OPTIMIZATION OF OPERATION OF ENERGY SUPPLY SYSTEMS WITH CO-GENERATION AND ABSORPTION REFRIGERATION

by

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Co-generation systems, together with absorption refrigeration and thermal storage, can result in substantial benefits from the economic, energy and environmental point of view. Optimization of operation of such systems is important as a component of the entire optimization process in pre-construction phases, but also for short-term energy production planning and system control. This paper proposes an approach for operational optimization of energy supply systems with small or medium scale co-generation, additional boilers and heat pumps, absorption and compression refrigeration, thermal energy storage and interconnection to the electric utility grid. In this case, the objective is to minimize annual costs related to the plant operation. The optimization problem is defined as mixed integer non-linear and solved combining modern stochastic techniques: genetic algorithms and simulated annealing with linear programming using the object oriented "ESO-MS" software solution for simulation and optimization of energy supply systems, developed as a part of this research. This approach is applied to optimize a hypothetical plant that might be used to supply a real residential settlement in Niš, Serbia. Results are compared to the ones obtained after transforming the problem to mixed 0–1 linear and applying the branch and bound method.

Key words: *optimization, co-generation, absorption refrigeration, genetic algorithms, simulated annealing, linear programming*

Introduction

Energy supply systems, which integrate the technologies of co-generation, absorption refrigeration and thermal storage, can provide substantial benefits from the economic, energy, and environmental point of view [1]. It is very convenient to use cogenerated heat during summer months for cooling with absorption refrigerators and thermal storage for peak shaving enabling co-generation and other components to operate continuously at nominal conditions. Co-generation systems can be operated in variety of modes, like baseload, load following, peak shaving and economic dispatch mode, and any analysis of co-generation system economics must consider these operating modes [2], because the operation cost of the plant largely depends on the planning method and operation regime [3]. The importance of implementation and optimization of co-generation systems is stressed in [4, 5]. Optimization

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of such systems is usually divided into three segments: (1) operation regimes, (2) design parameters, and (3) plant synthesis, *i. e.* superstructure, and each should include the previous [6]. Optimizations of operation and design are often integrated [3, 7, 8]. Optimization of operation parameters is important as a component of the entire optimization process, but also for short-term energy production planning and system control.

Depending on the level of details considered, requirements for accuracy of the results, and available computer resources, different models are defined and suitable optimization methods are chosen. Lozano *et al*. [9] define a simple linear model only with continuous variables representing energy flows suitable to be solved using techniques of linear programming (LP). A linear problem is also defined by Cardona *et al*. [10]. Using binary (0–1) or integer variables to bound the load level and define the on/off status of the units results in more sophisticated models, but also in the necessity for more complicated and time and resources consuming optimization methods. The mixed integer linear problem (MILP) is often defined, as in [3, 7, 8, 11-13]. MILP are usually solved with the branch and bound method (BBM), but Sakawa *et al*. [13] have shown that for larger problems a combination of genetic algorithms (GA) and LP could be much more effective providing highly accurate approximate solutions. Most authors just assume the steady-state operation of components and do not consider their transient behavior and increased energy consumption and costs during startups and shutdowns, even if that might improve the realistic aspects of the results, as concluded by Weber *et al*. [14]. When included, this usually leads to non-linear models [15]. If non-linearities are only related to binary variables, such problems might be transformed to MILP by introducing new decision variables and inequality constraints, as shown in [16]. Non-linear models are also used when there are case specific non-linear constraints or objective functions, and they might be solved using Lagrange multipliers [17], sequential quadratic programming [18], *etc*. For solving complex non-linear and mixed integer non-linear problems, authors sometimes decide to use GA [14, 19-21], proven to be convenient and effective for energy systems optimization, while simulated annealing (SA) is used on rare occasions [22]. Objective functions are usually economic, environmental, or related to primary energy. Multiobjective optimization is used in [14, 23, 24], *etc*.

When considering the whole year, rather than only one or several consecutive days, the entire period is usually represented by few typical days, because it is time consuming, although not impossible, to treat 8760 hours in an optimization problem. Three typical days are used in [18], 6 in [25], 12 in [1, 3, 7, 14], while there are 24 typical days in [8]. Ortiga *et al*. [26] suggest the methodology for choosing typical days.

This paper proposes an operational optimization approach for energy supply systems with small or medium scale co-generation, absorption refrigeration and thermal energy storage, which represents an extension of the work published in [27, 28]. The optimization problem is defined as mixed integer non-linear and solved combining modern stochastic techniques: genetic algorithms and simulated annealing with linear programming, exploiting the advantages of both. Results are compared to the ones obtained by transformation of the same problem to MILP and the use of BBM. All the methods used are described in the optimization and mathematical programming literature, such as [29]. The approach presented here might be extended to the design level of optimization.

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Description of the energy supply plant

The energy supply plant considered in this paper consists of co-generation units (CG), additional heating units, *i. e.* hot water boilers (CH), heat pumps (HP), heat thermal energy storage (TS), and components for refrigeration, *i. e.* compression (CR) and absorption chillers (AR). Energy flows are shown in fig. 1.

Generated electricity is used for onsite consumers' demand satisfaction and operation of components and auxiliary equipment (pumps, fans, *etc.*). Energy exchange via electrical transformers (ET) with the national grid is foreseen.

In this approach, different temperature levels of obtained and consumed heat are recognized. High and medium temperature outputs from CG and CH could be used to satisfy all the consumers' heat demands, to charge TS or to run AR, while lower temperature output from HP could only be used by consumers that require low temperature heat. CR and AR are used for satisfying cooling demands.

The presented model is suitable when reciprocating engines, gas turbines, or fuel cells are used as CG.

