac{mh}{+(1+\gamma_1)\gamma_2 M^2} \right] \sinh M \end{array} \right]} \left(\begin{array}{l} \cosh 2M \left(\frac{p_1}{-2\gamma_2 p_2 M} \right) - \sinh 2M \left(\frac{p_2}{-2\gamma_2 p_1 M} \right) + \\ + p_9(1+4\gamma_2) - p_{10}(1+3\gamma_2) + \\ + p_{11}(1+2\gamma_2) - p_{12}(1+\gamma_2) + p_{13} + \\ \left\{ \begin{array}{l} -p_1 + 2M p_2 \gamma_2 + p_3(1+2\gamma_2) - \\ -M p_4 \gamma_2 - p_5 + M p_6 \gamma_2 + \\ + p_{12} \gamma_2 - p_{13} - \end{array} \right\} \\ + \cosh M \left\{ \begin{array}{l} \left[\begin{array}{l} r_1(1+8\gamma_1) + r_2(1+7\gamma_1) + \\ + r_3(1+6\gamma_1) + r_4(1+5\gamma_1) + \\ + r_5(1+4\gamma_1) + r_6(1+3\gamma_1) + \\ + r_7(1+2\gamma_1) \end{array} \right] \end{array} \right\} + \\ + \sinh M \left\{ \begin{array}{l} -M p_1 \gamma_2 + 2p_2 + M p_3 \gamma_2 - p_4(1+2\gamma_2) + \\ + (p_{12} + p_5)/M - M p_5 \gamma_2 + \\ + p_6(1+\gamma_2) - M p_{13} \gamma_2 - \\ -M \gamma_2 \left[\begin{array}{l} r_1(1+8\gamma_1) + r_2(1+7\gamma_1) + \\ + r_3(1+6\gamma_1) + r_4(1+5\gamma_1) + \\ + r_5(1+4\gamma_1) + r_6(1+3\gamma_1) + \\ + r_7(1+2\gamma_1) \end{array} \right] \end{array} \right\} \end{array} \right),$$

$$j_2 = -j_1(1+\gamma_1) - \left[\frac{r_1(1+8\gamma_1) + r_2(1+7\gamma_1) + r_3(1+6\gamma_1) +}{+r_4(1+5\gamma_1) + r_5(1+4\gamma_1) + r_6(1+3\gamma_1) + r_7(1+2\gamma_1)} \right],$$

$$j_3 = j_2 - p_1 - p_7 - p_{13}, j_4 = \frac{1}{M}(j_1 mh - 2M p_2 - p_5 - M p_8 - p_{12})$$

Results and discussion

The problem of hydromagnetic two-phase flow and heat transfer in an inclined channel in the presence of velocity/thermal slip conditions is studied analytically. The approximate analytical solutions of coupled nonlinear equations (9)-(11) with boundary conditions (12) and (13) are obtained using regular perturbation method for small values of ε upto of order one. Throughout the computations the following parameters $Gr = 5$, $m = 0.5$, $k = 1$, $h = 1$, $n = 1.5$, $\phi = \pi/6$, $Re = 5$, $\varepsilon = 0.05$, $Pr = 5$, $b = 1$ are fixed in this study. In order to analyze the effects of various involving parameters, for example, velocity slip parameters γ_1 , γ_2 , thermal slip parameters λ_1 , λ_2 and magnetic parameter M on the fluid velocity and temperature profile, figs. 2-8 are plotted. It is worth mentioned to note that the present results are compared with the existing results of [19] without velocity and thermal slip conditions, and we found them to be in excellent agreement. Therefore, it may be concluded that the development analytical solutions for the velocity and temperature profiles can be used with great confidence in this paper.

