HEAT AND MASS TRANSFER DURING CONTACT DRYING OF A SHEET OF MOIST MATERIAL

by

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Proposed is a model for heat and mass transfer during drying of a sheet of moist material placed on the heating surface of certain temperature while over the opposite free surface of the sheet the air of defined parameters is blown. All thermophysical and transport properties of the dried material are considered to be fully dependent on temperature and moisture content. The model is solved numerically by use of finite difference explicit numerical scheme. The results of temperature and moisture distributions inside the layer of the material obtained by the model with variable coefficients are compared, for the same values of process parameters, to the results obtained through simultaneous solving of the similar mathematical model with constant thermophysical coefficients in governing equations, as well as with the results acquired experimentally.

Introduction

The contact drying is the combined conductive – convective drying process in which the heat is supplied to wet material conductively out of possession of hot solid surface and the moisture evaporated from the material is removed convectively into the surroundings. High heat flux rates that could be transferred to the material conductively make possible intensive drying of thin materials, so the contact drying is extensively used in industry. In spite of the wide use of the contact drying, there are very few serious theoretical and experimental studies over this process. The previous analytical investigations [2, 3, 4, 7, 8] have not succeed, even for the simplest geometries, in building the mathematical model of the process that could endure the experimental verification for various materials. This is primarily due to the great complexity of mechanisms of heat and mass transfer inside the colloidal capillary-porous bodies and diversity of their priorities in different materials. Analytical solution of the system of transport equations in the referred mathematical models is additionally complicated by the difficulties arising from the asymmetry of boundary conditions in combined conductive - convective drying process. In the papers of wider aim, that are not focused just on modeling the contact drying of particular material [7, 8], introducing of certain inadmissible assumptions and simplifications of the initial systems of equations, necessary to make their analytical solution possible – afflict the physical conformity of the model and lead to incorrect solutions of drying dynamics. One of these commonly used assumptions that are especially harmful in analyzing such high intensity drying processes, is the postulate that the values of thermophysical and transport properties of the moist material are unconditionally constant during the process. In this paper the system of equations describing combined heat and mass transfer in the layer of moist material, as well the corresponding equations for initial conditions and boundary conditions at contact and free surface, are formed following, in general, the Luikov's theoretical approach, but considering all thermophysical coefficients to be fully dependent on temperature and moisture content inside the material during the process. Of course, solving of such a system by use of analytical procedures is impossible, so the adequate numerical methods are utilized. For proper verification of the proposed model it is necessary to compare the results obtained through its numerical solving with the experimental results obtained for material with previously defined dependencies of relevant thermophysical properties on temperature and moisture content. Considering that there is no similar experimental data in literature, an experimental investigation of simple geometry drying process, with the material that could be assumed representative of wide family of colloidal capillary-porous substances used in technical practice, is accomplished.

Mathematical model

The simple, one dimensional case of the sheet of moist material placed on the hot plate of constant temperature, whilst over the opposite free surface of the sheet convective air stream of certain parameters is blown, is observed. The temperature of the heating surface and the resulting temperatures inside the sample during the process are well under the level that causes boiling of liquid moisture within the material, so the influence of total pressure gradient on the heat and mass transfer could be neglected [6, 9, 11]. Consequently the local values of temperature and moisture content inside the material could be considered the relevant potentials for heat and mass transfer in the capillary porous body during drying process. As all the thermophysical coefficients are acknowledged to be fully dependent on temperature and moisture content, and as the shrinking of the material during drying might be neglected, the resulting equations for heat and mass transfer inside the material are:

$$c(t,u)\rho_0 \frac{\partial t}{\partial \tau} = \frac{\partial}{\partial x}\left[\lambda(t,u)\frac{\partial t}{\partial x}\right] + \varepsilon(t,u)\rho_0 r(t,u)\frac{\partial u}{\partial \tau} \tag{1}$$

$$\frac{\partial u}{\partial \tau} = \frac{\partial}{\partial x}\left[a_m(t,u)\frac{\partial u}{\partial x} + a_m(t,u)\delta(t,u)\frac{\partial t}{\partial x}\right] \tag{2}$$