Thermal output from gas turbines exhaust gasses is at high temperature. Thermal output from reciprocating en-

Figure 1. Energy flows chart for energy supply plant and consumers

gines is obtained in three qualitatively different forms: (1) high temperature output from exhaust gasses (XG), (2) medium temperature output from engine cooling, and (3) low temperature output from charge air cooling. The last one is assumed to be completely rejected to the environment. If not required and not being able to be stored, medium and high temperature outputs can also be partly rejected to the environment using coolers with fans and forced convection, which results in additional consumption of electrical energy for fans operation. XG useful output that cannot be used could intentionally be wasted if a part of the XG flow is allowed to bypass the heat exchanger.
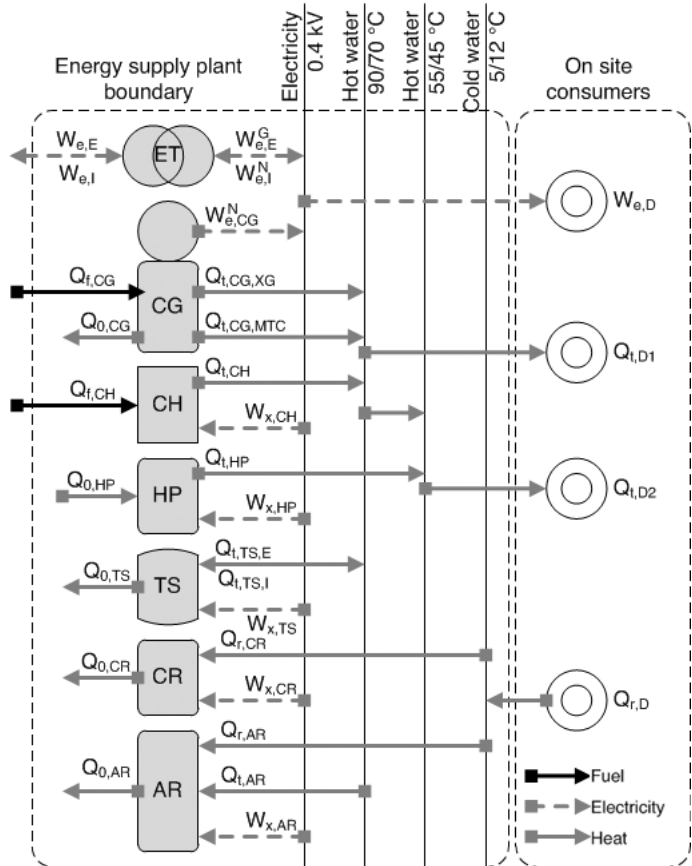
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Mathematical model and problem formulation

Inputs and decision variables

Inputs to the optimization problem are: energy demand profiles, ambient conditions (air temperature, pressure, *etc*.), system configuration, components design parameters and performance curves, prices of electrical energy and fuel, conversion factors, specific constraints, input parameters for solvers, initial values of integer decision variables, *etc*.

Table 1. Decision variables summary

Variable	Symbol	Type	Unit	Lower and upper bounds
CG on/off variable	$\delta_{CG,k}^{i,j}$	0–1	–	–
CG generated electrical power	$\dot{W}_{e,CG,k}^{i,j}$	Real	[kW]	$\delta_{CG,k}^{i,j} \cdot \dot{W}_{e,CG,k,min}^{i,j} \leq \dot{W}_{e,CG,k}^{i,j} \leq \delta_{CG,k}^{i,j} \cdot \dot{W}_{e,CG,k,max}^{i,j}$
CG used thermal output from XG	$\dot{Q}_{t,CG,XG,k}^{i,j}$	Real	[kW]	$0 \leq \dot{Q}_{t,CG,XG,k}^{i,j} \leq \dot{Q}_{t,CG,XG-tot,k}^{i,j}$
CG rejected thermal output	$\dot{Q}_{0,CG,k}^{i,j}$	Real	[kW]	$0 \leq \dot{Q}_{0,CG,k}^{i,j} \leq \dot{Q}_{t,CG,MTC,k}^{i,j} + \dot{Q}_{t,CG,XG,k}^{i,j}$
CH on/off variable	$\delta_{CH,k}^{i,j}$	0–1	–	–
CH thermal output	$\dot{Q}_{t,CH,k}^{i,j}$	Real	[kW]	$\delta_{CH,k}^{i,j} \cdot \dot{Q}_{t,CH,k,min}^{i,j} \leq \dot{Q}_{t,CH,k}^{i,j} \leq \delta_{CH,k}^{i,j} \cdot \dot{Q}_{t,CH,k,max}^{i,j}$
HP on/off variable	$\delta_{HP,k}^{i,j}$	0–1	–	–
HP thermal output	$\dot{Q}_{t,HP,k}^{i,j}$	Real	[kW]	$\delta_{HP,k}^{i,j} \cdot \dot{Q}_{t,HP,k,min}^{i,j} \leq \dot{Q}_{t,HP,k}^{i,j} \leq \delta_{HP,k}^{i,j} \cdot \dot{Q}_{t,HP,k,max}^{i,j}$
CR on/off variable	$\delta_{CR,k}^{i,j}$	0–1	–	–
CR refrigeration effect	$\dot{Q}_{r,CR,k}^{i,j}$	Real	[kW]	$\delta_{CR,k}^{i,j} \cdot \dot{Q}_{r,CR,k,min}^{i,j} \leq \dot{Q}_{r,CR,k}^{i,j} \leq \delta_{CR,k}^{i,j} \cdot \dot{Q}_{r,CR,k,max}^{i,j}$
AR on/off variable	$\delta_{AR,k}^{i,j}$	0–1	–	–
AR refrigeration effect	$\dot{Q}_{r,AR,k}^{i,j}$	Real	[kW]	$\delta_{AR,k}^{i,j} \cdot \dot{Q}_{r,AR,k,min}^{i,j} \leq \dot{Q}_{r,AR,k}^{i,j} \leq \delta_{AR,k}^{i,j} \cdot \dot{Q}_{r,AR,k,max}^{i,j}$
Electricity import allowed	$\delta_I^{i,j}$	0–1	–	$\delta_I^{i,j} + \delta_E^{i,j} \leq 1$; $\delta_E^{i,j} \leq \sum_{k=1}^{n_{CG}} \delta_{CG,k}^{i,j}$
Electricity export allowed	$\delta_E^{i,j}$	0–1	–	(optional)
Gross imported electrical power (losses excluded)	$\dot{W}_{e,I}^{i,j}$	Real	[kW]	$\dot{W}_{e,I}^{i,j} \leq \delta_I^{i,j} \cdot \dot{W}_{e,I,max}^{i,j}$
Net exported electrical power (losses included)	$\dot{W}_{e,E}^{i,j}$	Real	[kW]	$\dot{W}_{e,E}^{i,j\,G} \leq \delta_E^{i,j} \cdot \sum_{k=1}^{n_{CG}} W_{e,CG,k}^{i,j\,N}$
TS charging rate	$\dot{Q}_{t,TS,I}^{i,j}$	Real	[kW]	$0 \leq \dot{Q}_{t,TS,I}^{i,j} \leq \dot{Q}_{t,TS,I,max}^{i,j}$
TS discharging rate	$\dot{Q}_{t,TS,E}^{i,j}$	Real	[kW]	$0 \leq \dot{Q}_{t,TS,E}^{i,j} \leq \dot{Q}_{t,TS,E,max}^{i,j}$
TS average temperature at the beginning of hour j	$t_{TS}^{i,j}$	Real	[°C]	$70\,°C \leq t_{TS}^{i,j} \leq 90\,°C$