Figure 2 shows the effect of magnetic field on the fluid velocity $u(y)$ in the case of no-slip condition $\lambda_1 = \lambda_2 = 0$ (dashed lines) and with velocity slip condition $\lambda_1 = \lambda_2 = 0.1$, respectively. It is found from this fig. that by increasing the values of magnetic field, the fluid velocity is decreased in both cases. This is because for the investigated problem, the application of the magnetic field normal to the flow direction has the tendency to slow down the movement of the fluids in the channel because it gives rise to the resistance force called the Lorentz force which acts opposite to the flow direction. It is also evident from this fig. that the magnitude of the fluid velocity is smaller in the phase II as compared with the velocity distribution of phase I, because we have considered the electrically conducting fluid. Figure 3 shows the changes in the fluid velocity $u(y)$ of phase I for several values of velocity slip parameter γ_1 at the upper wall of channel, by keeping the fixed values of γ_2 at the lower wall of channel. We observed from this fig. that the fluid velocity is increased with an increase in the velocity slip parameter γ_1 . But the increment in the magnitude of fluid velocity $u(y)$ is larger in phase I. The influence of the velocity slip parameter γ_2 at the lower wall of the channel on the fluid velocity $u(y)$ of phase

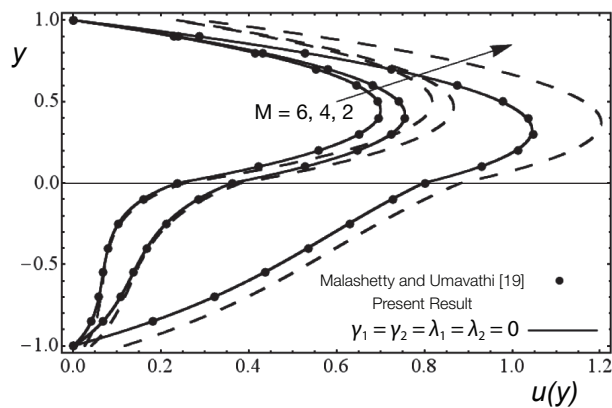


Figure 2. Variation of velocity profiles for several value of M ; Solid lines for no-slip and dash lines for slip $\gamma_1 = \gamma_2 = 0.1$

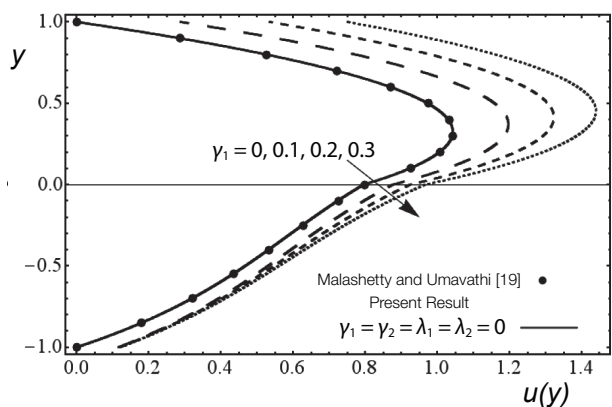


Figure 3. Variation of velocity profiles for several value of γ_1 with $\gamma_2 = \lambda_1 = \lambda_2 = 0.1$

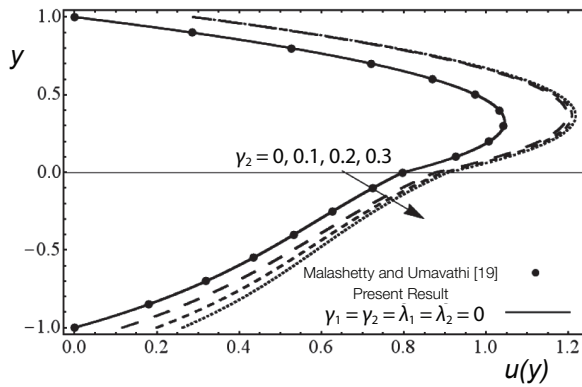


Figure 4. Variation of velocity profiles for several value of γ_2 with $\gamma_1 = \lambda_1 = \lambda_2 = 0.1$

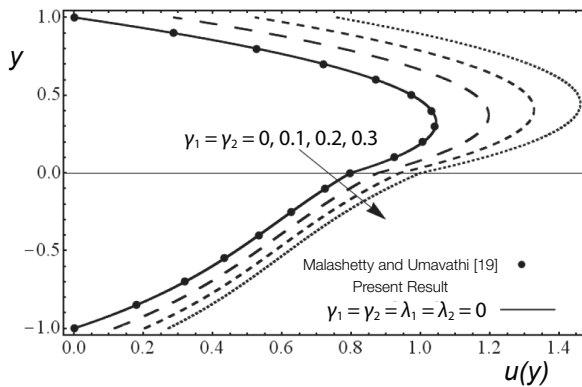


Figure 5. Variation of velocity profiles for several value of γ_1 and γ_2 with $\lambda_1 = \lambda_2 = 0.1$

