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CALCULATION OF $t_{8/5}$ BY RESPONSE SURFACE METHODOLOGY FOR ELECTRIC ARC WELDING APPLICATIONS

by

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One of the greatest difficulties traditionally found in stainless steel constructions has been the execution of welding parts in them. At the present time, the available technology allows us to use arc welding processes for that application without any disadvantage. Response surface methodology is used to optimise a process in which the variables that take part in it are not related to each other by a mathematical law. Therefore, an empiric model must be formulated. With this methodology the optimisation of one selected variable may be done. In this work, the cooling time that takes place from 800 to 500ºC, $t_{8/5}$, after TIG welding operation, is modelled by the response surface method. The arc power, the welding velocity and the thermal efficiency factor are considered as the variables that have influence on the $t_{8/5}$ value. Different cooling times, $t_{8/5}$, for different combinations of values for the variables are previously determined by a numerical method. The input values for the variables have been experimentally established. The results indicate that response surface methodology may be considered as a valid technique for these purposes.

Key words: *cooling time $t_{8/5}$ response surface methodology (RSM), welding, thermal modelling, heat affected zone (HAZ).*

Introduction

The cooling time between 800 and 500 ºC ($t_{8/5}$) predicts the influence that the heat source has on a welded joint from the centre of the bead towards the base metal. Variations in the $t_{8/5}$ can be appreciated if it is determined empirically or also if it is modelled analytically [1]. A numerical method employed in the modelling of the cooling time is the finite difference method, that can be applied for three-dimension analysis by means of the Douglas-Gunn method to obtain the $t_{8/5}$when the heat source is moving [2,3].

The experimental calculation of $t_{8/5}$ in structures that are susceptible to be repaired [4] confirms that the hardness obtained in the heat affected zone (HAZ) is similar to the corresponding $t_{8/5}$ [5, 6]. The time $t_{8/5}$ can also be determined analytically as a function of the equivalent carbon [7], that it is an index of the toughness and hardness of the steels. The welded joints carried out by resistance spot welding (RSW) and shielded metal arc welding (SMAW) cause thermal deformations; these can be modelled [8,9]and such deformations turn

out to be proportional to the temperatures reached in each process. They are also dependent on the cooling time, which causes a variation in the hardness of the HAZ. A consequence of the appearance of the deformations is the residual stresses in welded joints simulated for finite elements in a stainless steel pipe [10].

In this research work, response surface methodology (RSM) is used as a tool for the modelling of the $t_{8/5}$. RSM belongs to a group of techniques known as design of experiments (DOE) and which are used to calculate the experiments necessary for a trial according to the variables defined. Some research work exists which optimises the welding parameters in a GMAW process [11], determining the critical variables. Different analytical methods can be employed in the optimisation of the cooling time and in particular, RSM allows us to optimise a response function subjected to different independent variables.

The principal objective of any experimental design applied to the optimisation of a process is to study the influence of the different operating variables or experimental factors, both in relation to the variability of the responses as well as to their central tendency, always carrying out the minimum number possible of experiments. It is a matter of establishing the mathematical method which relates them. To do so, a sequential strategy is carried out with the maximum information being obtained in different phases with a minimum of resources [12].

Analytical and numerical methods can be used to solve the differential equations which define the thermal diffusion process to calculate the field of temperatures in a welded joint. The advantage of analytical methods over experimental methods is that it is possible to adjust the welding parameters aimed at avoiding the undesired effects in the HAZ, using simple approaches. In the current work, a mathematical method based on the finite differences is applied to know the field of temperatures in arc welding using the GMAW procedure of an AISI 304 austenitic stainless steel. The parameter $t_{8/5}$ is obtained from the model for different combinations of technological variables of the process and finally the expression of the behaviour of the $t_{8/5}$ is obtained as a function of the variable considered by means of the RSM application

Numerical method to calculate the temperature distribution of TIG welding

The numerical method that was used to calculate the temperature distribution of a TIG welding observes the phase transformation solid to liquid, determining the liquid fraction in each point of the bead. The procedure herein considered is established from a double ellipsoidal heat source[13-18]. The ellipsoidal non dimension heat source parameters u_a and u_c were defined from the welding variables according to eq. (1) and (2), where v_s is the welding rate and α is the thermal diffusivity; a_h and c_h are the ellipsoidal semi-axes. Mean values of a_h and c_h usually adopted for a TIG welding source are 0.32 mm and 0.64 mm, respectively.

$$u_a = \frac{v_s \cdot a_h}{2 \cdot \alpha \cdot \sqrt{6}} \tag{1}$$

$$u_c = \frac{v_s \cdot c_h}{2 \cdot \alpha \cdot \sqrt{6}} \tag{2}$$

The size of the heat and the melting area were defined according to the parameters obtained from the above equations. This work analyses the heat input corresponding to a welding of a stainless steel plate AISI 304 with the properties indicated in tab. 1 [19].

