BEJAN'S CONSTRUCTAL THEORY AND OVERALL PERFORMANCE ASSESSMENT
The Global Optimization for Heat Exchanging Finned Modules

by

Giulio LORENZINI* * and Simone MORETTI

Department of Industrial Engineering, University of Parma, Parma, Italy

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Bejan's Constructal theory has provided a dramatic contribution to the design of natural and industrial systems, mainly because all sort of flows tend to follow precise rules which Bejan was able to explain. Recently, for what concern heat removal systems, the Authors have extended the classical Constructal optimization to a broad range of 3-D geometries, also applying such results to the optimal choice of the suitable thermal carrier fluid. The definition of the Global Optimization was centred on the concepts of Relevance and Overall Performance Coefficient, which are technical parameters. This paper follows the principle of the Constructal Theory in the assessment of the Global Optimization to highlight how the coupling of the two may provide a tool becoming part of everyday decision making processes in future design. The results show not only which is the best performing module but also proves that, via the superimposition of a suitable Relevance value, the Overall Performance Coefficient allows for the correct characterization of the whole heat transfer process, so confirming the interesting perspective of the novel approach proposed.

Key words: *Bejan's Constructal theory, global optimization, numerical models, heat removal, extended surfaces*

Introduction

Bejan's Constructal theory has shown what is common in natural and industrial flow processes. The need for optimal heat removals is nowadays very pushing as, especially in electronics, not only the heat to be removed tends to grow with the power of the devices but it is inversely proportional to the volume available for the heat exchangers. This is reflected also in literature: many international researches have recently investigated the problem of heat exchange through finned surfaces or cavities [1-13] entering often in particular details for what concern a common industrial requirement: water evaporation performance improvement process [14-20]. In this way experimental studies [21, 22] and experimental-theoretical studies [23-25] are to be mentioned. Among numerical studies facing the heat transfer aspects in extended surfaces in presence of different boundary conditions and constraints the works in [26, 27] need to be cited. Many works have recently investigated the optimization of the heat removal in specific heat exchangers. In particular the Authors started optimizing, employing Bejan's Constructal theory, a single fin with respect to dimensionless conductance [28, 29] and went on optimizing the fin modules as

* Corresponding author; e-mail: giulio.lorenzini@unipr.it

heat removal is not just a matter of geometrical optimization of each single fin but also of volume optimization intended as relation between heat removed and space occupied by the heat exchanger [30-33]. The further fundamental step was that of defining the concept of Global Optimization of such optimized finned modules [34], taking into account the pressure losses with air flowing on the exchanger surface in laminar condition. Such criterion is based on the definition of Relevance, *i. e.* weight of heat removal maximization with respect to pressure loss minimization, and overall performance coefficient (OPC), *i. e.* the general optimization parameter. The extension of such approach was successively done imagining the possibility of a hybrid gas-liquid cooled heat exchanger [35]. From the main results in [34] and in [35] comes the need for putting together in synergy both the results, geometry and flow optimization, coming from the concept of Global Optimization. Thus the present paper takes into account the ideas of Bejan's Constructal theory and of Global Optimization: the first is based on the Constructal law, which is a law of Physics; the second provides a practical mean to help the engineer decide the optimal geometrical and flow configuration of a heat exchanger involved in a technical process. The two aspects are closely examined for what pertains fins shape, fins module configuration and thermal carrier fluid choice, water and air in this case as typically representing liquids and gases. One of the main aims is that of showing the usefulness of the OPC, and thus of the Global Optimization, defined as a practical decision making parameter for heat exchanging processes. The study, being numerical (again Consol Multiphysics, version 3.5a [36] was employed) and parametrically dimensionless, also provides a generally applicable approach that opens new possibilities in the heat removal optimization panorama.

Systems and conditions investigated

Geometries selected for the analysis come from previously done investigations described in other works mainly from the same authors [1, 29-35], where Bejan's Constructal theory is applied to obtain high performing shapes. The starting point for the definition of this kind of geometries can be found in [1], where Bejan and Almogbel study the trend of the performance parameter, the dimensionless conductance q^*, in function of some fundamental ratios between the fin main dimensions. Such parameter was defined as:

$$q^* = \frac{q_\mathrm{L}}{k(T_\mathrm{R} - T_\infty)} \tag{1}$$

where q_L is the thermal power per unit length through the root of the heat exchanging profile, k – the thermal conductivity, T_R, and T_∞ are the fin root temperature and the local undisturbed one, that is the undisturbed flow inlet temperature, respectively.

