RECONSTRUCTIVE SCHEMES FOR VARIATIONAL ITERATION METHOD WITHIN YANG-LAPLACE TRANSFORM WITH APPLICATION TO FRACTAL HEAT CONDUCTION PROBLEM

by

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Original scientific paper
DOI: 10.2298/TSCI120826075L

A reconstructive scheme for variational iteration method using the Yang-Laplace transform is proposed and developed with the Yang-Laplace transform. The identification of fractal Lagrange multiplier is investigated by the Yang-Laplace transform. The method is exemplified by a fractal heat conduction equation with local fractional derivative. The results developed are valid for a compact solution domain with high accuracy.

Key words: *fractal heat conduction equation, local fractional variational iteration method, Yang-Laplace transform, local fractional derivative*

Introduction

Heat transfer problems can be mathematically modelled by ordinary and partial differential equations [1], integral equations [2], and integro-differential equations [3]. Commonly heat conduction problems are solved either analytically or analytical and numerically applying various methods, among them: finite difference techniques (FDT) [4], regression analysis (RA) [5], Adomian decomposition method (ADM) [6], homotopy analysis method (HAM) [7], differential transformation method (DTM) [8], boundary element method (BEM) [9], the heat-balance integral method (HBIM) [10-12], *etc*.

The variational iteration method (VIM) was first proposed by He [13] and was successfully applied to deal with heat conduction equations [14-17]. In 2010, the fractional variational iteration method (FVIM) via modified Riemann-Liouville derivative was conceived [17]. On the other hand, the local fractional variational iteration method via local fractional calculus [18-20] was developed in [21] and successfully applied to differential equations on Cantor sets [22-24]. In this context, recently, a method combining the variational iteration method and Laplace transform method was suggested [25, 26] and a modification via fractional calculus and Laplace transform was conceived by Wu [27].

Fractal heat conduction problems describe heat transport in inhomogeneous materials as fibrous materials, textiles, coal deposits, and other discontinuous media where homogenizations are unacceptable because this approach neglects important physical characteristics of the transport processes. The non-smoothness raises problems and avoids application of the classical calculus of integer and fractional order. To avoid this problem, the local fractional

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calculus [20, 21] was successfully applied to ordinary and partial differential equations with either fractal media or with fractal boundary conditions. Solutions of such local problems have been developed the local fractional Fourier series method [28], Yang-Fourier and Yang-Laplace transforms [29], local fractional variational iteration method [21-24], *etc*.

As a stand of portion of development of solutions to fractal heat conduction problems, the present communication refers to a coupling method combining the features of the variational iteration method and the Yang-Laplace transform [29] local fractional derivatives.

Reconstructive processes

Let us consider the following local fractional partial differential equations:

$$L_\alpha u - R_\alpha u = g(x) \tag{1}$$

where $u = u(x)$, L_α is the linear local fractional operator, R_α is the linear local fractional operator of order less than L_α, and $g(x)$ is a source term of the non-differential function.

According to the rule of local fractional variational iteration method, the correction local fractional functional for eq. (1) is constructed as [21-24]:

$$u_{n+1}(x) = u_n(x) + {}_0I_x^{(\alpha)}\left\{\frac{\lambda(t)^\alpha}{\Gamma(1+\alpha)}[L_\alpha u_n(t) + R_\alpha \tilde{u}_n(t) - \mathrm{g}(t)\right\} \tag{2}$$

In eq. (2) the local fractional integral operator is defined as [18-20]:

$${}_\mathrm{a}I_\mathrm{b}^{(\alpha)} f(x) = \frac{1}{\Gamma(1+\alpha)}\int_a^b f(t)(\mathrm{d}t)^\alpha = \frac{1}{\Gamma(1+\alpha)}\lim_{\Delta t\to 0}\sum_{j=0}^{j=N-1} f(t_j)(\Delta t_j)^\alpha \tag{3a}$$

and the partition of the interval $[a, b]$, is:

$$\Delta t_j = t_{j+1} - t_j,\ \ \Delta t = \max\{\Delta t_1, \Delta t_2, \Delta t_j, \ldots\}\ \text{ and }\ j = 0, \ldots, N-1,\ \ t_0 = a, t_N = b \tag{3b}$$

In eq. (2) $\lambda(t)^\alpha/\Gamma(1+\alpha)$ is a fractal Lagrange multiplier.