There are two types of decision variables: (a) integer, *i. e.* 0–1 on/off variables, $\delta_k^{i,j}$, indicating if the unit k is operating during the j-th hour of the i-th day (value 1), or not (value 0), and (b) real non-negative continuous variables related to energy outputs of the components and the average temperatures of TS medium for each observed time interval (hour), *i. e.* dispatch problem variables. Some real decision variables are equal to zero if the unit is off or otherwise bounded with predefined minimal and maximal values that might depend on input variables such as air temperature, pressure, *etc*. Decision variables are summarized in tab. 1.

Performance characteristics of the components

For all the components where energy conversion occurs, under the assumption of steady operation, linear equations are used to represent relations between energy inputs and outputs, similarly as in [3, 7, 13]. General relationship between energy output, $\dot{X}_k^{i,j}$, a real decision variable, where $\delta_k^{i,j} \dot{X}_{k,\min}^{i,j} \leq \dot{X}_k^{i,j} \leq \delta_k^{i,j} \dot{X}_{k,\max}^{i,j}$, and input, $\dot{Y}_k^{i,j}$, of the unit k, during j-th hour of the i-th day, both in [kW], is given in eq. (1):

$$\dot{Y}_k^{i,j} = a_k^{i,j} \dot{X}_k^{i,j} + b_k^{i,j} \delta_k^{i,j} \tag{1}$$

where $\delta_k^{i,j}$ is the 0–1 on/off variable, while $a_k^{i,j}$ and $b_k^{i,j}$ are the linear regression coefficients usually derived from manufacturers' data. Both coefficients might depend on input variables, such as air temperature and pressure, *etc*., so they are also considered as inputs to the optimization problem.

Additional energy input related to transient startup and shutdown operation of the unit k, during the j-th hour of the i-th day, $Y_{s,k}^{i,j}$, in kWh, is calculated using eq. (2):

$$Y_{s,k}^{i,j} = \delta_k^{i,j} (\sigma_k^{i,j} Y_{\sigma,k}^{i,j} + \varsigma_k^{i,j} Y_{\varsigma,k}^{i,j}) \tag{2}$$

where $Y_{\sigma,k}^{i,j}$ and $Y_{\varsigma,k}^{i,j}$ are the additional energy inputs for cold startup and for shutdown, both in kWh, while $\sigma_k^{i,j}$ and $\varsigma_k^{i,j}$ are the startup and shutdown correction factors, depending on the unit turning on and off schedule. Energy needed for startup can be well approximated as a (nearly) exponential function of the time the unit was off, τ_s, being zero for $\tau_s = 0$. Thus, $\sigma_k^{i,j}$ might be calculated as: $\sigma_k^{i,j} = 1 - \exp(a \cdot \tau_s)$, where a is the negative regression coefficient, $\tau_s = \tau[(1 - \delta_k^{i,j-1}) + (1 - \delta_k^{i,j-1})(1 - \delta_k^{i,j-2}) + \ldots]$ and $\tau = 1$ hours is the length of each observed time interval. For simplicity, these factors are calculated similarly as in the eQUEST software [30]: $\sigma_k^{i,j}$ has nearly exponential dependence on the number of hours the unit was off, *i. e.* it is equal to 0 if it was on during the previous hour, 0.5 if it was off for 1 hour (hot startup), 0.8 for 2 hours (warm startup), and 1 for 3 or more hours (cold startup), while $\varsigma_k^{i,j}$ is equal to 0 if the unit is going to be on during the next hour, and 1 otherwise.

