NUMERICAL STUDY ON HYGROSCOPIC CAPILLARY-POROUS MATERIAL DRYING IN A PACKED BED

by

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During the non-hygroscopic and/or hygroscopic capillary-porous materials drying the first part of the drying curve has the similar form. In the case of the hygroscopic capillary-porous material drying, the drying rate drops more sharply as deep inside the material the sorption state is ensured, i. e. when the partial pressure of water vapor at this place becomes less than the partial pressure of free water. This drying period disappears at the moment when all the material layer is in the hygroscopic regime (the second critical point appears). Results of modeling and numerical simulation for the case of the hygroscopic capillary-porous material drying in a packed bed are shown in the paper. A mathematical model describing unsteady simultaneous one-dimensional heat and mass transfer between gas phase and dried material during drying process in a packed bed is described. Heat and mass transfer between solid phase and gas phase takes place at the surface of the solids (assuming the state of thermodynamic equilibrium). Heat transfer (temperature gradient) inside the dried material is neglected. Mass transfer coefficients inside dried material are defined based on experimental investigation of drying kinetics for an elementary packed bed of a given material, thus enabling better results in the drying rate evaluation for the case of the great number of real (colloidal capillary-porous) materials. Verification of the model was done successfully on the basis of the available experimental data for the hygroscopic capillary-porous material (potato cubes) drying. Numerical analysis of the influence that relevant parameters have onto the drying process in the case of potato cubes (4.0 mm, 8.0 mm and 12.0 mm) was carried out and shown in this paper.

Introduction

Digital computers and developed numerical mathematics are widely used for the purpose of solving heat and mass balance equations for chemical engineering processes. Contemporary research of various processes is mostly directed to modeling and numerical simulation. Results obtained on the basis of the reliable mathematical model and numerical analysis enable prediction of the relevant process parameters required for design of particular device and optimization of the process itself. A good model (numerical simulation) can successfully substitute lengthy and expensive experimental research.

During past years a lot of experimental and theoretical investigations directed to the convective drying process of particulate real (colloidal capillary-porous) materials in a packed bed have been carried out in the Laboratory for Thermal Engineering and Energy, Belgrade. Some of the developed and successfully verified mathematical models describing the drying process in a packed bed have been already reported [9, 10]. The common idea of all the models is that drying process is calculated for the elementary bed of immobile particles and generalized for the whole bed. The new approach in the basic drying equation definition has enabled development of all the models, being continually improved and checked for new materials in order to obtain higher precision of the calculated data.

The model and some of the obtained results on numerical simulation of the hygroscopic capillary-porous material (potato cubes) drying in a packed bed are shown in this paper. The new approach in drying equation definition, although discussed in some of the previous publications [10], is also explained in this paper.

Typical drying curves for hygroscopic capillary-porous materials

The drying curve Fig. 1 is initially of similar form during the early stages of drying while the moisture still exerts its full vapor pressure. The drying rate drops more sharply as the material at progressively deeper positions below the exposed surface enters the

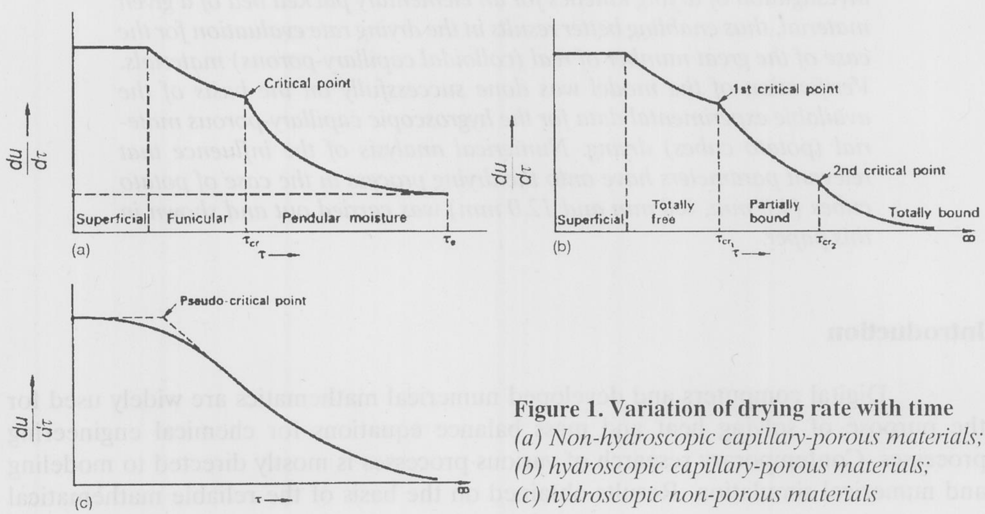


Figure 1. Variation of drying rate with time
(a) Non-hydroscopic capillary-porous materials;
(b) hydroscopic capillary-porous materials;
(c) hydroscopic non-porous materials