When $\delta^\alpha \tilde{u}_n$ is considered as a restricted local fractional variation [20], *i. e.* $\delta^\alpha \tilde{u}_n = 0$, we obtain the following fractal Lagrange multiplier:

$$\frac{\lambda(t)^\alpha}{\Gamma(1+\alpha)} = -\frac{(x-t)^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]} \tag{4}$$

where L_α in eq. (1) are $k\alpha$ times local fractional partial differential equations.

Therefore, eq. (2) becomes:

$$u_{n+1}(x) = u_n(x) - {}_0I_x^{(\alpha)}\left\{\frac{(x-t)^{)k-1)\alpha}}{\Gamma[1+(k-1)\alpha]}[L_\alpha u_n(t) + R_\alpha \tilde{u}_n(t) - \mathrm{g}(t)]\right\} \tag{5}$$

For initial value problems of eq. (1), we can start with:

$$u_0(x) = u(0) + \frac{x^\alpha}{\Gamma(1+\alpha)}u^{(\alpha)}(0) + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}u^{(2\alpha)}(0) + \cdots + \frac{x^{k\alpha}}{\Gamma(1+k\alpha)}u^{(k\alpha)}(0) \tag{6}$$

where local fractional derivative is given by [18-20]:

$$\mathrm{D}_x^{(\alpha)} f(x_0) = f^{(\alpha)}(x_0) = \left.\frac{\mathrm{d}^\alpha f(x)}{\mathrm{d}x^\alpha}\right|_{x=x_0} = \lim_{x\to x_0}\frac{\Delta^\alpha[f(x) - f(x_0)]}{(x-x_0)^\alpha} \tag{7a}$$

with $\Delta^\alpha[f(x) - f(x_0)] \cong \Gamma(1+\alpha)[f(x) - f(x_0)]$, and local fractional derivative of high order is denoted as [20]:

$$f^{(k\alpha)}(x) = \overbrace{\mathrm{D}_x^{(\alpha)} \ldots \mathrm{D}_x^{(\alpha)}}^{k \text{ times}} f(x) \tag{7b}$$

The Yang-Laplace transform is defined as [18, 19, 29]:

$$\widetilde{L}_\alpha \{f(x)\} = f_s^{\widetilde{L},\alpha}(s) = \frac{1}{\Gamma(1+\alpha)} \int_0^\infty E_\alpha(-s^\alpha x^\alpha) f(x) (\mathrm{d}x)^\alpha, \ 0 < \alpha \le 1 \tag{8a}$$

and its inverse formula is [18, 19, 29] :

$$f(x) = \widetilde{L}_\alpha^{-1} \{f_s^{\widetilde{L},\alpha}(s)\} = \frac{1}{(2\pi)^\alpha} \int_{\beta - i\infty}^{\beta + i\infty} E_\alpha(s^\alpha x^\alpha) f_s^{\widetilde{L},\alpha}(s) (\mathrm{d}s)^\alpha \tag{8b}$$

where $f(x)$ is local fractional continuous, $s^\alpha = \beta^\alpha + i^\alpha \infty^\alpha$ and $\mathrm{Re}(s^\alpha) = \beta^\alpha$.