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Convective heat transfer and heat loss were neglected as an early approach. The process corresponds to a butt welded joint executed in natural position and the dimensions of the plate to be welded were 0.3 m long, 0.075 m width and 0.002 m thick.

Table 1. Physical properties of the stainless steel AISI 304

T_o	20	[°C]	Room temperature
k	25	[J m^{-1} s^{-1} K^{-1}]	Fourier conductivity
T_f	1400	[°C]	Fusion temperature
C_p	630	[J kg^{-1} K^{-1}]	Specific heat
ρ	7500	[kg m^{-3}]	Density
ΔH	272000	[J kg^{-1}]	Latent heat

Equation (3) corresponds to the heat expression for a two-dimensional flow. Equation (4) expresses the value of the sum of the input heat and the solid-liquid latent heat, Q, that is a function of the arc voltage, V, the arc current, I, the thermal efficiency, η, the density, ρ, and the latent heat, ΔH_x, according to the liquid fraction state, x, for the considered time.

$$\frac{\partial T}{\partial t} = \frac{k}{\rho \cdot c_p} \cdot \left(\frac{\partial^2 T}{\partial^2 x^2} + \frac{\partial^2 T}{\partial^2 y^2} \right) + \frac{Q}{\rho \cdot c_p} \tag{3}$$

$$Q = \eta I V - \rho \Delta H_x \tag{4}$$

The implemented numerical method solves eq. (3) taking into account adiabatic boundary conditions. Time evolution is established from the Crank-Nicolson planning and the partial derivatives are formulated with discretization by finite differences dividing the 2D geometry of the plate in Δx-side squares. The partial derivatives are calculated according to eq. (5) and (6). In those expressions, n represents the evolution of the time and the i,j subscripts indicate the direction in the plane for the element x.

$$\left(\frac{\partial T}{\partial x} \right)^n_j = \frac{1}{2} \left(\frac{T^n_{i+1,j} - T^n_{i-1,j}}{\Delta x} \right) \tag{5}$$

$$\left(\frac{\partial^2 T}{\partial x^2} \right)^n_{i,j} = \left(\frac{T_{i+1,j} + T_{i-1,j} - 2T_{i,j}}{\Delta x^2} \right) \tag{6}$$

Taking into account the mentioned above, the equation (3) turns into:

$$T^{n+1}_{i,j} = \frac{1}{1+4\alpha} \left(T^{n+1}_{i,j} (1-4\alpha) + \alpha \left(T^n_{i-l,j} + T^n_{i+l,j} + T^n_{i,j-1} + T^n_{i,j+1} + + T^{n+1}_{i-l,j} + T^{n+1}_{i+l,j} + T^{n+1}_{i,j-1} + T^{n+1}_{i,j+1} \right) - Q \right) \tag{7}$$

Thermal diffusivity can be calculated from the material physical and dimensional properties:

$$\alpha = \frac{k \Delta t}{4 \rho C_p \Delta x^2} \tag{8}$$

Based on the Crank-Nicolson model, a simulation program was made, [16]. Running the program, the distribution of temperatures for each welding established conditions were obtained, Fig. 1. The cooling time measurement into the range of temperatures 800-500°C was considered for a point located at distance of 1.5 mm from the axis of bead in all cases, Tab 2, [20].
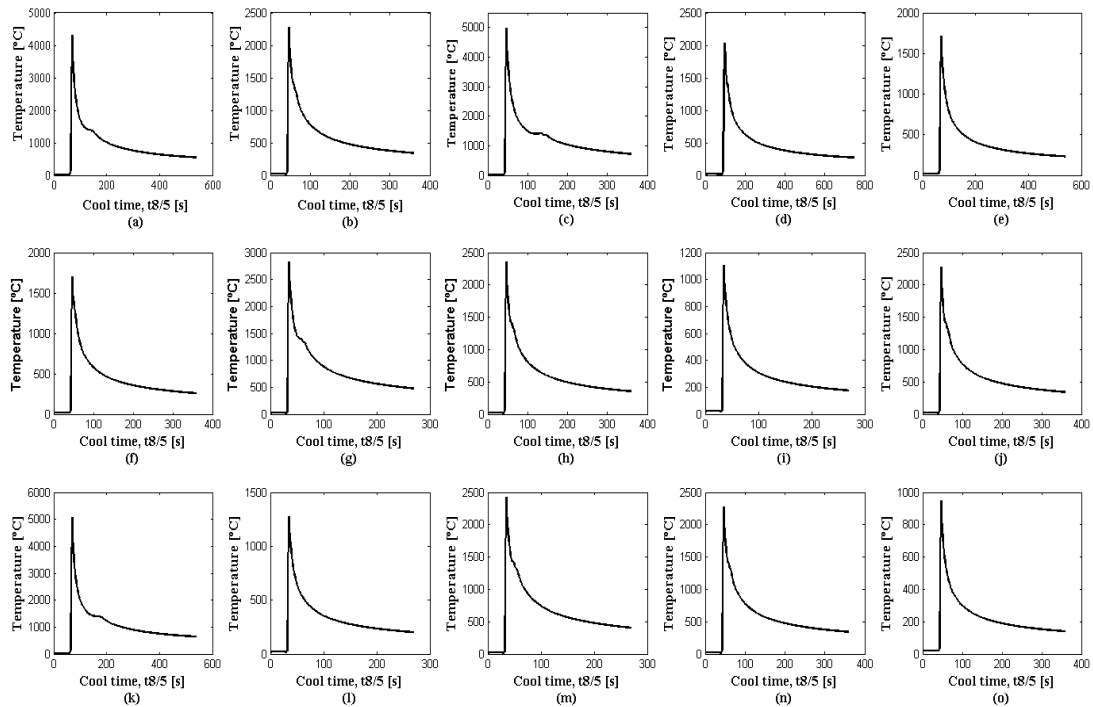
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Figure 1.Distribution of temperatures obtained by numerical modelling for the values of the corresponding variables for 15 simulations.

Table 2. Results obtained for the $t_{8/5}$ by numerical simulation[20]

	$t_{8/5}$[s]		$t_{8/5}$[s]		$t_{8/5}$[s]
Simulation 1	265.68	Simulation 2	86.41	Simulation 3	162.63
Simulation 4	108	Simulation 5	54	Simulation 6	48.96
Simulation 7	131.78	Simulation 8	93.61	Simulation 9	16.2
Simulation 10	86.41	Simulation 11	272.16	Simulation 12	24.84
Simulation 13	95.05	Simulation 14	86.41	Simulation 15	11.52

Optimization method

The RSM is applied together with a factorial experimental design. The focus consists in using the design of experiments to determine the variables which present a significant influence on the response of interest. Once those variables have been identified, an approximate estimation of the response surface is obtained by means of factorial models. This response surface is used as a guide to gradually vary the controllable factors which affect the response in order to improve its value. The response surface is given by a second order quadratic equation, where ε is the experimental error, $x_1, x_2, ..., x_k$ are the independent variables, and $\beta_0, \beta_1, ..., \beta_k$ are the regression parameters of the surface estimated from the experimental data [21] indicated in equation (9) in a compact manner.

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{j=2}^{k} \sum_{i=1}^{k} \beta_{ij} x_i x_j + \varepsilon \qquad (9)$$

The factors in this work considered as variables of the welding process with an influence on the variable of response are: the power of the electric arc, P; the welding feed, v_s; and the thermal efficiency factor of the welding process, η. The response of the method will be affected if the levels of the factors are changed. The variable of response is the cooling time $t_{8/5}$ and the objective pursued is to optimise this value.

The response function can be represented with a polynomial equation, which in the case studied is fitted to a second degree polynomial. In accordance with the RSM, the model obtained is the indicated in eq.(10), that it has been developed for the indicated variables.

$$t_{8/5} = \beta_o + \beta_1 P + \beta_2 v_s + \beta_3 \eta + \beta_4 P v_s + \beta_5 P \eta + \beta_6 v_s \eta + \beta_7 P^2 + \beta_8 v_s^2 + \beta_9 \eta^2 + \varepsilon \qquad (10)$$

The relationship between the factors and levels considered is shown in Tab3.