Optimised Y-shaped fins used throughout the two analysis we are going to compare [28], were obtained in previous works and are based on precise geometrical ratios among their dimensions ($t_1/t_0 = 5$, $L_1/L_0 = 0.07$), where t_0 is the thickness of the arms, t_1 – the thickness of the base, L_0 – the length of the arms, and L_1 – the length of the base.

Starting from these results, the true innovations that featured the works [28] have to be found in the application of the so called Performance coefficient P_{ij}, that will be deepened in the section *The overall optimisation*, and that enabled the model user to select proper configurations for defined boundary conditions. Parallel to that, another novel choice has to be found in the heat exchanging fluid. In [35] water (Pr ≈ 7.01, T = 293.15 K) is used parallel to air (Pr ≈ 0.71), with the purpose to evaluate the performances difference in terms of heat exchange and pressure losses.

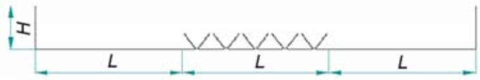

Figure 1. Example of geometrical model simulated (duct) and its characteristic dimensions

The module represented in fig. 1 is realized on a properly defined base plate, which has the feature to give a periodical trend to an heat exchanger realized using a multi-module approach [30, 31].

The channel where this module is enclosed is sized in order to not influence the target parameters of the simulation. It has been verified that side effects result negligible already for lengths of the channel smaller than the one chosen for this investigation ($3L = L_D$). This parameter has been held constant throughout all the performed simulations, varying, instead, the height of the channel H, through the variation of the dimensionless parameter $\theta = H/L_D$.

Furthermore, in order to make comparable the results of the simulations performed on the two different fluids, it has been superimposed a condition of laminar flow, by limiting the upper value of the Reynolds number to 1500.

For what concern pressure losses, the evaluation has been conducted starting from the mean inlet and outlet pressures p_{in} and p_{out}, made dimensionless by means of the atmosphere pressure $p_0 = 1.013 \cdot 10^{-1}$ MPa.

The applied boundary condition are the same used in the previous works [1, 28-35]: T_R is equal to 373.15 K and T_∞ is equal to 297.15 K. Such values have been chosen because quite typical of the most of industrial application. The generality of the two studies, anyhow, do not suffer of their dimensional nature, since the global performance parameter, the dimensionless conductance q^*, is defined as a function of such parameters, as described in eq. (1). Also the material that compose the fins has been selected following the same philosophy. Aluminum, in fact, is the most common choice in the industry of heat exchangers, thanks to its great performance in terms of heat transmission, availability, lightness, and easiness of production. Volumetric mass and viscosity, required to correctly compute the flow and temperature fields within the domain, were initially computed at a pressure value p_0, in case of air as fluid. In case of water the only depending variable affecting directly these dimensions remains the punctual temperature T.

Method and tests performed

The tool used to perform the analysis has been the finite element software Comsol Multiphysics 3.5a. In both the works we are here going to compare, the main analyze started on a preliminary evaluation of the mesh quality level to be imposed. It has been selected an optimized mean mesh element value, able to give an acceptable accuracy of the output, together with a reasonable computing time. The numerical platform, moreover, include a key feature, called Mesh Adaptivity, which increase automatically on incremental steps the mesh grid refinement level, in the areas where are found stronger gradients. This improvement is applied after a first evaluating numerical simulation, where the software identify the critical areas. This approach allow to increase the accuracy of the analyze selectively, only where it gives really an added value, without any waste of computational power.