sorptive state, and the partial pressure of the moisture there present becomes less than that for unbound moisture. This second stage of drying ends when the whole bed is within the hygroscopic region, and a second "critical point" may be detected. The humidity potential gradually falls as the material dries and is zero when all the material is in equilibrium with the surrounding air. The drying rate dwindles proportionally to be zero when drying is complete, strictly requiring infinite time.

Mathematical model

Consideration is given to convective drying as unsteady problem of heat and mass transfer caused by fluid flow (drying agent) through a bed of particulate solids. The differential equation for conservation of the general dependent variable ϕ in the case of the unsteady one-dimensional problem can be written in the following form:

$$\frac{\partial}{\partial \tau}(\rho\phi) + \mathrm{div}(\rho\vec{U}\phi) = \mathrm{div}(\Gamma_{\phi}\mathrm{grad}\phi) + S_{\phi} \qquad (1)$$

The terms in the differential equation on the left side are the unsteady term and the convection term, while on the right side are the diffusion term, where Γ_{ϕ} represents the diffusion coefficient, and the source term. All kinds of transport laws must be incorporated inside the source term, which represents generation and dissipation of the variable ϕ. Forms of the variables Γ_{ϕ} and S_{ϕ} depend on the physical meaning of the variable ϕ. In the case of the continuity equation $\phi = 1$ and $\Gamma_{\phi} = 0$. In convective drying theory, the moisture content and temperature of a material to be dried as well as the humidity and temperature of a drying agent represent particular cases of the general dependent variable ϕ.

The basic assumptions adopted in the mathematical model describing the unsteady simultaneous one-dimensional heat and mass transfer between a gas phase and a material during convective drying in packed bed are:

- each phase is considered as a quasi-homogeneous phase with effective transport coefficients,
- one-dimensional model is used,
- all solids are of the same size, shape and density at one moment of time,
- heat and mass transfer between the solid phase and the gas phase are taking place at the surface of the solids, assuming the state of thermodynamic equilibrium,
- moisture transfer inside the dried material is calculated using an original technique based on the modified equation from the drying theory [2],
- heat transfer inside the dried material (temperature gradient) is neglected.

On the basis of mentioned assumptions the system of partial differential equations for the case of convective drying in packed bed (the simplest case) can be defined in the following form:

Mass balance:

– gas humidity conservation:

$$\rho_g V_g \left(\frac{\partial g}{\partial \tau} + U_g \frac{\partial g}{\partial x} \right) = S_M \tag{2}$$

where:

$U_g = U_0/\varepsilon$ – real gas velocity,
$S_M = \rho_g k_{mg} a_b V(g_s - g)$ – mass (humidity) source.

– material moisture content conservation:

$$\rho_{md} V_m \frac{\partial u}{\partial \tau} = -S_M \tag{3}$$

Heat balance:

– gas enthalpy conservation:

$$\rho_g V_g C_g \left(\frac{\partial T_g}{\partial \tau} + U_g \frac{\partial T_g}{\partial x} \right) = h_{mg} a_b V (T_m - T_g) + \\ + h_{wg} a_w V (T_W - T_g) + C_V (T_g - T_p) S_M \tag{4}$$

– material enthalpy conservation:

$$\rho_{md} V_m C_p \frac{\partial T_m}{\partial \tau} = h_{mg} a_b V (T_g - T_m) + \\ + h_{wm} a_w V (T_w - T_m) - r S_M \tag{5}$$

The defined of coupled partial differential equations (2–5) has to be completed with the drying equation. It is not easy to determine exactly the drying rate for a particular convective drying process. On the basis of the original approach defined by Stefanović and Nešić [7], developed in order to determine the drying process of an arbitrary bed for any real material, the transport phenomena inside and outside the material are discussed separately. This is because of the fact that during convective drying of any real material two different types of resistance exist due to:

– internal moisture transport (from the material interior to its surface),
– external moisture transport (convection from the material surface to the drying agent).