Table 3.Levels of the welding process variables

Factors	Levels		
	-1	0	1
P [W]	910.00	1620.00	2530.00
V$_s$[m/s]	3.30E-03	3.73E-03	4.16E-03
η	0.4	0.6	0.8

With the combination of the values of variables indicated in Tab 1 a matrix of experiments has been made, taking into account the codified values of the variables according to the levels (-1, 0,1). These levels correspond to the minimum, mean, and maximum values of P, v_s, η, that have been selected for the experimental TIG welding of a stainless steel plate with a thickness lower than 3 mm, [20]. Therefore, the input values for the simulation of the thermal map are real values used in welding procedures for metallic structures. The experiments selected for obtaining the matrix of experiments are shown in Tab 4. The values corresponding to each variable in the different experiments considered are indicated in Table 5.

Table 4.Codificationof the factors - levels

Trials	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
P	0	0	1	0	-1	-1	1	1	-1	0	1	0	0	0	-1
v$_s$	-1	0	0	-1	-1	0	1	0	1	0	-1	1	1	0	0
η	1	0	1	-1	0	1	0	-1	0	0	0	-1	1	0	-1

The RSM is expressed as a matrix form according to that established in eq. (11). In this way, the relationship which exists between the cooling time in the temperature range 800-500ºC and the welding parameters used to carry out the experiments, is selected from the matrix of experiments. The matrix X includes the values of the selected technological variables and the values calculated for their combination in the corresponding quadratic terms; that is, the values of P, v_s, η and their combinations, according to that indicated in eq. (12).

$$[t_{8/5}] = [X][\beta] \qquad (11)$$

$$t_{8/5} = a + bP + cv + d\eta + eP^2 + fv^2 + g\eta^2 + hPv + kP\eta + lv\eta + m \qquad (12)$$

The independent terms *a to k* are those must be calculated if the combination of the design of experiments is considered, and which constitute the elements of the matrix β[21,22]. This matrix is called the matrix of coefficients and to obtain it, it is necessary to transpose the matrix X and multiply it by itself to be able to solve the equation. This is because X is not a square matrix and, therefore, has no inverse solution.

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Table5.Values of the matrix of experiments

Trials	P[W]	v_s[m/seg]	η
1	1620.00	0.0020833	0.8
2	1620.00	0.00312465	0.6
3	2530.00	0.00312465	0.8
4	1620.00	0.0020833	0.4
5	910.00	0.0020833	0.6
6	910.00	0.00312465	0.8
7	2530.00	0.004166	0.6
8	2530.00	0.00312465	0.4
9	910.00	0.004166	0.6
10	1620.00	0.00312465	0.6
11	2530.00	0.0020833	0.6
12	1620.00	0.004166	0.4
13	1620.00	0.004166	0.8
14	1620.00	0.00312465	0.6
15	910.00	0.00312465	0.4

The $t_{8/5}$ matrix contains the values of the cooling time calculated by simulation according to the indicated in the epigraph 2.The procedure mentioned above was applied for each simulated experiment. The coefficients calculated to build up the matrix *β* are written in eq. (13) in which the $t_{8/5}$ cooling time is expressed according to RMS.

$$t_{8/5} = -51.2278 + 0.2251P - 1.6 \cdot 066 \cdot 10^5 v_s + 510.43\ \eta - 28.80 \cdot pv_s + 0.0370 \cdot p \cdot \eta$$
$$-1.0499 \cdot 10^{-5} v_s \cdot \eta - 2.1984 \cdot 10^{-5} p^2 + 3.5188 \cdot 10^7 v_s^2 - 29.3260\eta^2 \tag{13}$$

Discussion

A characteristic ANOVA variance analysis is carried out to decide if the expression obtained, eq. (13), explains the variations existing for the $t_{8/5}$, Table 6. Fig 2 depicts the calculated results.

Table 6.Variance analysis of the model obtained by RSM.

Variance Analysis	Degrees of freedom	Sum of squares	Mean square	F-quotient	P-value
Model	2	$2.799 \cdot 10^7$	13995177.9	111.2	0
Residue	42	$5.2861 \cdot 10^7$	125860		
Total	44	$3.327 \cdot 10^7$			

$R^2 = 84.11\%$ & R^2 (adjust) = 83.36%

The analysis of Tab. 6 allows the statistic *F* and its level of significance, which is the probability of rejecting the null hypothesis as being true. The level of significance allows us to acceptor to reject the null hypothesis, that is to say, the independence between the variables electric power, welding speed and thermal efficiency of the welding procedure, without having to compare the *F* value with a real value from the statistical tables of the Snedecor distribution function.