For the uniform values of temperature and moisture content inside the material before the strat up of drying, the initial conditions ($\tau = 0$) are:

$$t(x, 0) = t_0 \tag{3}$$

$$u(x, 0) = u_0 \tag{4}$$

If the values of convective heat and mass fluxes (j_q and j_m) at the free surface of the material are known, the boundary conditions at the free surface could be expressed as:

$$j_q(L,\tau) = -\lambda(t,u)\left(\frac{\partial t}{\partial x}\right)_{x=L} - r(t,u)[1-\varepsilon(t,u)]\, j_m(L,\tau) \tag{5}$$

$$j_m(L,\tau) = -a_m(t,u)\rho_0\left(\frac{\partial u}{\partial x}\right)_{x=L} - a_m(t,u)\delta(t,u)\rho_0\left(\frac{\partial t}{\partial x}\right)_{x=L} \tag{6}$$

The boundary condition for heat transfer at the contact surface is obtained over the known temperature of the heating surface. If the contact resistance to heat transfer in the zone of junction of these two surfaces might be neglected, the boundary condition is:

$$t(0,\ \tau) = t_h \tag{7a}$$

Nevertheless, in most of the cases the value of contact resistance to heat transfer could not be disregarded. Analytical defining of the exact value of this contact resistance, for different periods of the process, are impossible concerning complexity of heat transfer processes at the contact of dry, solid, metal heating surface and deformabile, moist surface of the dried material, that is additionally complicated with phase changes and mass transfer in the contact zone. For these reasons the contact resistance is introduced through an empirical parameter $\Delta t(\tau)$ that shows the value of temperature drop at contact surface due to the contact resistance during the process.

$$t(0,\ \tau) = t_h - \Delta t(\tau) \tag{7b}$$

The boundary condition for the mass transfer at the contact surface is derived assuming there is no moisture transfer from the material to the heating surface through the contact surface.

$$\left(\frac{\partial u}{\partial x}\right)_{x=0} + \delta(t,u)\left(\frac{\partial t}{\partial x}\right)_{x=0} = 0 \tag{8}$$

If all thermophysical coefficients are considered to be constant, the proposed equations for heat and mass transfer (1, 2) regularly simplifies to the known Luikov's system of transfer equations [5, 6, 7, 8] along with the appropriate initial and boundary conditions. The proposed model with variable coefficients is solved numerically by means of nonlinear finite difference explicit scheme. Simultaneously, using the same method, the simplified model with constant coefficients is solved.

Experimental

The material used in experimental part of the investigation is specified mixture of quartz sand and bentonite, proved [9, 11, 12] to optimally affiliates thermophysical properties of wide family of colloidal capillary-porous substances used in technical practice. The mechanical properties of this material and its ability to absorb – desrob moisture without changing the volume, as well as the fact that the earlier experimental investigations [9] defined its equilibrium states and all its relevant thermophysical and transport coefficients as functions of temperature and moisture content, makes quartz sand-bentonite mixture attractive for wide range of experimental use.

The experiments were performed on the laboratory pilot facility schematically displayed on Fig. 1. A sample of moist material having dimensions $L \times 80 \times 80$ mm (1) was placed on the electrically heated, thick copper plate (2), with the known temperature regulation (3). Over the opposite, free surface of the sample air of the known temperature and relative humidity was blown by means of axial fan (4). Depending on the sample thickness, the relative height of the heated plate with sample was regulated in order to