Results and preliminary discussion

In both the works [32, 33], the aim is to evaluate the trend of the dimensionless conductance q^* and of the pressure losses Δp^*, subjected to the variation of the channel height. Such operation is done by varying the value of the θ parameter, defined in the section *Systems and conditions investigated*. In the first work [32], moreover, enters in the evaluation also the shape

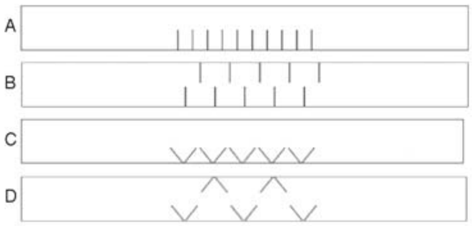

Figure 2. Type of shapes evaluated – Shape A, B, C, and D

of the fins as fundamental parameter. As can be observed from fig. 2 there are basically four different shapes of fins. The distinctive parameter featuring the second study [33] is instead the type of fluid used during the numerical evaluations. In fact, also water is simulated through the finite element software, enabling a view of the heat exchanging and fluid dynamic phenomena from an additional perspective.

From the graphs presented in figs. 5 to 8 it can be observed how these parameters vary in function of the punctual value of Reynolds number, for the different shapes evaluated throughout the two mentioned works [32, 33].

Case 1: $\theta = 0.1$

Minimal channel height H, which results 1/10 of the channel length L_D. It can be observed a notable resistance to fluid streaming, confirmed by higher trends of dimensionless pressure losses, which are maximal if compared with the other cases of θ.

The increased friction on the channel walls and fins external layers generates a positive effect with respect to the other performance parameter: the dimensionless conductance. Such trends have been evaluated inside the whole range Re = 0-1500. A sample of temperature gradients and velocity field can be observed in figs. 3 and 4.

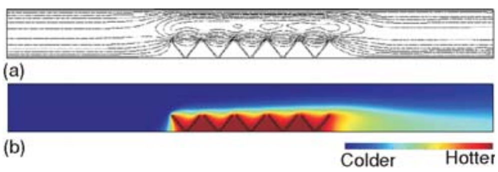

Figure 3. Example of velocity (a) and temperature (b) fields (shape *C*; θ = 0.1) air cooling (for color image see journal web site)

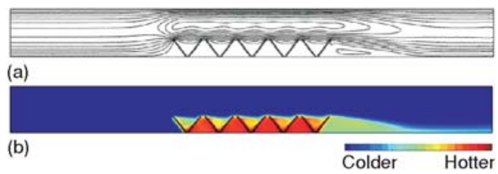

Figure 4. Example of velocity (a) and temperature (b) fields (shape *C*; θ = 0.1) water cooling (for color image see journal web site)

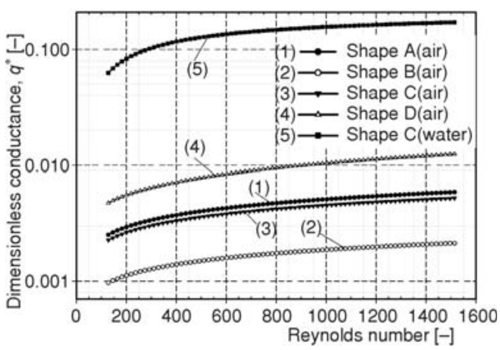

Figure 5. Dimensionless conductance q^* in function of Re number for θ = 0.10

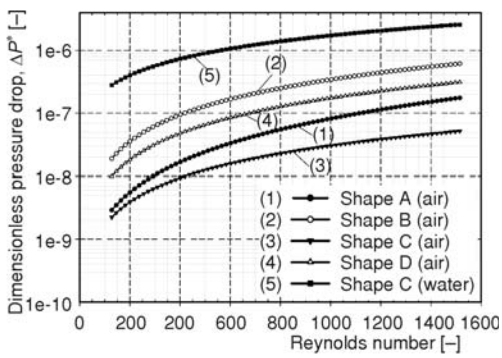

Figure 6. Dimensionless pressure loss Δp^* in function of Re number for θ = 0.10

Looking to some details relative to the case 1, we can state that for Re = 1500 the highest thermal performances results equal to $q^* = 1.71872 \cdot 10^{-1}$, and it is relative to the Shape C water cooled. Concerning pressure losses, for the same Reynolds value we obtain $\Delta p^* = 5.09563 \cdot 10^{-8}$ using Shape C and air, figs. 5 and 6.

Case 2: $\theta = 0.15$

The height of the channel increases slightly, influencing the values of thermal and fluid performances. It can be observed a noticeable reduction in terms of dimensionless conductance, parallel to a positive effect on pressure losses, which result strongly reduced along the channel path.