In the particular case herein studied, given that the level of significance, the *p-value*, is 0.0248 it is accepted that the variation obtained in the $t_{8/5}$ values is significantly greater than the unexplained variation and, therefore, the viability of the model presented in eq. (13) is justified. In this sense, it can also be established that there is not a dependence relationship

between the technological variables considered in this work. From eq. (13) and the values of each of the factors in each of its levels, the corresponding value of $t_{8/5}$can be calculated for each of the experiments considered, according to that indicated in Fig. 2. The difference which exists between the values of t$_{8/5}$ obtained by simulation and those corresponding to the calculation carried out using RSM can be considered as the error in the RSM function, that is, the distance of the blue line to the red one in Fig.2.
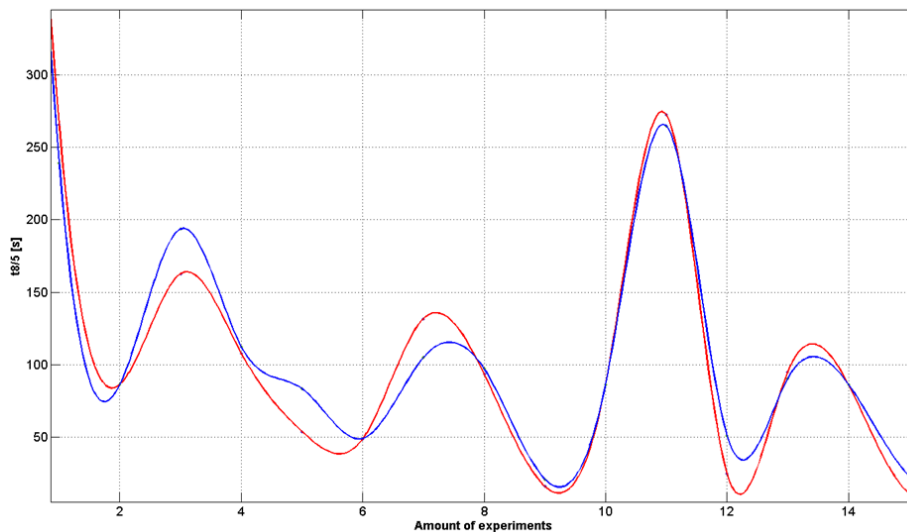
Figure 2. Comparison of t$_{8/5}$values calculated by FEM (red) and by response surface method (blue).

According to the mentioned above, the best fit of the response surface is obtained, in absolute value terms, for experiments 10 and 14, corresponding to values of *P*, v_s and η of 1620 W, 3.12·10^{-3} m/s and 0.6, respectively, if the relative error is taken into account. The relative error is expressed as a percentage with respect to the simulated value.

In order to visualise the influence of the process variables, Fig.3 shows the response surface of the cooling time t$_{8/5}$ obtained by difference finites method, as a function of the involved factors selected by pairs. Figure 4 depicts the contours map of the cooling time as a function of the electric power, welding speed and thermal output. Figures 5 and 6 show the response surfaces and contour maps corresponding to t$_{8/5}$ obtained by RSM.

A relationship between the response method $t_{8/5}$ and the factors analysed separately, *P*, v_s , η , is indicated in Fig.7 from the TIG welding process. It can be observed that the model provides the expected tendencies in the welding process, that is to say that as the power of the electric arc of the welding machine is increased and the thermal output of the process increases, so the cooling time after welding increases."+" symbol lines represent the confidence intervals, and "□" symbol dotted lines represent the evolution of every variable in relation to cooling time t$_{8/5}$. Mean values of the input variables in the process are 1673.33 W, 0.00312 m/s and 0.6 for *P*, v_s and η, respectively. The mean value of $t_{8/5}$ is 106.0077 ± 25.181 s.

From the written above, it could be established that for the different outputs, the greatest values of speed experimented, together with those of lowest power, would lead to the lowest values of $t_{8/5}$. As can be seen in Fig. 2, the lowest value for cooling time is obtained, in

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all cases, for the lowest power applied. The same does not occur with the welding speed, which adopts a different optimal speed depending on the output considered.