Total energy input of the unit k, during j-th time interval, of the i-th day is calculated as in eq. (3) and total energy input for n units of the same type as in eq. (4), both in [kWh]:

$$Y_k^{i,j} = Y_{s,k}^{i,j} + \dot{Y}_k^{i,j} \tau = Y_{s,k}^{i,j} + (a_k^{i,j} \dot{X}_k^{i,j} + b_k^{i,j} \delta_k^{i,j}) \tau \tag{3}$$

$$\sum_{k=1}^{n} Y_k^{i,j} = \sum_{k=1}^{n} \left[Y_{s,k}^{i,j} + (a_k^{i,j} \dot{X}_k^{i,j} + b_k^{i,j} \delta_k^{i,j}) \tau \right] \tag{4}$$

If there are the same units (in terms of capacities and performances) assumed to operate in the same regime, and if $\dot{X}_k^{i,j}$, bounded with $\gamma_k^{i,j} \dot{X}_{k,\min}^{i,j} \leq \dot{X}_k^{i,j} \leq \gamma_k^{i,j} \dot{X}_{k,\max}^{i,j}$, is the total energy output of all these units, the total input when γ units are on ($\gamma \leq n$) is:

$$\sum_{k=1}^{n} Y_k^{i,j} = \sum_{k=1}^{n} Y_{s,k}^{i,j} + a_k^{i,j} \dot{X}_k^{i,j} + b_k^{i,j} \gamma_k^{i,j} \quad \tau \tag{5}$$

Annual operation and maintenance (OM) costs of each unit k can be expressed as the sum of fixed and variable costs, $Z_{c,k}$ and $Z_{v,k}$ [31], and variable costs are often expressed in terms of energy output and/or number of hours of operation:

$$Z = Z_{c,k} + Z_{v,k} = Z_{c,k} + \sum_{i=1}^{n_{td}} \left[d_i \sum_{j=1}^{24/\tau} \alpha_k^{i,j} \dot{X}_k^{i,j} + \beta_k^{i,j} \delta_k^{i,j} \quad \tau \right] \tag{6}$$

where n_{td} is the number of typical days, d_i is the number of days represented with i-th typical day, and α_k and β_k are the linear regression coefficients. Additional OM costs related to startups and shutdowns can also be foreseen.

Co-generation components. For each CG k, for each hour j of each day i, both thermal outputs, total from XG, $\dot{Q}_{t,CG,XG-tot,k}^{i,j}$, and from engine cooling, $\dot{Q}_{t,CG,MTC,k}^{i,j}$, are presented as linear functions of generated electrical power, eqs. (7) and (8):

$$\dot{Q}_{t,CG,XG-tot,k}^{i,j} = a_{t,CG,XG,k}^{i,j} \dot{W}_{e,CG,k}^{i,j} + b_{t,CG,XG,k}^{i,j} \delta_{CG,k}^{i,j} \tag{7}$$

$$\dot{Q}_{t,CG,MTC,k}^{i,j} = a_{t,CG,MTC,k}^{i,j} \dot{W}_{e,CG,k}^{i,j} + b_{t,CG,MTC,k}^{i,j} \delta_{CG,k}^{i,j} \tag{8}$$

Fuel input is also assumed to linearly depend on generated electrical power for the steady regime. If the unit was recently turned on or should be turned off at the end of the observed time interval, the total fuel input, in kWh, should also include additional energy related to startup and/or shutdown, as shown in eq. (9):

$$Q_{f,CG,k}^{i,j} = Q_{s,f,CG,k}^{i,j} + (a_{f,CG,k}^{i,j} \dot{W}_{e,CG,k}^{i,j} + b_{f,CG,k}^{i,j} \delta_{CG,k}^{i,j}) \tau \tag{9}$$

Auxiliary electrical power used for pumps, fans, *etc.*, in kW, depends on the generated electrical power and thermal power rejected to the environment by forced convection, as given in eq. (10):

$$\dot{W}_{x,CG,k}^{i,j} = a_{0,CG,k}^{i,j} \dot{Q}_{0,CG,k}^{i,j} + a_{x,CG,k}^{i,j} \dot{W}_{e,CG,k}^{i,j} + b_{x,CG,k}^{i,j} \delta_{CG,k}^{i,j} \tag{10}$$

Finally, net thermal output, $Q_{t,CG,k}^{i,j}$, and net electrical output, $W_{e,CG,k}^{i,j\,N}$, of the unit k during the j-th hour of the i-th day, expressed in kWh, can be calculated using eqs. (11) and (12):

$$Q_{t,CG,k}^{i,j} = (\dot{Q}_{t,CG,XG,k}^{i,j} + \dot{Q}_{t,CG,MTC,k}^{i,j} - \dot{Q}_{0,CG,k}^{i,j}) \tau \tag{11}$$

$$W_{e,CG,k}^{i,j\,N} = (\dot{W}_{e,CG,k}^{i,j} - \dot{W}_{x,CG,k}^{i,j}) \tau \tag{12}$$

Auxiliary heating components. Total CH fuel and auxiliary electrical energy inputs are given in eqs. (13) and (14), and total HP electrical input, is given in eq. (15), all in kWh:

$$Q_{f,CH,k}^{i,j} = Q_{s,f,CH,k}^{i,j} + (a_{f,CH,k}^{i,j} \dot{Q}_{t,CH,k}^{i,j} + b_{f,CH,k}^{i,j} \delta_{CH,k}^{i,j}) \tau \tag{13}$$

$$W_{x,CH,k}^{i,j} = W_{s,x,CH,k}^{i,j} + (a_{x,CH,k}^{i,j} \dot{Q}_{t,CH,k}^{i,j} + b_{x,CH,k}^{i,j} \delta_{CH,k}^{i,j}) \tau \tag{14}$$

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$$W_{x,HP,k}^{i,j} = W_{s,x,HP,k}^{i,j} + (a_{x,HP,k}^{i,j} \dot{Q}_{t,HP,k}^{i,j} + b_{x,HP,k}^{i,j} \delta_{HP,k}^{i,j})\tau \tag{15}$$