For such a second case, relative to a Reynolds number of 1500, it can be obtained a dimensionless conductance up to $q^* = 9.8093 \cdot 10^{-2}$ for the Shape C with water as coolant fluid. For what relates to the pressure losses in this case study, the best performance for Re = 1500 assumed values on $\Delta p^* = 2.950619052 \cdot 10^{-8}$ with air wetting Shape C fins.

Case 3: $\theta = 0.2$

Channel height increased further on with respect to Case 2. The values relative to dimensionless conductance, if compared with the ones of case with $\theta = 0.1$, seem to respect the ratio that features duct heights of these two cases.

The best configuration offers a performance headed on $q^* = 7.7600 \cdot 10^{-2}$ at Re = 1500 again for the Shape C and water to cool down the surfaces. With reference to pressure losses, trends are different. In fact, also if they remain close to be linear, the values result quite lower than the half of the ones observed in case 1. For Re = 1500 it has been obtained a value of pressure losses on $\Delta p^* = 1.22719 \cdot 10^{-8}$ referring to Shape C and air as coolant fluid.

Case 4: $\theta = 0.25$

Fourth case completes the analyze. The limit of $\theta = 0.25$ has been chosen to constrain the investigation to a fixed value, since performance indicators tends to the same asymptote, by increasing θ further on.

Qualitative trends of pressure losses and thermal fields can be observed in figs. 7 and 8. As it was expected, such case produces the lowest values both looking to pressure losses or dimensionless conductance, for a Reynolds value of Re = 1500. Dimensionless conductance arrives in fact to a value of $q^* = 6.2652 \cdot 10^{-2}$ for the Shape C when it is used water in the duct, while

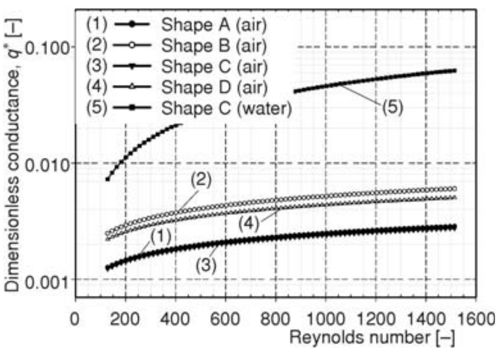

Figure 7. Dimensionless conductance loss q^* in function of Re number for $\theta = 0.25$

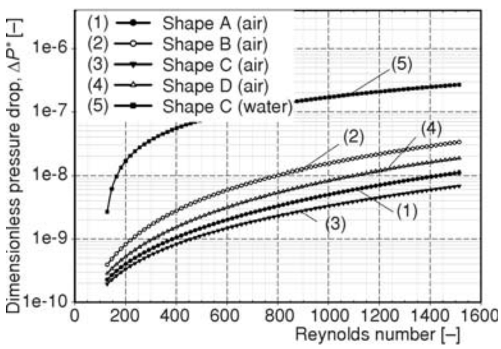

Figure 8. Dimensionless pressure drop Δp^* in function of Re number for $\theta = 0.25$

dimensionless pressure drops to a value of $\Delta p^* = 6.75242\cdot10^{-9}$ for Shape C and air as thermal exchanging fluid.

As it can be observed, the choice of Shape C as optimized has been properly done conducting the test on [33]. In fact, Shape C is the one which grant the best performances in terms of dimensionless conductance and pressure losses. Using water or air as coolant fluid, the two performance parameter can be maximized or minimized.

The global optimization

The results of this investigation, performed through the finite element method, show different trends for different type of cooling fluids. An ideal system should grant optimal performances from a thermal point of view, parallel to negligible dissipative effects generated by pressure losses. Systems commonly used in practical applications, for physical reasons, do not allow to obtain described behaviors, but are optimized case by case to push one effect instead of the concurrent one.

With the aim to consider both contributes, it has been defined the performance parameter OPC. Its value can be used to guide the designer through the choice of the proper solution to its particular case. The idea is to grant the user the possibility to decide for a minimization of pressure losses (accounted by Δp^*) or an increasing of the heat exchange rate (accounted by q^*). The parameter α, called Relevance, has been chosen as a real value included in the range 0 to 1, defined with the purpose to immediately give an idea of the importance of one particular phenomenon. Actually, when α get closer to 0, the minimization of pressure losses will result for the application extremely important, and the influence of heat exchange rate will be secondary. From the other side, by imposing an α close to 1, it would lead to the opposite approach: maximization of heat transfer rate, with consequent effects due to increased pressure losses to be neglected.