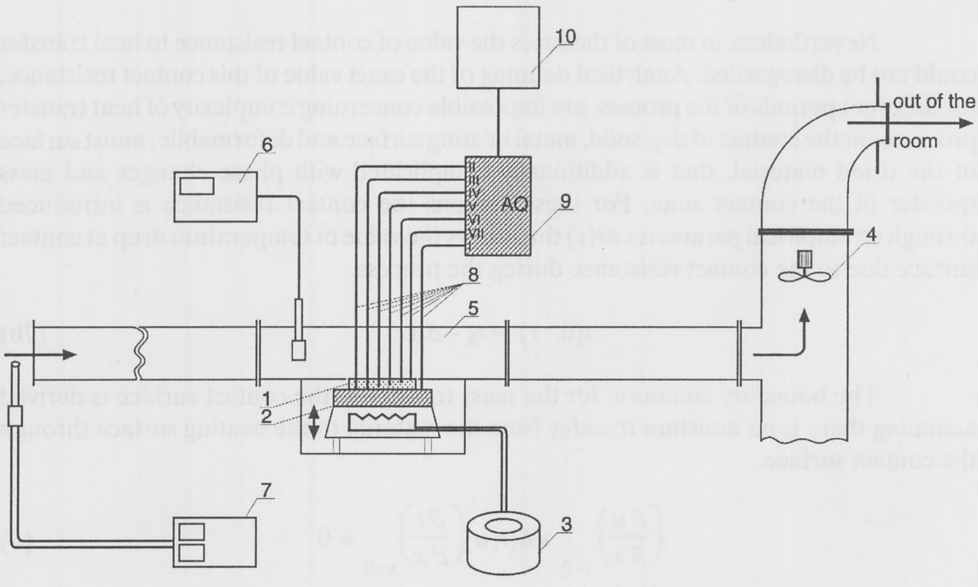

Figure 1. Scheme of the experimental facility

level the free surface of the sample with the bottom of air channel (5). Geometry of an air channel (the length of air channel before the sample is 1500 mm and its cross section is 250×100 mm) provided stabile velocity profile over the sample. Air velocity over the sample was measured by anemometer (6). All presented experiments were performed with average air velocity of 2 m/s.

For different values of sample thickness (L = 10; 15 or 20 mm), initial moisture content (u_0 = 0.175; 0.200 or 0.233 kg/kg) and hot surface temperature (t_h = 50; 65 or 80 °C), temperature profiles and average moisture content inside the sample for various duration of drying process were observed.

Inlet temperature and relative humidity of an air stream during the experiments were measured by Rotronic sonde (7) and maintained at values 20 °C for temperature, and 50% for relative humidity.

The temperatures inside the sample during the process were measured by means of fine wire (0.1 mm in diameter) Cr-Ni thermocouples (8), connected with the system for data acquisition (9) and processing (10). Thermocouples were inserted into the sample horizontally, following the isothermal surfaces. Inside the sample four thermocouples were positioned (three for the sample thickens of 10 mm) in the arrangement shown on Fig. 2. Additionally, two thermocouples were placed at the border surfaces of the sample (one at the contact surface and another at the free surface) and one thermocouple was placed just under the surface of the copper plate. Signals of all seven thermocouples were recorded with sample period of 10 seconds.

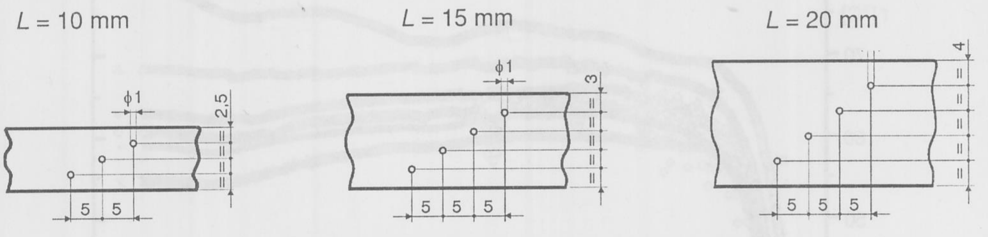

Figure 2. Positions of thermocouples inside the sample

The average moisture content of the sample after certain drying time was determined by measuring the mass of the sample before and after dehydration and the mass of the dry base of the sample after complete dehydration in furnace.

Results

The analysis and comparison of experimental results and results obtained on the basis of mathematical modeling of the process are performed for large number of different process conditions. The conclusions will be presented through an exemplary case of experimental and numerical results for temperature and moisture distributions obtained for the following process conditions: temperature of the heating surface: 80 °C; velocity of air stream over the free surface of the sample: 2.0 m/s; temperature of air stream: 20 °C; relative humidity of air stream: 50%; thickness of the layer of moist material: 15 mm; initial temperature of the material: 20 °C; initial moisture content of the material: 0.2 kg/kg.

Experimentally obtained temperature change at different distances from the heating surface inside the sample, during the observed drying experiment is showed on Fig. 3. During the worming up period of the material, temperature at all points of the sheet increases rapidly. Following is a period of relatively stabile temperature in the observed zone of the sample. After that period until the end of the process a slight but constant temperature decrease is registered. Temperature of the thermocouple placed close to the contact surface ($x \approx 0$) increases during all stages of the process. Except the temperature oscillations caused by the imperfectly constant temperature of the heating surface, the experimentally obtained temperature curves shows no bounces and jumps that could indicate an existence of any distinctly shaped evaporation front inside the sample.

