From the Guest Editor of Part One

FRACTIONAL CALCULUS TO HEAT, MOMENTUM, AND MASS TRANSFER PROBLEMS

Fractional Calculus is a hot topic encompassing a broad list of problems such new analytical and, numerical technique, efficient solution of complex problems in modelling of transient heat and flow problems. In contrast to the well-known integer counterparts, the fractional derivatives and integrals are not local [1-3] widely encountered in applications to transient rheology [4, 5], heat [6, 7] and mass transfer [8, 9], non-linear diffusion in porous and granular media [10], Stefan problem [11-13] manifest this technique as a power tool for efficient engineering solutions of complex problems.

This issue of the journal *Thermal Science* conveys strong, reliable, efficient, and promising developments of articles on analytical and numerical problems in fractional calculus for solution problems in heat, mass, and momentum transfer.

The analytical works in this collection are presented by an exellent work on a transform converting fractional differential equations with the modified Riemann-Liouville derivative into partial differential equations [14] and it is concluded that the fractional order in fractional differential equations is equivalent to the fractal dimension. This fractional complex transform is efficiently to solve exactly time-fractional differential equations with the modified Riemann-Liouville derivative [15] incorporating suitable boundary/initial conditions. Moreover, this fractional complex transform is used to convert time-fractional heat conduction equations (with the modified Riemann-Liouville derivative) into ordinary differential equations, and exact solutions can be easily obtained [16]. The solutions demonstrated by these articles developed by He and colleagues are straightforward and the can be used as paradigms for efficient analytical solution of fractional differential equations.

The exact analytical solutions of fractional momentum equation describing transient flows are seriously represented in this collection. First of all, Siddique *et al.* [17] solved a rotational flow of a fractional Maxwell fluid in an infinite circular cylinder caused by a torsional variable time-dependent shear stress that is prescribed on the boundary of the cylinder the Laplace and Hankel transforms. The analytical solution developed by Mahmood [18] is related to a torsional oscillatory flow of a fractional Oldroyd-B fluid between two infinite coaxial circular cylinders. The motion of fluid is due to the simple harmonic sine oscillations of both cylinders around their common axis, with different angular frequencies. The exact solutions (using the Laplace and Hankel transform, too) of the velocity field and associated shear stress are presented under integral and series forms in terms of generalized G and R functions. These solutions satisfies all imposed initial and boundary conditions and allows the cases of ordinary Maxwell fluids, generalized second grade, classical second grade as well as those for Newtonian fluids to be obtained as limiting cases of the general solutions developed.

The approximate analytical technique of the heat-balance integral method is successfully applied to solve the Stokes' first problem of a viscoelastic generalized second grade fluid has been developed [19]. The solution has been performed by a parabolic profile with an unspecified exponent allowing optimization through minimization of the L_2 norm over the domain of the penetration depth. The closed form solution explicitly defines two dimensionless similarity variables $\xi = y/(\nu t)^{1/2}$ and $D_0 = \chi^2 = (p/\nu t^\beta)^{1/2}$, responsible for the viscous and the elastic responses of the fluid to the step jump at the boundary. The closed-form approximate solution is physically relevant and allows demonstrating the effect of the Deborah number on the developed velocity and stress fields.

The heat conduction problems, with and without phase change, form the second principle group of contributions (see ref. 15, also commented above) to development of efficient analytical methods employing the power of the fractional calculus. Protic *et al.* [20] have attached the classical ground heat flux problem determination from known ground surface temperature time-dependant functions. The fractional heat-equation is solved for two, most frequently encountered harmonic surface temperature functions and the results are compared with known analytic solutions employing integer-derivative models. The thermal impedance estimation at the interface of two contraction body was efficiently developed by Hristov [21] by semi-derivatives and semi-integrals in the Riemann-Liouville sense. The example solved clearly reveals that the fractional calculus is more effective in calculation the thermal resistances than the entire domain solutions.

The third groups of good examples of applied fractional calculus problems are those devoted to development of efficient numerical methods. Beibalaev *et al.* [22] developed a finite difference approximation for the Caputo fractional derivative of the $4-\beta$, $1<\beta\leq 2$ order for solving the Dirihlet's problem of the Poisson's equation. Both the stability of difference problem in its right-side part and the convergence has been proved. A numerical example was developed by applying both the Liebman and the Monte-Carlo methods.

Guo *et al.* [23] studied the fractional Langevin equation with fractional Caputo derivative and developed a numerical algorithm to obtain solutions in two cases: without force and with constant external force. The simulations allow finding the displacement and the mean square displacements. Cheng *et al.* [24] developed a numerical algorithms based on fast convolution for the fractional integral and fractional derivative, both in Riemann-Liouville and Caputo senses, with examples demonstrating the efficiency of the derived method.

Last, the good example provided by Wu *et al.* [25] about an explosion-proof textile with hierarchical Steiner tree structure shows the high efficiency of the fractal stricture of designed material. The example does not provide a mathematical model for either heat transfer of blast reaction of the textile, but the information in it is a challenging for development of suitable fractional models and testing to experimental data.

This special issue provides a collection of solved problems, all of them employing fractional calculus, especially collected to demonstrate the efficiency and the power of this technique for solving real problems in transport of heat, mass and momentum. We believe that the articles will serve as intriguing and motivating texts for further development of novel analytical and numerical solution to applied problems described by fractional models.

Last but not least, I like to express my gratitude to all authors who contributed the collection with nice examples, trusted me in the initial call and through the long way to the final versions of the manuscripts. The work of the reviewers offering their time to increase the quality of the contents and the editors of the journal Thermal Science, as well, for the support of our initiative is highly appreciated.

Clarifications

The generalized G and R functions are [26]:

$$\mathrm{G}_{\mathrm{a,b,c}}(d,t) = \sum_{j=0}^{\infty} \frac{(c)_j d^j t^{(j+c)a-b-1}}{j!\Gamma[(j+c)a-b]}$$

$$R_{a,b}(c,d,t) = \sum_{j=0}^{\infty} \frac{c^j (t-d)^{(j+1)a-b-1}}{\Gamma[(j+1)a-b]}$$

where $(c)_j$ is the Pochhammer polynomial [26] and Γ is the gamma function.

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