Refrigeration components. Total AR thermal and auxiliary electrical inputs, are given in eqs. (16) and (17), while total CR electrical input is given in eq. (18), in kWh:

$$Q_{t,AR,k}^{i,j} = Q_{s,t,AR,k}^{i,j} + (a_{t,AR,k}^{i,j} \dot{Q}_{r,AR,k}^{i,j} + b_{t,AR,k}^{i,j} \delta_{AR,k}^{i,j})\tau \tag{16}$$

$$W_{x,AR,k}^{i,j} = W_{s,x,AR,k}^{i,j} + (a_{x,AR,k}^{i,j} \dot{Q}_{r,AR,k}^{i,j} + b_{x,AR,k}^{i,j} \delta_{AR,k}^{i,j})\tau \tag{17}$$

$$W_{x,CR,k}^{i,j} = W_{s,x,CR,k}^{i,j} + (a_{x,CR,k}^{i,j} \dot{Q}_{r,CR,k}^{i,j} + b_{x,CR,k}^{i,j} \delta_{CR,k}^{i,j})\tau \tag{18}$$

Connection to electricity grid. Two main types of energy losses in transformers are considered when the transformer is energized: core (no load) loss that is roughly constant and winding (load) loss that depends on the load with a small temperature correction [32]. Relations between gross and net imported and exported electrical power are given in eq. (19) and electrical energy exchanged with the grid during hour *j* of day *i*, in kWh, in eq. (20):

$$\dot{W}_{e,I}^{i,j\,N} = a_I^{i,j} \dot{W}_{e,I}^{i,j} + b_I^{i,j} \delta_I^{i,j}, \quad \dot{W}_{e,E}^{i,j\,G} = a_E^{i,j}(\dot{W}_{e,E}^{i,j} + b_E^{i,j} \delta_E^{i,j}) \tag{19}$$

$$W_{e,grid}^{i,j} = (\dot{W}_{e,I}^{i,j\,N} - \dot{W}_{e,E}^{i,j\,G})\tau \tag{20}$$

Thermal energy storage. TS is modeled as a hot water tank with temperature varying from 70°C (empty TS) to 90°C (full TS). TS medium (water) average temperature is not assumed steady during observed time intervals and its change during the *j*-th hour is calculated from the energy balance that includes heat loss to the environment:

$$m_{TS} c_{TS} \frac{dt_{TS}}{d\tau} = \dot{Q}_{t,TS,I} - \dot{Q}_{t,TS,E} - U_{TS} A_{TS} \ t_{TS} - t_0 \tag{21}$$

where m_{TS} is the TS medium mass, c_{TS} – the TS medium specific heat, U_{TS} – the TS envelope overall heat transfer coefficient, and A_{TS} – the TS envelope area, all assumed constant, while t_0 is the surrounding air temperature. From eq. (21), it follows:

$$t_{TS}^{i,j+1} = t_{TS}^{i,j} \exp\left(-\frac{U_{TS} A_{TS}}{m_{TS} c_{TS}}\tau\right) + \left(\frac{\dot{Q}_{t,TS,I}^{i,j} - \dot{Q}_{t,TS,E}^{i,j}}{U_{TS} A_{TS}} + t_0^{i,j}\right)\left[1 - \exp\left(-\frac{U_{TS} A_{TS}}{m_{TS} c_{TS}}\tau\right)\right] \tag{22}$$

Total thermal energy exchanged with the thermal storage and auxiliary electrical energy required by TS during hour *j* of the day *i*, in kWh are given in eqs. (23) and (24):

$$Q_{t,TS}^{i,j} = (\dot{Q}_{t,TS,E}^{i,j} - \dot{Q}_{t,TS,I}^{i,j})\tau \tag{23}$$

$$W_{x,TS}^{i,j} = \dot{W}_{x,TS}^{i,j}\tau = (a_{TS,I}^{i,j} \dot{Q}_{t,TS,I}^{i,j} + a_{TS,E}^{i,j} \dot{Q}_{t,TS,E}^{i,j})\tau \tag{24}$$

Demand satisfaction related constraints

The set of constraints given in (in)equalities (25)-(28) represents the so-called demand satisfaction constraints and have to be valid for each day *i* and hour *j*:

$$W_{e,D}^{i,j} = \dot{W}_{e,grid}^{i,j} + \sum_{k=1}^{n_{CG}} W_{e,CG,k}^{i,j\,N} - \sum_{k=1}^{n_{CH}} W_{x,CH,k}^{i,j} - \sum_{k=1}^{n_{HP}} W_{x,HP,k}^{i,j} -$$

$$-\sum_{k=1}^{n_{CR}} W_{x,CR,k}^{i,j} - \sum_{k=1}^{n_{AR}} W_{x,AR,k}^{i,j} - W_{x,TS}^{i,j} \tag{25}$$

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$$Q_{\mathrm{t,D1}}^{i,j} \le \sum_{k=1}^{n_{\mathrm{CG}}} Q_{\mathrm{t,CG},k}^{i,j} + \sum_{k=1}^{n_{\mathrm{CH}}} \dot{Q}_{\mathrm{t,CH},k}^{i,j}\tau + Q_{\mathrm{t,TS}}^{i,j} - \sum_{k=1}^{n_{\mathrm{AR}}} \dot{Q}_{\mathrm{t,AR},k}^{i,j}\tau \tag{26}$$

$$Q_{\mathrm{t,D1}}^{i,j} + Q_{\mathrm{t,D2}}^{i,j} = \sum_{k=1}^{n_{\mathrm{CG}}} Q_{\mathrm{t,CG},k}^{i,j} + \sum_{k=1}^{n_{\mathrm{CH}}} \dot{Q}_{\mathrm{t,CH},k}^{i,j}\tau + Q_{\mathrm{t,TS}}^{i,j} + \sum_{k=1}^{n_{\mathrm{HP}}} \dot{Q}_{\mathrm{t,HP},k}^{i,j}\tau - \sum_{k=1}^{n_{\mathrm{AR}}} \dot{Q}_{\mathrm{t,AR},k}^{i,j}\tau \tag{27}$$

$$Q_{\mathrm{r,D}}^{i,j} = \sum_{k=1}^{n_{\mathrm{CR}}} \dot{Q}_{\mathrm{r,CR},k}^{i,j}\tau + \sum_{k=1}^{n_{\mathrm{AR}}} \dot{Q}_{\mathrm{r,AR},k}^{i,j}\tau \tag{28}$$