The values for the error are high in many cases. This suggests a refining process in the RSM method consisting in designing new value levels for the variables of an optimised setting value for the $t_{8/5}$, that is to say, for a setting value for the variables P, v_s, η which minimises the $t_{8/5}$ in accordance with equation (13).
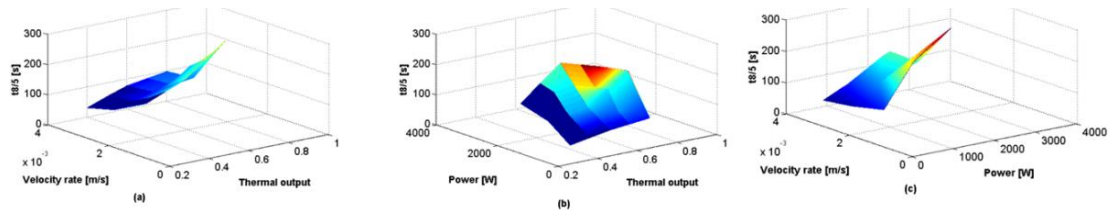
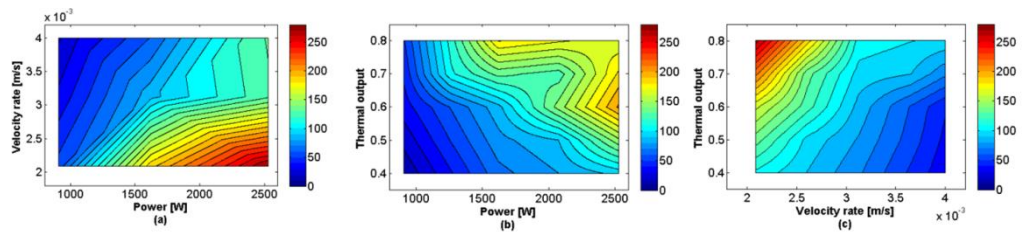Figure 3. $t_{8/5}$ as a function of thermal output, the welding speed and the electrical power.

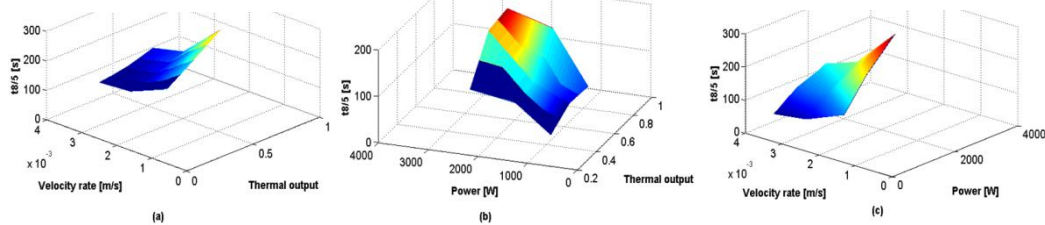Figure 4. Contour maps of $t_{8/5}$ corresponding to differences finites method.

Figure 5. $t_{8/5}$ calculated by RSM as a function of thermal output, welding speed and electrical power
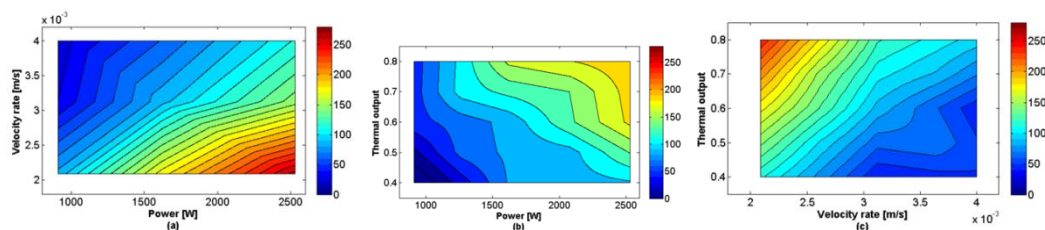

Figure 6. Graphic of contours of $t_{8/5}$ calculated by RSM

Conclusions

In the present work RSM has been applied to evaluate the different welding variables influence on the cooling time in the range of 800-500ºC, $t_{8/5}$, of electric arc welding processes. Specifically, an expression has been obtained which predicts the behaviour of the $t_{8/5}$ as a function of the electric arc power, the welding speed and the thermal efficiency. The ANOVA analysis has enabled to determine that those variables affect $t_{8/5}$ significantly. A RSM fit has been carried out from $t_{8/5}$ values obtained through a numerical modelling procedure consisting of