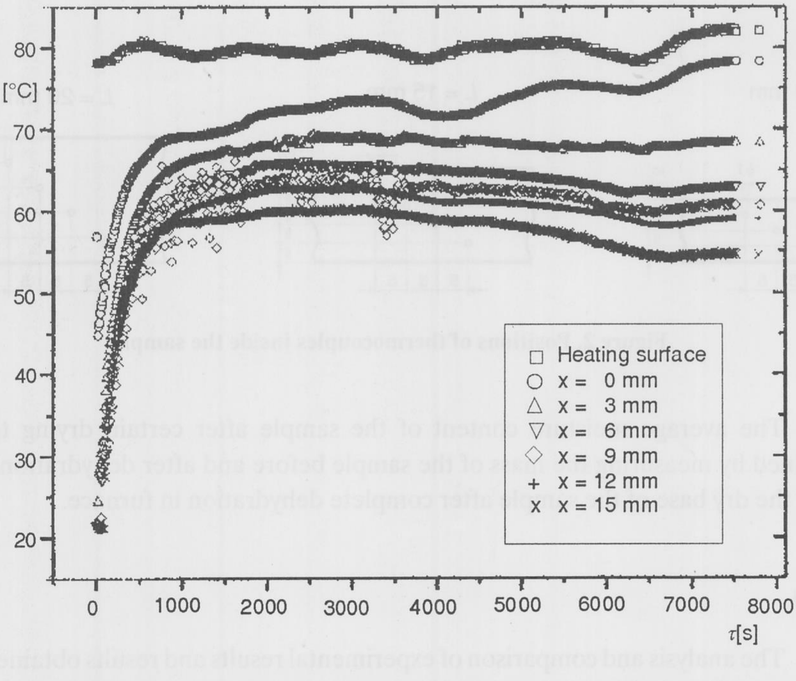

Figure 3. Experimentally obtained temperatures during the process (L=15 mm; u_0 = 0.200 kg/kg; t_h = 80 °C)

The temperatures calculated by numerical solving of both the models with constant and variable parameters, for the adopted process conditions, are shown on Figs. 4 and 5. The disagreements between the temperatures predicted by these two models are significant for all the periods of drying. Only during the initial – worming up period of drying the temperatures calculated by these two models have the similar values, which is the consequence of the fact that variable thermophysical coefficients, especially the moisture diffusivity, the thermogradient coefficient and the thermal conductivity as the most influential for the temperature distribution, in the initial periods of the process are still of the comparable level. Especially inferior and without physical sense are the temperatures obtained by the model with constant coefficients for the zones of the sheet near to the free surface for longer drying times (Fig. 4).

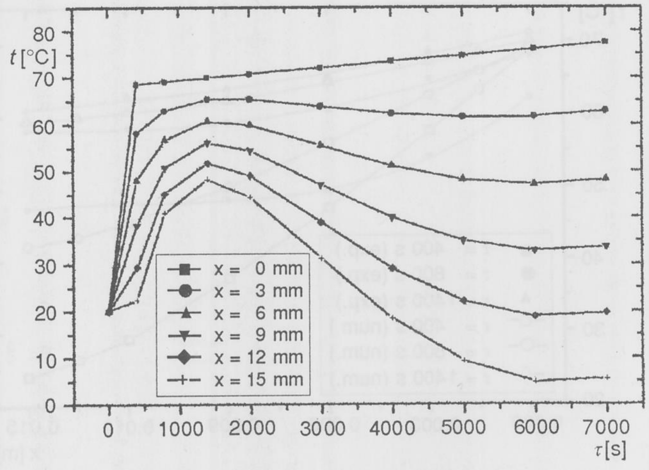

Figure 4. Temperatures calculated using the model with constant coefficients (L=15 mm; u_0 = 0.200 kg/kg; t_h = 80 °C)