The first constraint is related to electrical demand, $W_{\mathrm{e,D}}^{i,j}$, that has to be satisfied from the electrical grid import and CG, together with the satisfaction of other components of electricity use and grid export. Heating demand $Q_{\mathrm{t,D1}}^{i,j}$ is a higher temperature demand and has to be satisfied from CG, CH, and TS, together with the demand of AR, while heating demand $Q_{\mathrm{t,D2}}^{i,j}$ is a lower temperature demand and might be satisfied from HP. Finally, cooling demand $Q_{\mathrm{r,D}}^{i,j}$ has to be satisfied from CR and AR.

Objective function

The objective is to minimize annual costs related to operation of this energy supply system by determining the best operational regime, subject to the constraints described in this chapter, which is consistent with the maximization of operational profit [6]. Annual energy costs, *C*, and OM costs, *Z*, should be considered. Thus, the objective function is defined as:

$$f = \min(C + Z) \tag{29}$$

where *Z* is calculated according to eq. (6), and *C* according to the prices of imported and exported electrical energy and fuel for co-generation units and for boilers, respectively $\zeta_{\mathrm{e,I}}^{i,j}$, $\zeta_{\mathrm{e,E}}^{i,j}$, $\zeta_{\mathrm{f,CG}}^{i,j}$, and $\zeta_{\mathrm{f,CH}}^{i,j}$:

$$C = \sum_{i=1}^{n_{\mathrm{td}}}\left[d_i \sum_{j=1}^{24/\tau}\left(\zeta_{\mathrm{e,I}}^{i,j}\dot{W}_{\mathrm{e,I}}^{i,j}\tau - \zeta_{\mathrm{e,E}}^{i,j}\dot{W}_{\mathrm{e,E}}^{i,j}\tau + \zeta_{\mathrm{f,CG}} \sum_{k=1}^{n_{\mathrm{CG}}} Q_{\mathrm{f,CG},k}^{i,j} + \zeta_{\mathrm{f,CH}} \sum_{k=1}^{n_{\mathrm{CH}}} Q_{\mathrm{f,CH},k}^{i,j} \right) \right] \tag{30}$$

Other constraints and objectives

It is possible to include additional constraints related to equipment operation, *e. g.* in [10] the maximal hourly load level variation is defined for some components, or imposed by some legal or administrative demands. It is also possible to add constraints related to primary energy savings or (nearly) net zero energy or greenhouse gasses (GHG) emission. Primary energy consumption and GHG emission might be considered objectives for single or multiobjective optimization problems.

Optimization methods

From the presented mathematical model, the following might be concluded:
− the optimization problem defined is mixed integer (0–1) non-linear, with non-linearities occurring due to startup and shutdown transient behavior of the components,

- since all the non-linearities are related to 0–1 variables, the model might be transformed to MILP by introducing additional 0–1 decision variables and inequality constraints, as suggested in [16] or [29], and solved by using classical techniques for this type of optimization problems (*e. g.* BBM),
- decision variables related to different observed time intervals are connected, because of start-up and shutdown energy, thermal storage and, eventually, some additional constraints, so the problem cannot be decomposed into separate sub problems for each time interval (hour), and
- if the vector of all the integer decision variables is predefined, the problem becomes a linear continuous economic dispatch problem, suitable to be solved using LP techniques.

Besides solving the transformed MILP using BBM, another approach is used to solve the problem directly, as shown in fig. 2, characterized by the following:
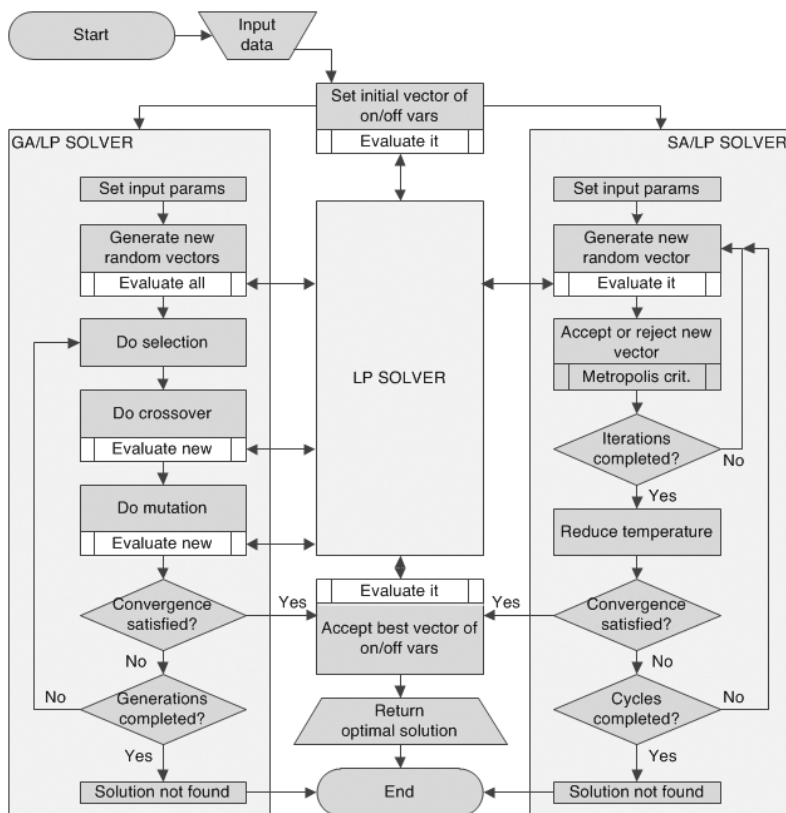