Thus, following eqs. (2) and (5), we obtain a new local fractional functional in the form:

$$u_{n+1}(x) = u_n(x) + {}_0 I_x^{(\alpha)} \left\{ \frac{\lambda (x-t)^\alpha}{\Gamma(1+\alpha)} [L_\alpha u_n(t) + R_\alpha \widetilde{u}_n(t) - g(t)] \right\} \tag{9}$$

We now take Yang-Laplace transform of eq. (2), namely:

$$\widetilde{L}_\alpha \{u_{n+1}(x)\} = \widetilde{L}_\alpha \{u_n(x)\} + \widetilde{L}_\alpha \left\{ {}_0 I_x^{(\alpha)} \left[\frac{\lambda (x-t)^\alpha}{\Gamma(1+\alpha)} [L_\alpha u_n(t) + R_\alpha \widetilde{u}_\alpha(t) - g(t)] \right] \right\} \tag{10a}$$

or

$$\widetilde{L}_\alpha \{u_{n+1}(x)\} = \widetilde{L}_\alpha \{u_n(x)\} + \widetilde{L}_\alpha \left\{ \frac{\lambda (x-t)^\alpha}{\Gamma(1+\alpha)} \right\} L_\alpha \{[L_\alpha u_n(x) + R_\alpha \widetilde{u}_\alpha(x) - g(x)]\} \tag{10b}$$

Here, we first take the variation, which is given by:

$$\begin{aligned} &\delta^\alpha \{\widetilde{L}_\alpha \{u_n(x)\}\} = \\ &= \delta^\alpha \{\widetilde{L}_\alpha \{u_{n+1}(x)\}\} + \delta^\alpha = \left\{ \widetilde{L}_\alpha \left\{ \frac{\lambda (x-t)^\alpha}{\Gamma(1+\alpha)} \right\} \widetilde{L}_\alpha \{[L_\alpha u_n(x) + R_\alpha \widetilde{u}_\alpha(x) - g(x)]\} \right\} \end{aligned} \tag{11a}$$

or

$$\delta^\alpha \left\{ \widetilde{L}_\alpha \{u_{n+1}(x)\} \right\} = \delta^\alpha \left\{ \widetilde{L}_\alpha \{u_{n+1}(x)\} \right\} + \delta^\alpha \left\{ \widetilde{L}_\alpha \left\{ \frac{\lambda (x)^\alpha}{\Gamma(1+\alpha)} \right\} \widetilde{L}_\alpha \{L_\alpha u_n(x)\} \right\} \tag{11b}$$

By using computation of (11b), we get:

$$\delta^\alpha \left\{ \widetilde{L}_\alpha \{u_{n+1}(x)\} \right\} = \delta^\alpha \left\{ \widetilde{L}_\alpha \{u(x)\} \right\} + \widetilde{L}_\alpha \left\{ \frac{\lambda (x)^\alpha}{\Gamma(1+\alpha)} \right\} \left\{ \delta^\alpha \widetilde{L}_\alpha \{L_\alpha u_n(x)\} \right\} \tag{12a}$$

where there is a following relation [18, 19]:

$$\widetilde{L}_\alpha \{u_n^{(k\alpha)}(x)\} = s^{k\alpha} \widetilde{L}_\alpha \{u_n(x)\} - s^{(k-1)\alpha} u_n(0) - s^{(k-2)\alpha} u_n^{(\alpha)}(0) - \cdots - u_n^{[(k-1)\alpha]}(0) \tag{12b}$$

Hence, from (12a) we get:

$$1 + \widetilde{L}_\alpha \left\{ \frac{\lambda (x)^\alpha}{\Gamma(1+\alpha)} \right\} s^{k\alpha} = 0 \tag{13a}$$

or

$$\widetilde{L}_\alpha \left\{ \frac{\lambda (x)^\alpha}{\Gamma(1+\alpha)} \right\} = -\frac{1}{s^{k\alpha}} \tag{13b}$$

Taking the inverse version of the Yang-Fourier transform, we have:

$$\frac{\lambda(x)^{\alpha}}{\Gamma(1+\alpha)} = \tilde{L}_{\alpha}^{-1}\left\{-\frac{1}{s^{k\alpha}}\right\} = -\frac{(x)^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]}, \quad k \in N \tag{14}$$