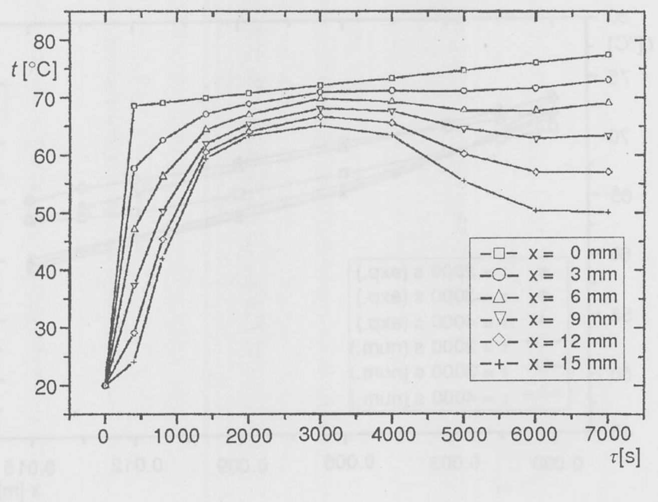

Figure 5. Temperatures calculated using the model with variable coefficients (L=15 mm; u_0 = 0.200 kg/kg; t_h = 80 °C)

The results obtained by the variable coefficients model show better agreement with experimentally obtained temperatures than the results obtained by the model with constant coefficients (Fig. 5). Temperature profiles inside the sheet obtained experimentally and by the model with variable coefficients are displayed on Figs. 6, 7 and 8. During the worming up period the numerically predicted temperatures increase slower than the temperatures determined in experiments. The main cause of this discrepancy is the adoption of linear time dependence of empirical parameter for contact resistance temperature drop $\Delta t(\tau)$, in the boundary condition for heat transfer at contact surface (7b) during the numerical calculation. The adopted linear dependence $\Delta t(\tau)$, hired in aim to simplify the numerical procedure, is accurate enough for all the periods of drying except the initial one when the temperature difference between the heating and the

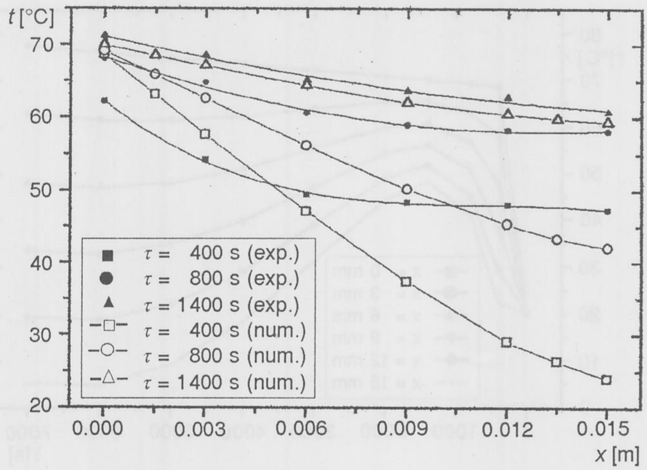

Figure 6. Temperature distributions obtained experimentally and numerically (model with variable coefficients) for different drying times

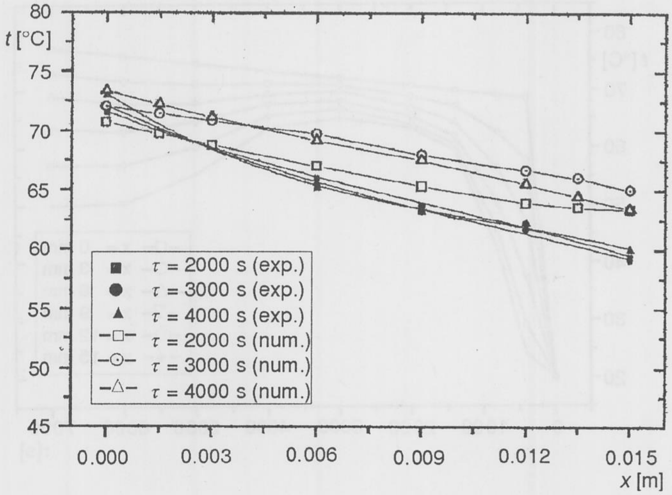

Figure 7. Temperature distributions obtained experimentally and numerically (model with variable coefficients) for different drying times

contact surface is greater than assumed. Thus for the calculations of temperature distributions during the worming up period the more exact function $\Delta t(\tau)$ should be used. Of course the accuracy of the predicted temperature fields is also affected by the precision of experimental determination of heat and mass fluxes at the free surface of the sheet.