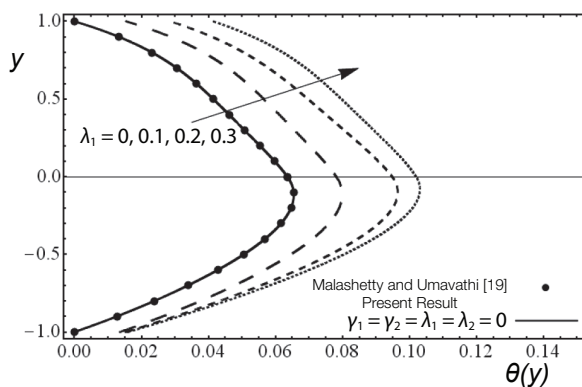


Figure 6. Variation of temperature profiles for several value of λ_1 with $\gamma_1 = \gamma_2 = \lambda_2 = 0.1$

II is given in fig. 4 by keeping the values of γ_1 fixed. From this fig., we can see the similar behavior of γ_2 on the velocity distribution as in the case of γ_1 . Figure 5 depicts the variation of both velocity slip parameters of γ_1 and γ_2 on the fluid velocity $u(y)$ in both phases of the channel at the same time. It can be seen from this fig. that the smaller the height of the lower phase, compared to the upper phase, the larger the flow field, and this is because of a magnetic field is applied in the lower phase of the channel. It is also found that the velocity is increased in two phases by increasing the values of velocity slip parameters γ_1 and γ_2 .

The effect of the thermal slip parameters λ_1 at the upper wall of the channel on the temperature profile $\theta(y)$ is shown in fig. 6, when the values of thermal slip parameter λ_2 at the lower wall of the channel is fixed. From this fig., it is evident that the change in the temperature of the fluid in the upper phase is larger, compared to the temperature in the lower phase by increasing the values of λ_1 . The temperature distribution $\theta(y)$ is increased by increasing the values of the thermal slip parameter λ_1 in both the phases. Figure 7 elucidates the influence of the phase thermal slip parameter λ_2 at the lower wall of channel on the temperature profile $\theta(y)$ of the phase II when λ_1 is fixed. From this fig., we can observe that the variation in the temperature distribution $\theta(y)$ is larger in the lower phase as compared with the upper phase. It is further noted that the temperature distribution $\theta(y)$ increases with an increase in the thermal slip parameter λ_2 in both phases, and the temperature is overshoot near the interface of the channel in the lower phase due to the magnetic field effect compared with the upper phase, because the magnetic field enhances to increase the temperature distribution of the fluid.

Figure 8 gives the variation of both thermal slip parameters λ_1 and λ_2 on the temperature profile $\theta(y)$ in the both lower and upper phases. It is noted from this fig. that the tem-

perature is increased by increasing the values of both λ_1 and λ_2 , respectively. It is further observed that the change in the temperature field is larger in the lower phase compared to the change in temperature in the upper phase as we increase the values of thermal slip parameters λ_1 and λ_2 .

Concluding remarks

In this paper, the problem of MHD two-phase fluid flow and heat transfer in an inclined channel with velocity/thermal slip conditions is studied. The obtained flow equations are coupled and non-linear. An approximate solution of these nonlinear equations is obtained using regular perturbation method with the ε as the perturbation parameter. From this study, the following observations have been made as follows:

- The fluid velocity is decreased with an increase in magnetic field in both cases of no-slip condition and with slip condition.
- The fluid velocity increases as we increase the values of the velocity slip parameters γ_1 and γ_2 .
- The temperature profile increases by increasing the thermal slip parameters λ_1 and λ_2 .
- The results of Malashetty and Umavathi [19] can be recovered by taking $\gamma_1 = \gamma_2 = \lambda_1 = \lambda_2 = 0$ as a special case.