Figure 2. Simplified algorithm of the presented optimization approach

- one of the stochastic methods: 0–1 GA or SA is used to determine integer (0–1) on/off decision variables,
- during each evaluation (*i. e.* calculation of value) of the objective function when running GA or SA, a linear economic dispatch problem is constructed considering current vector of integer decision variables predefined, and solved by using one of the LP techniques: revised or dual simplex method, interior point method, *etc.*, in order to determine the values of dispatch related real decision variables and the objective,

‒ for GA, if the linear problem constructed for a predefined vector of integer decision variables does not have a feasible solution, an appropriate penalty value, which is much worse than the expected feasible solutions objectives, is assigned to the objective; Sakawa *et al.* [13] define the objective penalty value according to the degree of infeasibility, while here it is assumed constant, and

‒ for SA, infeasible integer vectors are rejected without the implementation of the Metropolis criterion.

A similar idea is presented by Sakawa *et al.* [13], although the problem there is defined as mixed 0–1 linear, only the combination of GA (not SA) and LP is used, and the plant examined is much simpler, without thermal storage, co-generation, electricity production or exchange with the grid. Even in that, much simpler, case, it is shown that the combination of GA and LP is more effective in providing highly accurate approximate solutions with the higher benefits for larger problems.

The optimization problem is solved using the "ESO-MS" object oriented software solution built on the NET and Mono frameworks, developed by the first author of this paper. This solution contains a set of classes related to the components and the system, a LP solver able to use any of the mentioned LP methods, a MILP solver which uses BBM, GA and SA solvers that call the LP solver, referred to as GA/LP and SA/LP, respectively.

Results

A real residential settlement in Niš, Serbia was chosen to apply this approach. The hypothetical plant structure and design parameters of components were considered predefined. There were 1 CG of 803 kW, 1 CH of 500 kW, 1 HP of 650 kW, 3 ARs of 310 kW each, 2 CR of 350 kW each and a heat storage of 1865 kWh. Energy demand patterns were created for 4 typical days of the year and were based on the measured data. Typical days 1 and 2 represent 103 and 76 actual days during the heating season, respectively, while days 3 and 4 represent 106 midseason and 80 summer days. Energy flows of optimized energy supply plant and energy demands are shown in fig. 3. The price of natural gas was taken to be 4 cEUR/kWh (based on low heating value), the price of electricity imported from the grid was 10 cEUR/kWh during the day, *i. e.* from 7 h to 23 h and 2.5 cEUR/kWh during the night, while the price of exported electricity was 10.35 cEUR/kWh.

The problem was solved in 3 ways: (1) transforming the problem to MILP and using BBM, (2) using GA/LP and (3) using SA/LP approach. All the methods resulted in the same solution. The minimal value of the objective function, *i. e.* annual variable costs was 322128 EUR, which is 10.5% lower than 355874 EUR that correspond to the predefined heat load (including ARs) tracking operational strategy [2, 6]. CG should usually operate with the full load. Excess electricity should be exported to the grid, and excess heat, if any, should be stored in TS. HP should cover the low temperature heating demand when the electricity is cheaper or when there is not enough thermal energy from CG and TS. CH is unnecessary in this case. The cooling demand should be covered from AR whenever there is available thermal energy either directly from CG or from TS. During day 3, CR was not used. For other prices, design parameters, or energy demand, these results could be different.

Although this particular problem could have been solved for each typical day separately, it was also solved at once for all 4 days, which would be necessary if some special constraints were added or if consecutive days were examined. Convergence of the objective function towards its optimal value for both methods is shown in figs. 4 and 5.

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Figure 3. Energy flows of optimized energy supply plant and energy demands
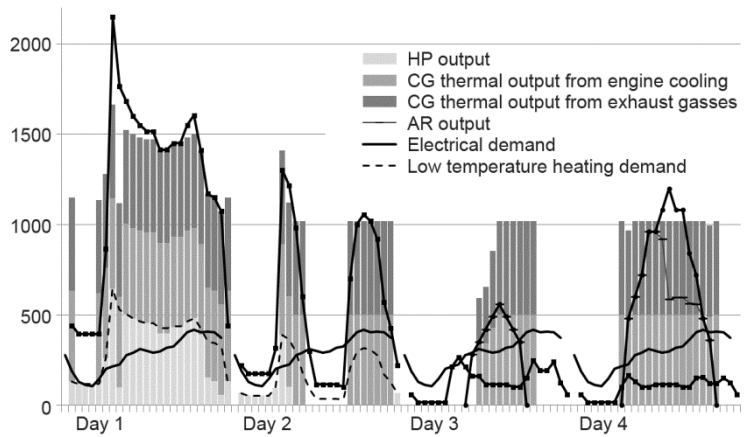