Internal moisture transport is more complex problem, as it is a result of several different mechanisms of moisture transport [4], namely:

– liquid transport due to the existence of capillary forces,
– liquid transport due to the existence of a moisture concentration gradient (diffusion of liquid phase in the case of solution existence in a material),
– liquid transport at pore surface due to surface tensions,

- vapor transport due to concentration gradients (vapor diffusion),
- liquid and vapor transport due to the existence of a temperature gradient (thermal diffusion),
- liquid and vapor transport due to the existence of the total pressure gradient.

On the basis of the simplified differential equation for drying rate [3] it can be expressed, for moisture transport inside the dried material, in the general form as:

$$\frac{\partial u}{\partial \tau} = K_i(u - u_{eq,s}) \tag{6}$$

The internal moisture transport coefficient (K_i) includes resistance of moisture transport inside the material, and can be defined on the basis of the material characteristics, separately from the drying conditions (drying agent velocity and temperature). An analysis of the data obtained by the experimental investigation of the drying process for potato cubes in packed bed [6] shows that internal moisture transport coefficient can be sufficiently exactly expressed as:

$$K_i = A_k t_m^{n_t} \left(\frac{u}{u_0}\right)^{n_u} \tag{7}$$

On the other hand, it is assumed that all amount of moisture transported from the material interior to its surface is transferred into the surrounding gas, permanently keeping the material surface and thin layer of gas close to that surface at the state of thermodynamic equilibrium. Drying rate for the case of external moisture transport (between the gas boundary layer at material surface and surrounding gas) can be expressed as:

$$\frac{\partial u}{\partial \tau} = k_{mg} a_b (g - g_s) \frac{\rho_g}{\rho_m} \tag{8}$$

The gas humidity at boundary layer (g_s) have to be defined from the state of equilibrium for the system formed by moist material in contact with humid gas. Defining the state of equilibrium for different materials and gases at certain temperature is based on experimental determination of the moisture content of the material being in thermal and hydrological equilibrium with the surrounding gas. For the purpose of numerical simulation of the drying process it is necessary to generalize the experimental data for moisture isotherms in the form of the empirical expression:

$$u_{eg} = \left[\frac{-\ln(1 - \varphi_s)}{CT^D}\right]^{\frac{1}{AT+D}} \tag{9}$$

All the coefficients (A, B, C and D) have to be defined on the basis of the experimentally obtained equilibrium values (u_{eq} and φ_s) for several corresponding temperatures (T).

Numerical procedure

Analysis of the partial differential equations (2–5) shows that all the dependent variables of interest obey a generalized conservation principle [5]. Each partial differential equation is discretized by means of the control-volume method (a flow field is divided by means of a vertical grid into a finite number of control volumes). Iterative line-by-line method is used for solving the obtained linearized algebraic equations, using recurrence formula during calculation of the variable's values for every line, following the procedure for all the lines in one direction. This method is called the Thomas algorithm or the TDMA (TriDiagonal-Matrix Algorithm). The designation TDMA refers to the fact that when the matrix of the coefficients of these equations is written, all the nonzero coefficients assign themselves along three diagonals of the matrix.

Results and discussion

Validation of described mathematical model defining the drying process and/or numerical simulation based on the model was done on the basis of the available experimental data for various materials (corn grain, poppy seeds, coal pieces, *etc.*) Data obtained by the numerical simulation on the basis of the model were already successfully compared with the data obtained by the experimental investigation for the case of the non-shrinking materials [8–10].

A few of the comparisons in the case of potato cubes drying in packed bed are shown in Figs. 2–4. Potato was chosen as a representative of the hygroscopic capillary-porous shrinking materials. It can be seen that the results look plausible.