Nomenclature

B_0	– magnetic field, [T]
b	– ratio of coefficient of thermal expansion of phases, $[= \beta_2/\beta_1]$, [-]
b_i	– velocity slip lengths
c_p	– specific heat at constant pressure, $[Jkg^{-1}K^{-1}]$
d_i	– thermal slip constants, [-]
Ec	– Eckert number, $[= \bar{u}_1^2/C_p(T_{w_1} - T_{w_2})]$
g	– acceleration due to gravity, $[LT^{-2}]$
h	– ratio of heights of phases, $[= h_2/h_1]$, [-]
h_i	– heights of the phases, [L]
K_i	– thermal conductivity of fluids of phases, $[Wm^{-1}K^{-1}]$
k	– ratio of coefficient of thermal conductivity of fluids of phases, $[= K_1/K_2]$, [-]
M	– Hartman number, $[= B_0 h_2 (\sigma/\mu_2)^{-1/2}]$
m	– ratio of coefficient of dynamic viscosity of phases, $[= \mu_1/\mu_2]$, [-]
n	– ratio of densities of fluids of phases, $[= \rho_2/\rho_1]$, [-]

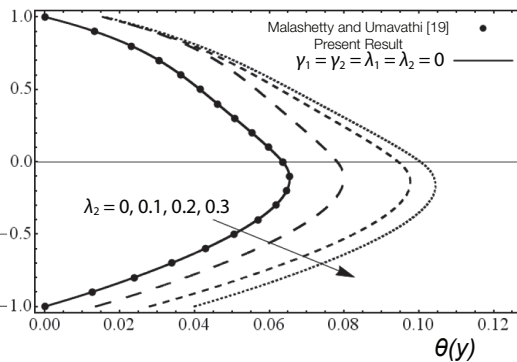


Figure 7. Variation of temperature profiles for several value of λ_2 with $\gamma_1 = \gamma_2 = \lambda_1 = 0.1$

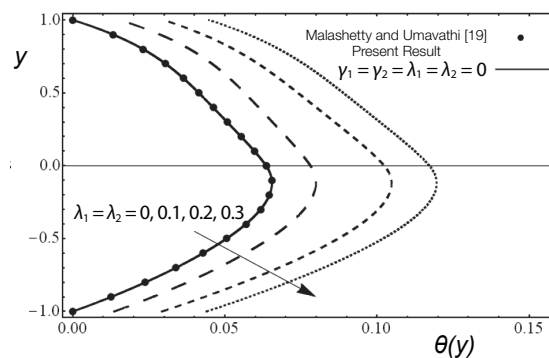


Figure 8. Variation of temperature profiles for several value of λ_1 and λ_2 with $\gamma_1 = \gamma_2 = 0.1$

P	– non-dimensional pressure gradient, [-]
Pr	– Prandtl number, [-]
Re	– Reynolds number, [-]
T_i	– temperature of the fluid of phases, [K]
T_{w_i}	– temperature at the surfaces of the plates of the phases, [K]
u_i	– velocity of phases x-direction, $[ms^{-1}]$
\bar{u}_1	– average velocity, $[ms^{-1}]$
u_i^*	– dimensionless velocities of phases, $[ms^{-1}]$
x, y, z	– spatial co-ordinates, [m]
y_i^*	– dimensionless variables

Greek symbols

β_i	– coefficient of thermal expansion, $[K^{-1}]$
γ_i	– velocity slip parameters of the phases, [-]
ε	– $[= Pr Ec]$
θ_i	– dimensionless temperatures of the phases, $[=(T - T_{w_2})/(T_{w_1} - T_{w_2})]$, [-]

λ_i	– thermal slip parameters of phases, [-]	ϕ	– angle of the channel with horizontal, [rad]
μ_i	– dynamic viscosities of the phases, [kgm ⁻¹ s ⁻¹]	Subscripts	
ν_i	– kinematic viscosities of the phases, [m ² s ⁻¹]	i	– for phase I $i = 1$, for phase II $i = 2$
ρ_i	– fluid densities of the phases, [kgm ⁻³]	w	– surface conditions
σ_2	– electrical conductivity of fluid of phase II, [sm ⁻¹]		

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