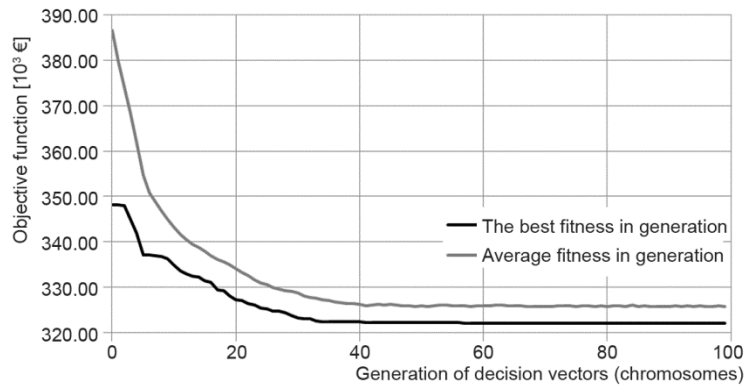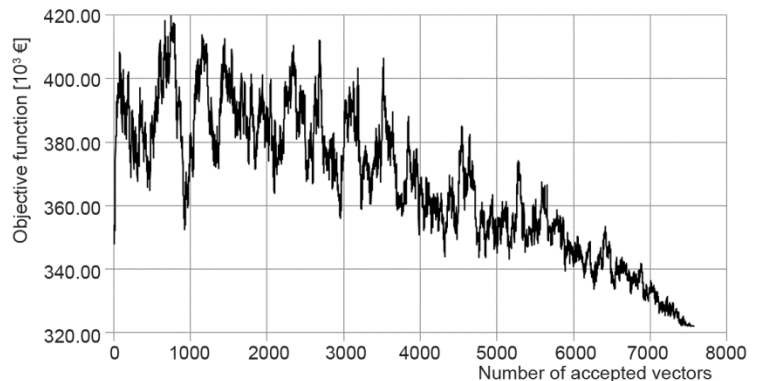**Figure 4. Convergence of the objective function when using GA and LP**

Figure 5. Convergence of the objective function when using SA and LP

The solution was found approximately after 150 minutes with the GA/LP solver and after 34 minutes with the SA/LP solver. There are possibilities for further improvements of both solvers' performances and input parameters. Although this was the case here, it is not realistic to expect that these solvers will always result with the exact solution due to their stochastic nature. The MILP solver, that was adjusted to return the exact solution without time limits, appeared to be more efficient for solving smaller problems, *i. e.* for each typical day separately, requiring only up to 3 min for one day. When used to solve the large problem, for

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4 days together, the solution time was 23 hours and 14 minutes, making it much less efficient than the GA/LP and SA/LP solvers.

Conclusions

This paper proposes an approach for optimization of operational regimes of energy supply plants with small or medium scale co-generation, absorption refrigeration and thermal energy storage. A mixed integer non-linear problem is solved combining modern stochastic techniques: genetic algorithms and simulated annealing with linear programming, exploiting the advantages of both. For the purpose of comparison, the problem is also converted to mixed integer linear and solved with the branch and bound method.

As an example, this approach is applied to optimize a hypothetical plant that might be used to supply a real settlement in Niš, Serbia. The application of genetic algorithms and simulated annealing combined with linear programming resulted in the same, realistic solution, equal to the one obtained using the branch and bound method. The proposed approach appeared to be less efficient for smaller and more efficient for larger problems than solving the transformed MILP. The solution was found quicker with simulated annealing, although the speed of both, GA/LP and SA/LP, solvers could probably be improved with better input parameters, further code optimization, and multithreading.

The approach presented here might be easily extended to design parameters optimization. Furthermore, new, case specific, constraints might be added, as well as other objective functions. The same approach might be applied to a plant with different structure, although that would require some changes in the mathematical model. Besides improving the solvers, extending the model to design and synthesis optimization levels and adding other objectives, it is also planned to include possibilities for consideration of renewable electricity generation technologies and cool thermal energy storage.

Nomenclature

A – area, [m^2]
a – regression coefficient, [–]
b – regression coefficient, [kW]
C – energy cost, [EUR]
c – specific heat, [kWhkg^{-1}K^{-1}]
d – number of days represented with a typical day, [–]
m – mass, [kg]
n – number of components or typical days, [–]
Q – thermal energy (fuel based on low heating value, heating or refrigeration), [kWh]
$\dot{Q}$ – thermal power, [kW]
t – temperature, [°C]
U – overall heat transfer coefficient, [kWm^{-2}K^{-1}]
W – electrical energy, [kWh]
$\dot{W}$ – electrical power, [kW]
Z – cost not related to energy flows, [EUR]

Greek symbols

α – regression coefficient, [EURkW^{-1}]
β – regression coefficient, [EURh^{-1}]

γ – number of units turned on, [–]
δ – on/off binary variable, [–]
ζ – energy unit price, [EURkWh^{-1}]
σ – startup correction factor, [–]
ς – shutdown correction factor, [–]
τ – length of time interval, [h]

Superscripts

G – gross
i – day index
j – hour index
N – net

Subscripts

0 – surrounding air, environment, thermal energy rejected to the environment
AR – absorption chiller
CG – co-generator
CH – hot water boiler
CR – compression chiller
c – constant, fixed
D – demand

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E	− export		ς	− shutdown
e	− electrical energy			

E   − export
e   − electrical energy
ET   − electrical transformer
f   − fuel energy
HP   − heat pump
I   − import
k   − component index
max   − maximal allowed value
min   − minimal allowed value
MTC − medium temperature circuit
r   − thermal energy, cold (refrigeration)
s   − startup and shutdown
t   − thermal energy, heat (heating)
TS   − thermal storage
v   − variable
x   − electrical energy input
XG   − exhaust gasses
σ − startup

ς − shutdown

Abbreviations

AR   – absorption chiller
BBM   – branch and bound method
CG   – co-generator
CH   – hot water boiler
CR   – compression chiller
ET   – electrical transformer
GA   – genetic algorithm
GHG   – greenhouse gasses
HP   – heat pump
LP   – linear programming
MILP – mixed integer linear problem
OM   – operation and maintenance
SA   – simulated annealing
TS   – thermal storage
XG   – exhaust gasses

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