Developed mathematical model gives a lot of the possibilities in predicting the relevant parameters for the drying process. Values of the time-averaged parameters, as

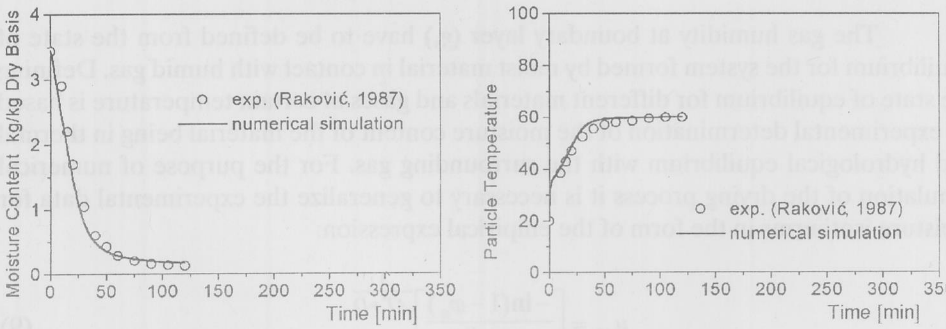


Figure 2. Comparison of simulated and experimental drying kinetics for potato cubes drying in packed bed
$a_0 = 4.0\ mm,\ u_0 = 3{,}44\ kg/kg,\ t_{m0} = 20\ °C,\ x_0 = 0.0055\ kg/kg,$
$t_{a0} = 60.0\ °C, U_0 = 1.054\ m/s,\ H_0 = 25\ mm$

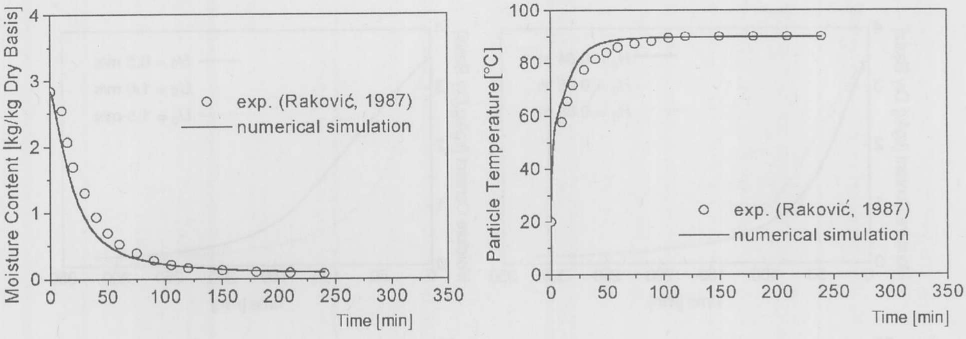


Figure 3. Comparison of simulated and experimental drying kinetics for potato cubes drying in packed bed
$a_0 = 8.0\ mm$, $u_0 = 2.83\ kg/kg$, $t_{m0} = 20\ ^oC$, $x_0 = 0.01\ kg/kg$, $t_{a0} = 90.0\ ^oC$, $U_0 = 1.058\ m/s$, $H_0 = 30\ mm$