The introduced OPC P_{jk} can be represented as follows:

$$P_{\mathrm{jk}} = \alpha \tilde{q}_{\mathrm{mjk}} + (1-\alpha)\frac{1}{\Delta\tilde{p}_{\mathrm{mjk}}} \tag{2}$$

In eq. (2) appear two new terms: $\tilde{q}_{\mathrm{mjk}}$ is the mean dimensionless relative conductance (with k type of coolant fluid, j-th θ) and $\Delta\tilde{p}_{\mathrm{mjk}}$ – the mean dimensionless relative pressure loss (with k type of coolant fluid, j-th θ), defined as:

$$\tilde{q}_{\mathrm{mjk}} = \frac{q^*_{\mathrm{mjk}}}{q^*_{\mathrm{mref}}}, \quad \Delta\tilde{p}_{\mathrm{mjk}} = \frac{\Delta p^*_{\mathrm{mjk}}}{\Delta^*_{\mathrm{mref}}} \tag{3}$$

where q^*_{mref} is the mean dimensionless conductance for the reference type of configuration that is the geometry for $\theta = 0.1$ cooled by air; Δp^*_{mA} – the mean dimensionless pressure loss for the same geometry and type of coolant. Both parameters were calculated in the whole Re range from 0 to1500 investigated. These two values were used as reference performances throughout the whole global performances evaluation. It was in fact possible to put these values in relation to every other (j, k)-th mean performance, q^*_{mjk} and $\Delta p^*_{\mathrm{mjk}}$ (j = 1, 2, 3, 4 being in 1-1 correspondence to $\theta = 0.10$, $\theta = 0.15$, $\theta = 0.2$, $\theta = 0.25$; k = 1, 2 representing, respectively, air and water).

In fig. 9 can be observed the trends of the most significant OPC curves, plotted varying the parameter α between 0 and 1 for the configurations described in the legend. The cases not drawn in the graph of fig. 9 have been excluded principally for clarity of exposition, and in second instance due to the low informative value they gave, considering that the offered performance level resulted always inside the boundaries represented by the visible curves. It is immediate, already from a first look to this image, to understand what should be the best configuration to choose, in function of the effect to be improved. Considering, for instance, a case in which it should be maximized the heat exchange, with not any particular constraint concerning available energy to generate fluid stream. In that condition it should be chosen the highest possible value of Relevance, that means 1. The best configuration will be the one relative to a cooling stream of water, with a minimal channel height ($\theta = 0.10$), and an exchanging profile composed by Shape C fins. This is the right choice when cooling fans are easily installable inside the system. The OPC results in that case equal to 33.4751698341.