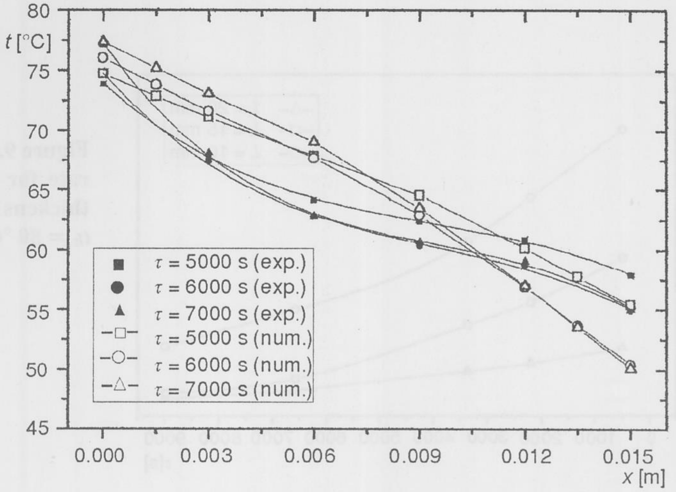

Figure 8. Temperature distributions obtained experimentally and numerically (model with variable coefficients) for different drying times

Experiments performed with samples having different thickness and different initial moisture content as well as at the different temperature of the heating surface, have the temperature distribution curves similar in shape with the curves presented. Of course, the temperature levels and duration of certain phases of the process depend on the process conditions.

The change of the overall drying rate during the process, for different process parameters are shown on Figs. 9, 10 and 11. The overall drying rate is determined on the basis of experimental data. As moisture leaves the sample only through the free surface, the value of overall drying rate is equal to the value of water vapor flux from the free surface and might be defined using the known average moisture content decrease during the process:

$$j_m(L,\tau) = \rho_0 L \left| \frac{d\bar{u}}{d\tau} \right| \tag{9}$$

The value of $d\bar{u}/d\tau$ in equation (9) is determined by derivation of experimentally obtained curve for average moisture content decrease during the observed process.

The thickness of the sample might be assumed the most determining process parameter in the case of contact drying. Thinner samples have higher drying rates (Fig. 9), because of the larger amount of the heat received from the heating surface and because the lower resistance to mass transfer, especially to the vaporized phase of the moisture flow, inside the sheet. During the drying time the overall drying rate of the thinner samples decline faster.

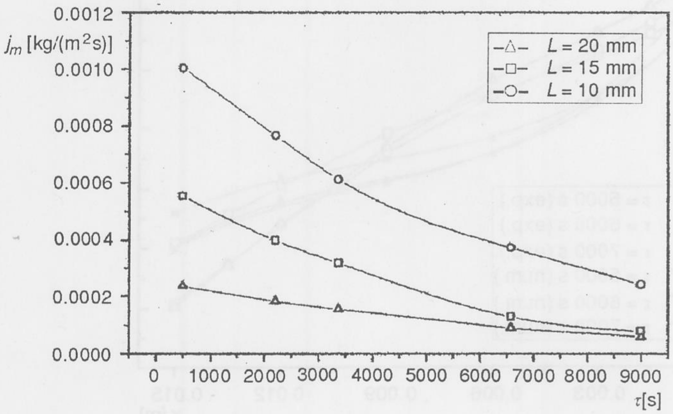

Figure 9. Overall drying rate for different sample thickens u_0 = 0.200 kg/kg; t_h = 80 °C)

Raising the temperature of the heating surface intensifies the heat supply to the material, so the internal moisture evaporation increases. Besides, the intensity of thermodiffusional movement of both liquid and vapor phase toward the free surface becomes larger. Consequently the drying rate gets higher (Fig. 10). In case of further increase of

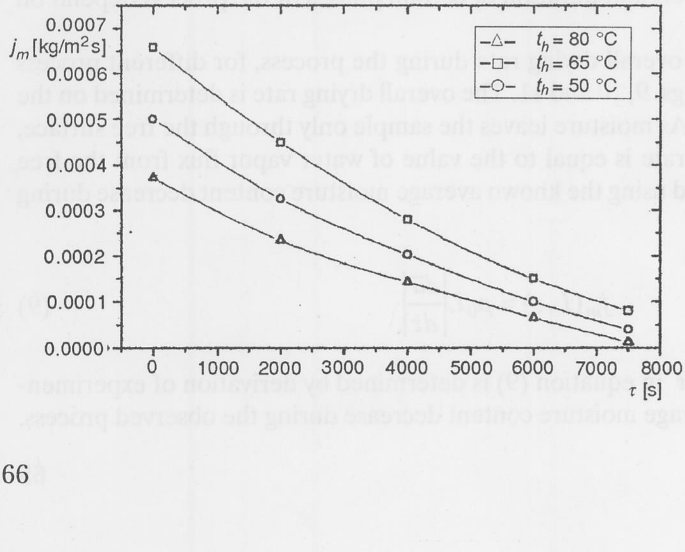

Figure 10. Overall drying rate for different temperatures of the heating surface (L = 15 mm; u_0 = 0.200 kg/kg)