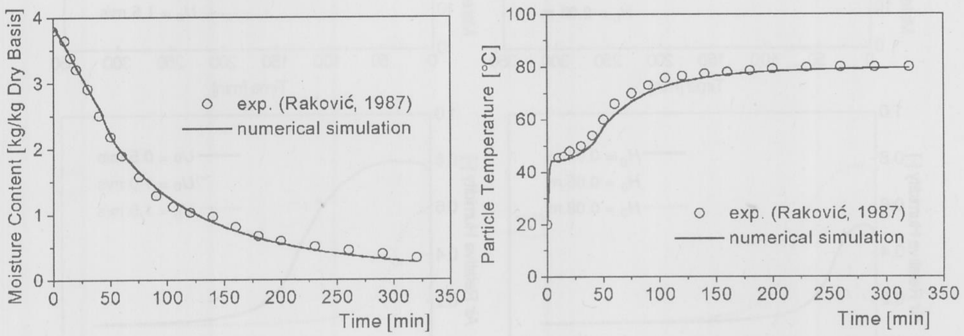
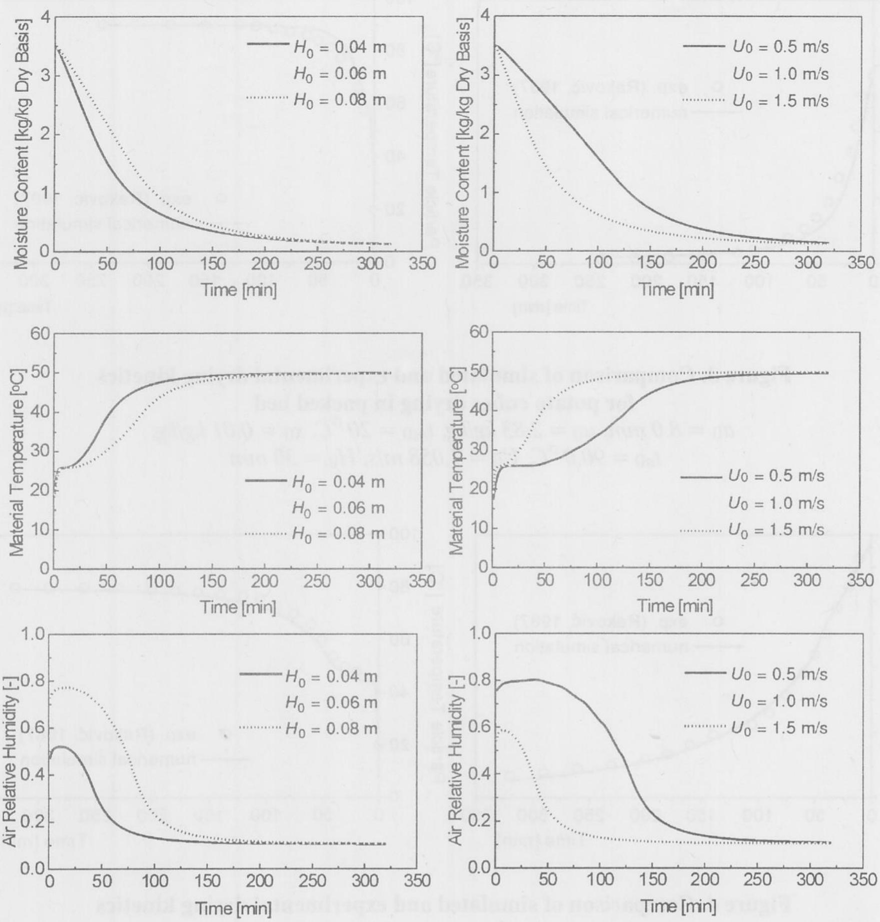


Figure 4. Comparison of simulated and experimental drying kinetics for potato cubes drying in packed bed
$a_0 = 12.0\ mm$, $u_0 = 3.79\ kg/kg$, $t_{m0} = 20\ ^\circ C$, $x_0 = 0.04\ kg/kg$, $t_{a0} = 80.0\ ^\circ C$, $U_0 = 1.015\ m/s$, $H_0 = 35\ mm$

well as parameters along the bed height for one moment of time, can be calculated. Just a few of the results that can be obtained are shown on Figs. 5 and 6.

A very good agreement of the results obtained by numerical simulation and experiment (Figs. 2–4) was already shown. Nevertheless, it is necessary to carry out an analysis about influences of the particular elements to the model accuracy.

The basis of shown mathematical model of drying process in a packed bed are the partial different equations of heat and mass balance having good enough accuracy. The only approximation is neglect of heat transfer between the bed (material and gas) and walls of the drying device. It is important then to pay attention at the other relations and parameters used in the model: – drying rate equations, – expression for moisture