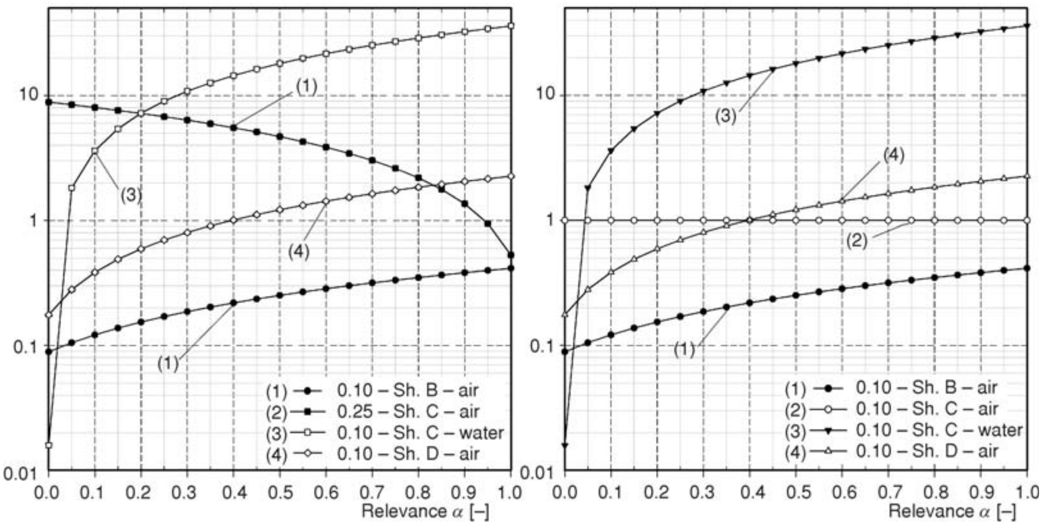

Figure 9. Overall performance coefficient *P* in function of Relevance α

In another typical situation, where available energetic resources are minimal, it will be critical to reduce to the minimum the dissipative effects due to an higher friction of fluid streams. In that situation, it is clear that the best configuration results the one with a low value of α, and the graph enclosed in fig. 9(a) leads to the choice of the configuration of Shape C fins that maximize channel height ($\theta = 0.25$), with air as cooling medium. In such conditions, the value of the OPC P_{jk} reach a level of 8.891148802, much higher than what is grantable by a water cooled system.

Usually in modern computing systems, another strong constraint is represented by the space at disposal for components designated to heat extraction. In such cases, the dimension of the channel gets a primary importance, and the evaluation should be done selecting between models with reduced channel heights. In fig. 9(b), are plotted curves relative to cases with $\theta = 0.10$. It can be observed that the most suitable solution to the task is the reference configuration cooled by air, where the OPC P_{jk} results equal to 1. For water cooled modules, when α tends to 0, we observe a worse behavior, that improve strongly for low α values. That means that, looking to a global optimization approach, it can be preferred to apply liquid cooling solution for high performing applications. In that case, already for an $\alpha = 0.1$, we assist to a strong increase of P_{jk} to a value of 3.618235021 for water cooled modules and $\theta = 0.10$.

For values of α included in the intermediate range between 0 and 1, we can identify several configurations on which one effect is improved versus the concurrent one, that should be discussed together with the real requirements of the application to be implemented.

Conclusions

Bejan's Constructal theory may be applied to every natural or industrial system and process in order to describe its present state and its probable evolutions and the heat transfer context is one of the fields where its contribution has been and is extremely deep. In the last years the Authors have applied Bejan's constructal theory *via* a CFD code approach to optimize the heat exchangers both for what pertains geometry and, more recently, flow conditions. This purpose was achieved also thanks to the definition and application of the Global Optimization, a technical criterion based on the definition of Relevance and of OPC. These two parameters are technical tools which help in the design of heat exchanger in terms of geometry and flow. For what relates to geometry optimization, the study proved that heat exchangers based on modules of Y fins located at the bottom of the duct surface are the best performing. For what relates to flow optimization, the study proved that water generally enhances heat removal but causes an increased pressure loss with respect to air, so affecting the global performances and the energy consumption, but for some applications that can be neglected. In fact, in some cases, water cooling represents the only solution available, considering the small space available and the high heat power to be removed. In general overall optimization clearly proved to be an extremely useful concept to design and use heat exchangers, in strict association to Bejan's Constructal theory, which will be the unavoidable starting point for any modelling developments in the future.

Nomenclature

D – characteristic length, [m]
H – duct height, [m]
K – thermal conductivity, [$\mathrm{Wm^{-1}K^{-1}}$]
L – optimised fin-module length, [m]
L_D – duct length (= $3L$), [m]
L_0 – optimised fin arms length, [m]
L_1 – optimised fin stem length, [m]
P_{cs} – duct cross-sectional perimeter, [m]
p_{in} – mean inlet pressure, [Pa]
p_{out} – mean outlet pressure, [Pa]
p_0 – atmospheric pressure, [Pa]
Δp^* – dimensionless pressure loss, [–]
q^* – dimensionless conductance, [–]
q_L – linear heat flux, [$\mathrm{Wm^{-1}}$]
Re – Reynolds number, [–]
S – duct cross-sectional area, [$\mathrm{m^2}$]
T_R – fin root temperature, [K]
T_∞ – undisturbed flow inlet temperature, [K]
t_0 – optimised fin arms thickness, [m]
t_1 – optimised fin stem thickness, [m]
u – inlet velocity (x-component), [$\mathrm{ms^{-1}}$]

Greek symbols

θ – duct shape ratio, [–]
ρ – volumetric mass of the refrigerant fluid, [$\mathrm{kgm^{-3}}$]

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