the heating surface temperature over 100 °C under the atmospheric pressure of the surrounding air, the drying rates incline rapidly because of quick arising of an additional driving force for the moisture movement – the gradient of total pressure inside the material. Increasing the initial moisture content of the sample gives higher drying rates (Fig. 11). For longer drying times the importance of the initial moisture content diminishes. The moisture content profiles inside the sample, for the adopted process conditions (L = 15 mm; u_0 = 0.200 kg/kg; t_h = 80 °C), obtained by solving the models with constant and variable parameters in governing equations are shown on Figs. 12 and 13.

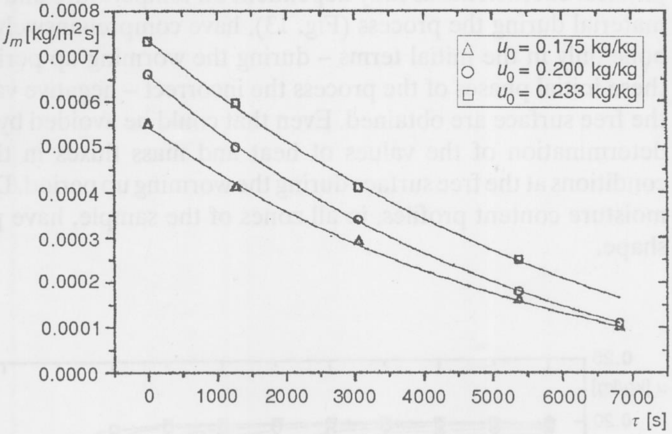

Figure 11. Overall drying rate for different initial moisture contents (L = 15 mm; t_h = 80 °C)

The parabolic shape of moisture content profiles obtained by the model with constant coefficients (Fig. 12) could have been expected, concerning the prior results [6,

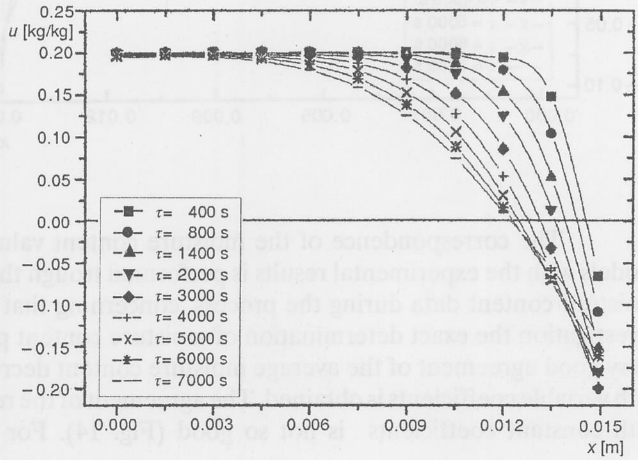

Figure 12. Moisture content profiles calculated using the model wit constant coefficients (L = 15 mm; u_0 = 0.200 kg/kg; t_h = 80 °C)

7, 8] of this approach. According to this model the lowering of moisture content of the sample is effected through decreasing the curve of parabolic moisture content profile and it's receding from the free surface. Consequently, in the zones near the free surface calculation gives physically incorrect – negative values of the moisture content. Besides, this model practically does not show the decline of moisture content near the contact surface during the process, which evidently exists.

The moisture content profiles predicted by the model that respects all thermo-physical coefficients as fully dependent on temperature and moisture content inside the material during the process (Fig. 13), have comprehensively different shape. It is parabolic only in the initial terms – during the worming up period of the material. Only in these initial phases of the process the incorrect – negative values of moisture content at the free surface are obtained. Even that could be avoided by more precise experimental determination of the values of heat and mass fluxes in the equations for boundary conditions at the free surface during the worming up period. During the rest of the process moisture content profiles, in all zones of the sample, have physically more satisfactory shape.