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the Finite Difference Method applied to the TIG welding of an austenitic stainless steel plate with a thickness lower than 3 mm and thus, the heat flow was considered as two-dimensional.
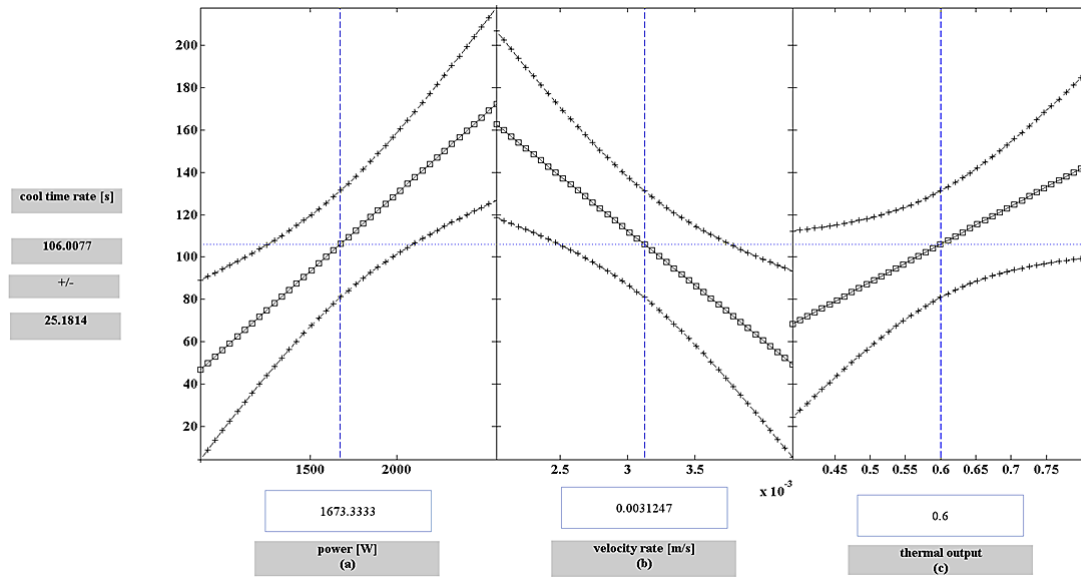

Figure7. Relationship cooling time-factors

An analysis of the obtained results was carried out by contrasting the values obtained with the numerical method and with RSM for the different variables considered. The analysis suggests carrying out a refining process in the setting values for the variables which lead to the minimum value of $t_{8/5}$, using RSM. Other variables of the process, such as the geometry of the joint and the existence of pre-heating, could be taken into account when modelling $t_{8/5}$.

The application to other materials and arc welding techniques must be done based on values obtained by suitably corrected numerical methods with the specific characteristics of the process and of the material, or from experimental data.

Nomenclature

a_h, c_h	- ellipsoidal semi-axes [mm]	u_a, u_c	- ellipsoidal non dimension heat source parameters
B	- width of the stainless steel plate to be simulated, [m]	y	- response surface
C_p	- Caloric capacity, [J kg^{-1} K^{-1}]	v_s	- welding speed, [m/s]
D	- thickness of the stainless steel plate to be simulated, [m]	*Greek letters*	
k	- Fourier conductivity, [J m^{-1} s^{-1} K^{-1}]	α	- thermal diffusivity [m^2/s]
L	- length of the stainless steel plate to be simulated, [m]	ΔH	- phase change enthalpy, [J kg^{-1}]
P	- power of the electric arc, [W]	β	- regression parameters of the surface estimated from experimental data
T_o	- initial temperature, [ºC]	ε	- experimental error
T_f	- fusion temperature, [ºC]	η	- thermal output of the welding procedure
$t_{8/5}$	- cooling time between 800 and 500 ºC, [s]	ρ	- density, [kg m^{-3}]

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References

[1] Lazic, V. N., et al, Theoretical-Experimental Determining of Cooling Time (T(8/5)) in Hard Facing of Steels for Forging Dies, *Thermal Science*, *14*, (2010), 1, pp. 235–246, DOI No. 10.2298/TSCI1001235L.

[2] Ivanovic, I. B.,et al, Numerical Study of Transient Three-Dimensional Heat Conduction Problem with A Moving Heat Source, *Thermal Science*, *15*, (2011),1 , pp. 257–266, DOI No. 10.2298/TSCI1001257I.