In view of eq. (14), we obtain:

$$\tilde{L}_{\alpha}\{u_{n+1}(x)\} = \tilde{L}_{\alpha}\{u_n(x)\} - \tilde{L}_{\alpha}\left\{{}_0 I_x^{(\alpha)}\left[\frac{(x-t)^{(\kappa-1)\alpha}}{\Gamma[1+(k-1)\alpha]}[L_{\alpha}u_n(t) + R_{\alpha}\tilde{u}_n(t) - g(t)]\right]\right\} \tag{15a}$$

Therefore, we have the following iteration algorithm:

$$\tilde{L}_{\alpha}\{u_{n+1}(x)\} = \tilde{L}_{\alpha}\{u_n(x)\} - \tilde{L}_{\alpha}\left\{\frac{x^{(k-1)\alpha}}{\Gamma[1+(k-1)\alpha]}\right\}\tilde{L}_{\alpha}\{[L_{\alpha}u_n(x) + R_{\alpha}u_n(x) - g(t)]\} \tag{15b}$$

From eq. (12b) we get:

$$\delta^{\alpha}\left\{\frac{s^{(k-1)\alpha}u_n(0) + s^{(k-2)\alpha}u_n^{(\alpha)}(0) + \cdots + u_n^{[(k-1)\alpha]}}{s^{k\alpha}}\right\} \tag{16a}$$

Hence, form eq. (16a) the initial value is determined by:

$$\begin{aligned} u_0(x) &= \tilde{L}_{\alpha}^{-1}\left\{\frac{s^{(k-1)\alpha}u(0) + s^{(k-2)\alpha}u^{\alpha}(0) + \cdots + u_n^{[(k-1)\alpha]}(0)}{s^{k\alpha}}\right\} = \\ &= u(0) + \frac{x^{\alpha}}{\Gamma(1+\alpha)}u^{(\alpha)}(0) + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}u^{(2\alpha)}(0) + \cdots + \frac{x^{k\alpha}}{\Gamma(1+k\alpha)}u^{(k\alpha)}(0) \end{aligned} \tag{16b}$$

Consequently, the Yang-Laplace transform for identification of fractal Lagrange multiplier is shown in eqs. (15a, b) which leads to the local fractional series solution:

$$u = \lim_{n\to\infty}\tilde{L}_{\alpha}^{-1}\{\tilde{L}_{\alpha}u_n\} \tag{17}$$

Application to fractal heat-conduction problem

In order to illustrate the proposed method, we give the fractal heat-conduction problem. As it was demonstrated earlier [20, 21], the heat equation on Cantor sets (local fractional heat equation) can be expressed as:

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = \frac{\partial^{2\alpha} u(x,t)}{\partial t^{2\alpha}} \tag{18a}$$

where $u(x, t)$ is a fractal heat flux.

The initial value is presented as [21]:

$$\frac{\partial^{\alpha} u(0,t)}{\partial x^{\alpha}} = E_{\alpha}(t^{\alpha}), \quad u(0,t) = 0 \tag{18b,c}$$

By using eq. (15a) we structure the iterative relation as:

$$\begin{aligned} \tilde{L}_{\alpha}\{u_{n+1}(x,t)\} &= \tilde{L}_{\alpha}\{u_n(x,t)\} - \tilde{L}_{\alpha}\left\{\frac{x^{\alpha}}{\Gamma(1+\alpha)}\right\}\tilde{L}_{\alpha}\left\{\left[\frac{\partial^{2\alpha}u_n(x,t)}{\partial x^{2\alpha}} - \frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}}\right]\right\} = \\ &= \tilde{L}_{\alpha}\{u_n(x,t)\} - \frac{1}{s^{2\alpha}}\left\{\left[s^{2\alpha}\tilde{L}_{\alpha}\{u_n(x,t)\} - s^{\alpha}u_n(0,t) - u_n^{(\alpha)}(0,t) - \frac{\partial^{\alpha}\tilde{L}_{\alpha}\{u_n(x,t)\}}{\partial t^{\alpha}}\right]\right\} = \\ &= \frac{1}{s^{\alpha}}u_n(0,t) + \frac{u_n^{(\alpha)}(0,t)}{s^{2\alpha}} + \frac{1}{s^{2\alpha}}\frac{\partial^{\alpha}\tilde{L}_{\alpha}\{u_n(x,t)\}}{\partial t^{\alpha}} \end{aligned} \tag{19a}$$