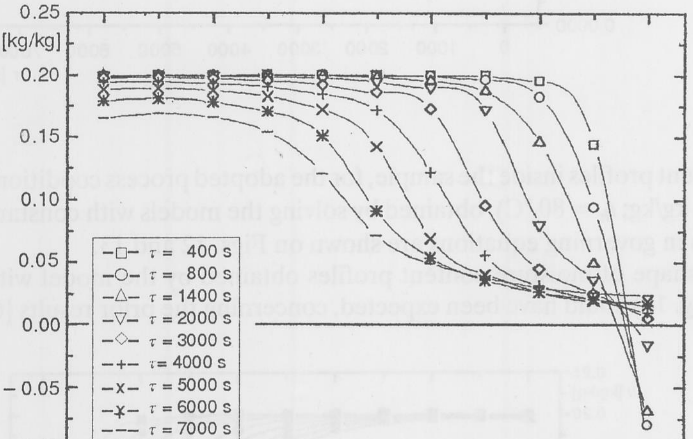

Figure 13. Moisture content profiles calculated using the model with variable coefficients

The correspondence of the moisture content values predicted by both of the models with the experimental results is performed trough the comparison of the average moisture content data during the process, concerning that in the experimental part of investigation the exact determination of moisture content profiles were not performed. Very good agreement of the average moisture content decrease predicted by the model with variable coefficients is obtained. The agreement of the results predicted by the model with constant coefficients is not so good (Fig. 14). For different values of process conditions the similar general conclusion about the correspondence of numerical and experimental results can be drown.

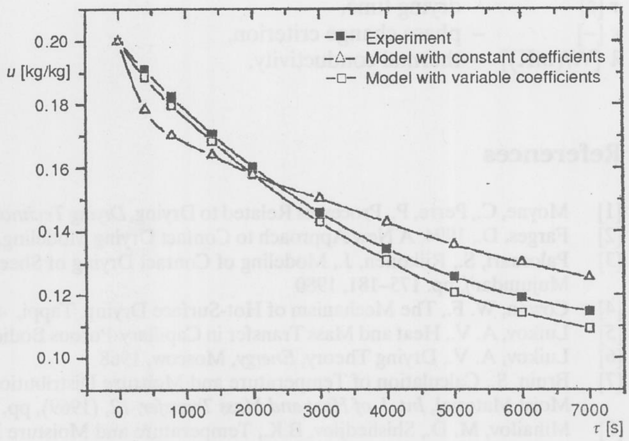

Figure 14. Decrease of the average moisture content during the observed drying process (L = 15 mm; u_0 = 0.200 kg/kg; t_h =80 °C)

Concluding remarks

The main aim of the performed investigation is to show the insufficiency of the use of constant values of thermophysical and transport properties of dried material in modeling the high intensity drying processes such as the process of combined conductive-convective drying, and to come up with a solution that would give a better results in drying dynamics prediction. Therefore, the mathematical model with variable thermophysic coefficients is proposed, dependent on the values of relevant potentials of heat and mass transfer inside the moist material, in governing equations. The confirmation of the mathematical model is carried out through the comparison with the results obtained experimentally. Experimental part of investigation is accomplished on the laboratory facility capable to maintain the required boundary conditions. The representative material, with previously defined dependencies of relevant thermophysical properties on temperature and moisture content was utilized. The values of the temperature and moisture content profiles inside the sample, predicted by the model with variable coefficients show much more physical sense and better agreement compared to experimental data than the corresponding results of the model with constant coefficients.

Nomenclature

a_m [m²/s]	– moisture diffusivity,	L [m]	– thickness of the sample,
c [J/(kgK)]	– heat capacity,	r [J/kg]	– latent heat of vaporization,
j_m [kg/(m²s)]	– mass flux; overall drying rate,	t [°C]	– temperature,
j_q [W/m²]	– heat flux,	t_0 [°C]	– initial temperature,
t_h [°C]	– temperature of the heating surface,	u [kg/kg]	– moisture content (dry basis),

u_0 [kg/kg] – initial moisture content (dry basis),
x [m] – distance from the heating surface,
$\Delta t(\tau)$ [°C] – empirical parameter for the contact resistance temperature drop,
δ [1/K] – thermo-gradient coefficient,
ρ_0 [kg/m^3] – density of dry material,
τ [s] – drying time,
ε [–] – phase change criterion,
λ [W/(mK)] – thermal conductivity,

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