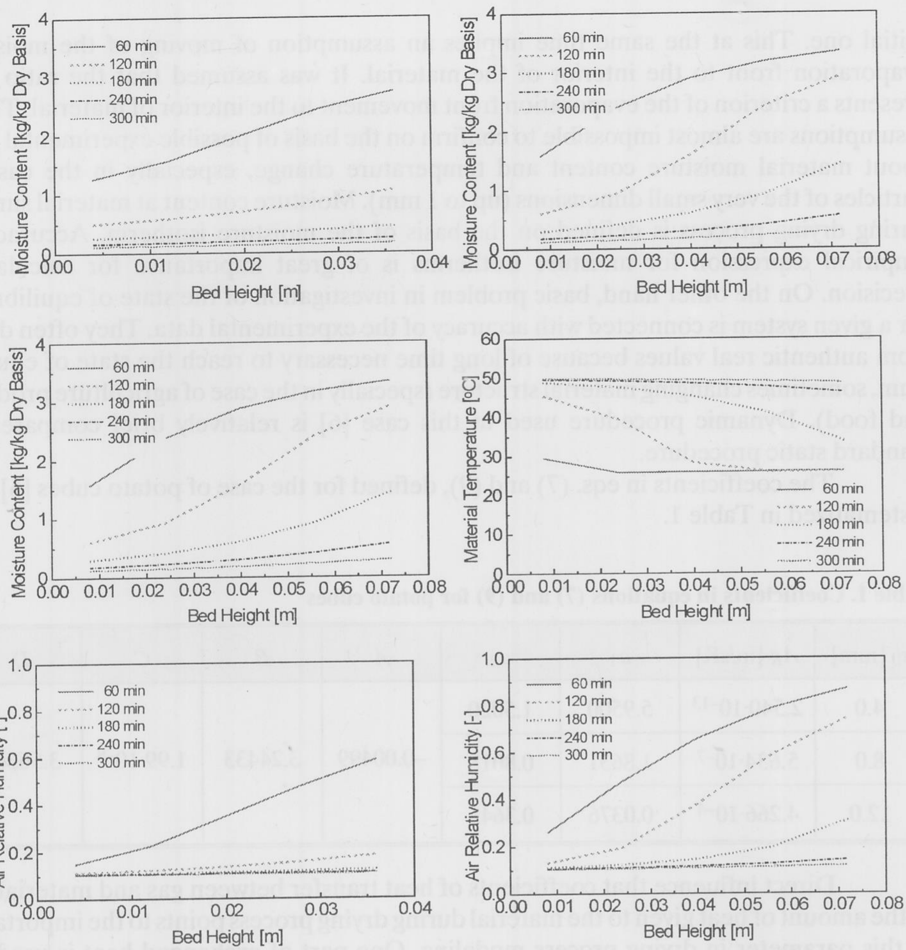
(a) t_{a0} = 50 °C, U_{a0} = 1.0 m/s (b) t_{a0} = 50 °C, H_0 = 0.06 m

Figure 5. Results of numerical analysis of drying kinetics in the case of potato cubes drying in packed bed

$a_0 = 12.0\ mm,\ u_0 = 3.5\ kg/kg,\ t_{m0} = 15\ °C,\ x_0 = 0.008\ kg/kg$

isotherms, – parameters characterizing the particulate solids and the bed, –- expressions for heat and mass transfer coefficients.

Drying rate was determined by simultaneous calculation of two coupled differential equations (6 and 8), in order to separate influences of the internal and external (convective) moisture transport. In the drying rate equation dealing with external moisture transport between the solid surface and the surrounding gas (drying agent), empirical expressions for the mass transfer coefficient in packed bed taken from literature [1]:



(a) H_0 = 0.04 m, U_{a0} = 0.5 m/s (b) H_0 = 0.08 m, U_{a0} = 1.0 m/s

Figure 6. Material moisture content, material temperature and outlet gas relative humidity along the bed height for potato cubes drying in packed bed
a_0 = 12.0 mm, u_0 = 3.5 kg/kg, t_{m0} = 15 °C, t_{a0} = 50 °C, x_0 = 0.008 kg/kg

$$\mathrm{Sh} = 1.83\mathrm{Re}^{0.485}\,\mathrm{Sc}^{0.33}, \quad \mathrm{Re} < 300$$
$$\mathrm{Sh} = 0.977\,\mathrm{Re}^{0.59}\,\mathrm{Sc}^{0.33}, \quad \mathrm{Re} > 300 \tag{10}$$

The internal moisture transport coefficient (K_i) in drying rate equation concerning internal moisture transport (eq. 6) is in form of eq. (7), was obtained on the basis of experimental data [6]. This dependence is proposed assuming that the surface layer of material is dried to the moisture content close to the equilibrium value at the period of decreasing drying rate, while central part of material keeps moisture content close to the initial one. This at the same time implies an assumption of moving of the moisture evaporation front to the interior of the material. It was assumed that the ratio u/u_0 presents a criterion of the evaporation front movement to the interior of material. Those assumptions are almost impossible to confirm on the basis of possible experimental data about material moisture content and temperature change, especially in the case of particles of the very small dimensions (up to 1 mm). Moisture content at material surface during drying process is defined on the basis of the moisture isotherm. Accuracy of empirical expression for moisture isotherms is of great importance for calculation precision. On the other hand, basic problem in investigation of the state of equilibrium for a given system is connected with accuracy of the experimental data. They often differ from authentic real values because of long time necessary to reach the state of equilibrium, sometimes changing material structure (specially in the case of agriculture products and food). Dynamic procedure used in this case [6] is relatively brief compared to standard static procedure.