In view of eqs. (17 b, c), the initial value reads:

$$u_0(x,t)=\frac{x^{\alpha}E_{\alpha}(t^{\alpha})}{\Gamma(1+\alpha)} \tag{19b}$$

and its Yang-Laplace transform is:

$$\widetilde{L}_{\alpha}\{u_0(x,t)\}=\frac{E_{\alpha}(t^{\alpha})}{s^{2\alpha}} \tag{19c}$$

where [19]

$$\widetilde{L}_{\alpha}\left\{\frac{x^{k\alpha}}{\Gamma(1+k\alpha)}\right\}=\frac{1}{s^{(k+1)\alpha}} \tag{19d}$$

Hence, we get the first approximation, namely:

$$\widetilde{L}_{\alpha}\{u_1(x,t)\}=\frac{1}{s^{\alpha}}u_0(0,t)+\frac{u_0^{(\alpha)}(0,t)}{s^{2\alpha}}+\frac{1}{s^{2\alpha}}\frac{\partial^{\alpha}\widetilde{L}_{\alpha}\{u_0(x,t)\}}{\partial t^{\alpha}}=E_{\alpha}(t^{\alpha})\left(\frac{1}{s^{2\alpha}}+\frac{1}{s^{4\alpha}}\right) \tag{20a}$$

Thus,

$$u_1(x,t)=\widetilde{L}_{\alpha}^{-1}\left\{E_{\alpha}(t^{\alpha})\left(\frac{1}{s^{2\alpha}}+\frac{1}{s^{4\alpha}}\right)\right\}=E_{\alpha}(t^{\alpha})\left(\frac{x^{\alpha}}{\Gamma(1+\alpha)}+\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}\right) \tag{20b}$$

The second approximation reads:

$$\widetilde{L}_{\alpha}\{u_2(x,t)\}=\frac{1}{s^{\alpha}}u_1(0,t)+\frac{u_1^{(\alpha)}(0,t)}{s^{2\alpha}}+\frac{1}{s^{2\alpha}}\frac{\partial^{\alpha}\widetilde{L}_{\alpha}\{u_1(x,t)\}}{\partial t^{\alpha}}=E_{\alpha}(t^{\alpha})\left(\frac{1}{s^{2\alpha}}+\frac{1}{s^{4\alpha}}+\frac{1}{s^{6\alpha}}\right) \tag{21a}$$

Therefore, we get:

$$u_1(x,t)=\widetilde{L}_{\alpha}^{-1}\left\{E_{\alpha}(t^{\alpha})\left(\frac{1}{s^{2\alpha}}+\frac{1}{s^{4\alpha}}+\frac{1}{s^{6\alpha}}\right)\right\}=E_{\alpha}(t^{\alpha})\left[\frac{x^{\alpha}}{\Gamma(1+\alpha)}+\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{x^{5\alpha}}{\Gamma(1+5\alpha)}\right] \tag{21b}$$

The other approximations are written as:

$$\begin{aligned}\widetilde{L}_{\alpha}\{u_3(x,t)\}&=\frac{1}{s^{\alpha}}u_2(0,t)+\frac{u_2^{(\alpha)}(0,t)}{s^{2\alpha}}+\frac{1}{s^{2\alpha}}\frac{\partial^{\alpha}\widetilde{L}_{\alpha}\{u_1(x,t)\}}{\partial t^{\alpha}}=\\&=E_{\alpha}(t^{\alpha})\left(\frac{1}{s^{2\alpha}}+\frac{1}{s^{4\alpha}}+\frac{1}{s^{6\alpha}}+\frac{1}{s^{8\alpha}}\right)\\&\quad\vdots\end{aligned} \tag{22}$$