[3] Yeh, R. H., et al, Transient three-dimensional analysis of gas tungsten arc welding plates, *Numerical Heat Transfer; Part A: Applications*, *51*, (2007) 6, pp. 573–592.

[4] Lazic, V. N., et al, Energetic Analysis of Hard Facing and Weld Cladding of An Air PoweredDrop Hammer Damaged Ram, *Thermal Science*, *14*,(2010),suppl, pp. S269–S284, DOI No.
10.2298/TSCI100501021L.

[5] J. C. L. X.YUE, Continuous Cooling Transformation Behavior in the CGHAZ of Naval Steels, *Welding Journal*, *91*,(2013), 3, pp. S67–S75.

[6] Leister, B.M. and DuPont, J.N., Fracture toughness of simulated heat affected zones in NuCu-140steel, *Welding Journal*, *91*, (2012) 2, p. 53s–58s.

[7]Cieslak, M. J., Hydrogen-Induced Cracking (Cold Cracking), in: *ASM handobook. Volume 6* (Eds. R. M. Nunes et al.), ASM International, New York, USA, 1993, pp. 241-245.

[8] Kalaba, D. V., et al, Thermomechanical Modelling the Resistance Welding of Pbsb Alloy, *Thermal Science*, *14*, (2010), 2, pp. 437–450, DOI No. 10.2298/TSCI1002437K.

[9] Al-Sa'ady, M. H., et al, Finite Difference Simulation of Low Carbon Steel Manual Arc Welding,*Thermal Science*, 15, (2011), 1, pp. 207–214, DOI No . 10.2298/TSCI100206055S.

[10] Yaghi, A., et al, Residual stress simulation in thin and thick-walled stainless steel pipe welds including pipe diameter effects,*International Journal ofPressure Vessels and Piping*, *83*,(2006),11–12, pp. 864–874.

[11]Y.T.iÇ, et al, Design of experiment and goal programing application for the GMAW process, *Welding Journal*, *91*,(2012), p. 106–S - 112-S.

[12]Montgomery, D.C. and Myers, R.H., *Response surface methodology: Process and product inoptimization using designed experiments*, Jhon Wiley&Sons, New York-USA, 1995.

[13] CHO, M. H., et al, Simulation of Weld Pool Dynamics in the Stationary Pulsed Gas Metal Arc Welding Process and Final Weld Shape, *Welding Journal*, *85*, (2006),12, p. 271–s - 283–s.

[14]Nguyen, N.T., et al, A analytical approximate solution for double ellipsoidal heat source in finite thick plate, *Welding Journal*, *83*, (2004),3, p. 82–s - 93–s.

[15]Ohta, A., et al, Analytical solutions for transient temperature of semi-infinite body subjected to 3-D moving heat sources, *Welding Journal*, *78* (1999), 8, pp. 265–274.

[16]Martínez-Conesa, E.J., et al, A mathematical approach based on finite differences method foranalyzing the temperature field in arc welding of stainless steel thin sheets,*Revista de Metalurgia*, *46*, (2010), 6, pp. pp. 511–519, DOI No 10.3989/revmetalmadrid.1021.

[17]Maljaars, J.,et al, Constitutive model for aluminum alloys exposed to fire conditions,*Metallurgical and Materials Transactions A-Physical Metallurgy and Materials Science*, *39A*,(2008), 4, pp. 778–789.

[18]May, J. E. and Menzemer, C. C., Strength of bolted aluminum alloy tension members," *Journal of Structural Engineering-Asce*, *131*, (2005), 7, pp. 1125–1134.

[19]Aalberg, A.,et al, Stiffened aluminium panels subjected to axial compression, *Thin-Walled Structures*, *39*, (2011), 10, pp. 861–885.

[20]Martínez Conesa, E.J.,et al, Optimization of t8/5 for stainless steel Arc Welding, *Dyna*, *84*,(2009), pp. 251–258.

[21]Box, G.E.P. and Wilson, K.B., On the Experimental attainment of optimum conditions, *Journal of the royal Statistical Society Series B-Statistical Methodology*, *13*,(1951), pp. 1–45.

[22] Kikuchi, N., and Suzuki, K.,A homogenization method for shape and topology optimization, *Computer methods in applied mechanics and engineering*, *93*, (1991), pp. 291–318.