The coefficients in eqs. (7) and (9), defined for the case of potato cubes [6], are systematized in Table 1.

Table 1. Coefficients in equations (7) and (9) for potato cubes

a_0 [mm]	A_K [m/sK]	a_1	a_2	A	B	C	D
4.0	$2.540 \cdot 10^{-13}$	5.9500	1.5020	–0.00499	3.24433	$1.99 \cdot 10^{-8}$	3.56361
8.0	$5.634 \cdot 10^{-7}$	1.8631	0.8018				
12.0	$4.266 \cdot 10^{-4}$	0.0376	0.3645				

Direct influence that coefficients of heat transfer between gas and material has to the amount of heat given to the material during drying process points to the importance of this parameter in drying process modeling. One part of exchanged heat is used for moist evaporation (through coupled drying rate equations) and the other part is used for heating the material. The ratio of those parts changes during drying process. The material temperature obtained from this balance effects the internal moisture transport coefficient as well as the amount of equilibrium moisture content at the solid surface, causing significant influence of heat transfer coefficient to the drying rate. Dependencies used for the heat transfer coefficients in packed bed used in this work are [1]:

$$\mathrm{Nu} = 1.95\,\mathrm{Re}^{0.49}\,\mathrm{Pr}^{0.33}, \quad \mathrm{Re} < 350$$
$$\mathrm{Nu} = 1.05\,\mathrm{Re}^{0.59}\,\mathrm{Pr}^{0.33}, \quad \mathrm{Re} > 350 \qquad (11)$$

Parameters defining characteristics of the solids and bed (equivalent diameter and specific surface of the solids, material specific heat capacity and density, bulk density and porosity of the bed), must be known and accurate enough. These parameters are of great importance for heat and mass balance equations (ρ, C_p, a_b), for heat and mass transfer coefficients (d_e, a_b) and momentum conservation equations (ε). It is well known that potato shrinks very significantly as a result of drying, due to its large moisture content. Dependencies defining particle diameter change due to moisture content variation can be found in literature [2]:

$$d = \left[\frac{6m_d}{\rho_m \pi}(1+u) \right]^{\frac{1}{3}} \tag{12}$$

The simplified dependence is successfully used in this model:

$$d_e = d_{ed} \left[\frac{\rho_{md}}{\rho_m}(1+u) \right]^{\frac{1}{3}} \tag{13}$$

Conclusions Results of theoretical investigation of heat and mass transfer during drying of potato cubes in packed bed are presented in this paper. Comparison of corresponding parameters for drying process obtained by calculations on the basis of developed model and experimental investigation (for applied parameter range) has shown a good agreement. Results obtained on the basis of the model can be used for calculations of the packed bed dryers (prediction of the mean moisture content or temperature of dried material with time, as well as the final target moisture and/or the residence time). It is necessary to carry out previously simple and relative short experimental determination of drying kinetics for corresponding materials in an elementary packed bed, in order to obtain experimental expression for internal moisture transport coefficient. State of equilibrium for a system formed by moist material in a contact with humid air at different temperatures is necessary to estimate experimentally for each of the materials in consideration, and than to define analytical dependence for moisture isotherms on the basis of obtained data. It is also necessary to define values of equivalent particle diameter as well as bed porosity. Mentioned experiments are the only experimental base for application of developed mathematical model.

Nomenclature

a [m]	– length
a_s [m²/m³]	– bed specific surface, $6(1-\varepsilon)/(\phi d_e)$
a_w [m²/m³]	– wall specific surface, A_w/V
C [J/kgK]	– specific heat capacity
d [m]	– diameter
g [kg/kg]	– gas humidity (wet basis)
h [W/m²K]	– specific heat capacity
H [m]	– bed height
k, K [m/s]	– mass transfer coefficient
r [J/kg]	– latent heat of water phase change (evaporation)
t,T [°C, K]	– temperature
u [kg/kg]	– material moisture content (dry basis)
U [m/s]	– velocity
V [m³]	– volume
x [kg/kg]	– air humidity (dry basis)

Greek symbols

ε [–] – void fraction
ρ [kg/m^3] – density
τ [s] – time
ϕ [–] – sphericity of a solids
φ [–] – air relative humidity

Subscripts

a – air
d – dry
e – equivalent
eq – equilibrium
g – gas
i – internal
m – material
s – surface
v – vapor
w – wall
0 – initial

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