Consequently, the local fractional series solution is:

$$\begin{aligned}u&=\lim_{n\to\infty}\widetilde{L}_{\alpha}^{-1}\{\widetilde{L}_{\alpha}u_n\}=\lim_{n\to\infty}\widetilde{L}_{\alpha}^{-1}\left\{E_{\alpha}(t^{\alpha})\left(\sum_{i=1}^{n}\frac{1}{s^{2i\alpha}}\right)\right\}=\\&=\lim_{n\to\infty}\left[E_{\alpha}(t^{\alpha})\sum_{k=0}^{n}\frac{x^{(2k+1)\alpha}}{\Gamma[1+(2k+1)\alpha]}\right]=E_{\alpha}(t^{\alpha})\sinh_{\alpha}(x^{\alpha})\end{aligned} \tag{23}$$

The same final result was developed by the local fractional variational method [21].

Conclusions

A local fractional continuous solution to fractal heat conduction problems was developed is obtained by a new approach coupling process the variational iteration method and the Yang-Laplace transform. The method is straightforward and well exemplified by a solution of fractal heat conduction with fractal Neumann condition.

Acknowledgments

The work was supported by National Natural Science Foundation of China (No. 61170317) and the National Natural Science Foundation of Hebei Province (No. A2012209043 and No. E2013209215)

Nomenclature

$\tilde{L}_\alpha$ – Yang-Laplace transform of $f(x)$
$\tilde{L}_\alpha^{-1}$ – inverse version of Yang-Laplace transform of $f_\omega^{F,\alpha}(\omega)$
t – time, [s]
$u(x,t)$ – the temperature function, [K]
x – space co-ordinate, [m]
α – time fractal dimensional order, [–]

References

[1] Widder, D. V., *The Heat Equation*, Academic Press, New York, USA, 1976
[2] Hsiao, G., MacCamy, R. C., Solution of Boundary Value Problems by Integral Equations of the First Kind, *SIAM Review, 15* (1973), 4, pp. 687-705
[3] Linz, P., *Analytical and Numerical Methods for Volterra Equations*, Siam, Philadelphia, Penn., USA, 1985
[4] Dehghan, M., The One-Dimensional Heat Equation Subject to a Boundary Integral Specification, Chaos, *Solitons, Fractals, 32* (2007), 2, pp. 661-675
[5] Ioannou, Y., *et al*., Approximate Solution to Fredholm Integral Equations Using Linear Regression and Applications to Heat and Mass Transfer, *Engineering Analysis with Boundary Elements, 36* (2012), 8, pp. 1278-1283
[6] Nadeem, S., Akbar, N. S., Effects of Heat Transfer on the Peristaltic Transport of MHD Newtonian Fluid with Variable Viscosity: Application of Adomian Decomposition Method, *Communications in Nonlinear Science and Numerical Simulation, 14* (2009),11, pp. 3844-3855
[7] Abbasbandy, S., The Application of Homotopy Analysis Method to Nonlinear Equations Arising in Heat Transfer, *Physics Letters A, 360* (2006), 1, pp. 109-113
[8] Joneidi, A. A., *et al*., Differential Transformation Method to Determine Fin Efficiency of Convective Straight Fins with Temperature Dependent Thermal Conductivity, *International Communications in Heat and Mass Transfer, 36* (2009), 7, pp. 757-762
[9] Simoes, N., *et al*., Transient Heat Conduction under Nonzero Initial Conditions: A Solution Using the Boundary Element Method in the Frequency Domain, *Engineering Analysis with Boundary Elements, 36* (2012), 4, pp. 562-567
[10] Hristov, J., Approximate Solutions to Fractional Sub-Diffusion Equations: The Heat-Balance Integral Method, *The European Physical Journal Special Topics, 193* (2011), 4, pp. 229-243
[11] Hristov, J., Heat-Balance Integral to Fractional (Half-Time) Heat Diffusion Sub-Model, *Thermal Science, 14* (2010), 2, pp. 291-316
[12] Hristov, J., Starting Radial Subdiffusion from a Central Point through a Diverging Medium (a Sphere): Heat-Balance Integral Method, *Thermal Science, 15* (2011), Suppl. 1, pp. S5-S20
[13] He, J.-H., Variational Iteration Method – a Kind of Non-Linear Analytical Technique: Some Examples, *International Journal of Non-Linear Mechanics, 34* (1999), 4, pp. 699-708
[14] Hristov, J., An Exercise with the He's Variation Iteration Method to a Fractional Bernoulli Equation Arising in Transient Conduction with Non-Linear Heat Flux at the Boundary, *International Review of Chemical Engineering, 4* (2012), 5, pp. 489-497
[15] Hetmaniok, E., *et al*., Application of the Variational Iteration Method for Determining the Temperature in the Heterogeneous Casting-Mould System, *International Review of Chemical Engineering, 4* (2012), 5, pp. 511-515
[16] Torvattanabun, M., *et al*., Efficacy of Variational Iteration Method for Nonlinear Heat Transfer Equations – Classical and Multistage Approach, *International Review of Chemical Engineering, 4* (2012), 5, pp. 524-528
[17] Wu, G. C., Lee, E. W. M., Fractional Variational Iteration Method and its Application, *Physics Letters A, 374* (2010), 25, pp. 2506-2509

[18] Yang, X. J., *Local Fractional Functional Analysis and Its Applications*, Asian Academic Publisher Limited, Hong Kong, 2011
[19] Yang, X. J., Local Fractional Integral Transforms, *Progress in Nonlinear Science, 4* (2011), pp. 1-225
[20] Yang, X. J., *Advanced Local Fractional Calculus and Its Applications*, World Science Publisher, New York, 2012
[21] Yang, X. J., Baleanu, D., Fractal Heat Conduction Problem Solved by Local Fractional Variation Iteration Method, *Thermal Science, 17* (2013), 2, pp. 625-628
[22] Yang, Y. J., *et al*., A Local Fractional Variational Iteration Method for Laplace Equation within Local Fractional Operators, Abstract and Applied Analysis, 2013 (2013), Article ID 202650
[23] Su, W. H., *et al*., Damped Wave Equation and Dissipative Wave Equation in Fractal Strings within the Local Fractional Variational Iteration Method, *Fixed Point Theory and Applications, 2013* (2013), 1, pp. 89-102
[24] He, J.-H., Liu, F.-J., Local Fractional Variational Iteration Method for Fractal Heat Transfer in Silk Cocoon Hierarchy, *Nonlinear Science Letters A, 4* (2013), 1, pp. 15-20
[25] Hesameddini, E., Latifizadeh, H., Reconstruction of Variational Iteration Algorithms Using the Laplace Transform, *International Journal of Nonlinear Sciences and Numerical Simulation, 10* (2009), 11-12, pp. 1377-1382
[26] Khuri, S. A., Sayfy, A., A Laplace Variational Iteration Strategy for the Solution of Differential Equations, *Applied Mathematics Letters, 25* (2012), 12, pp. 2298-2305
[27] Wu, G. C., Baleanu, D., Variational Iteration Method for Fractional Calculus – an Universal Approach by Laplace Transform, *Advances in Difference Equations, 2013* (2013), 1, pp.1-9
[28] Hu, M. S., *et al*., Local Fractional Fourier Series with Application to wave Equation in Fractal Vibrating String, *Abstract and Applied Analysis, 2012* (2012), Article ID 567401
[29] He, J.-H., Asymptotic Methods for Solitary Solutions and Compactons, *Abstract and Applied Analysis*, *2012* (2012), Article ID 916793

Paper submitted: August 26, 2012
Paper revised: April 24, 2013
Paper accepted: May 24, 2013