NUMERICAL SIMULATION OF REAL MATERIALS CONVECTIVE DRYING IN FIXED BED

by

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Original scientific paper
UDC: 519.876.5:66.0474/.5=20
BIBLID: 0354–9836, *1* (1997) 2, 59–70

Mathematical model describing unsteady one-dimensional heat and mass transfer between gas phase and dried material during convective drying process in fixed bed as well as the results of numerical simulation obtained on the basis of mentioned model for the case of real materials are shown in this paper. Heat and mass transfer between gas phase and material surface are calculated using well known expressions for the case of a fixed bed (available in the literature). Drying process inside dried material (internal moisture transport) is calculated using an original technique based on the modified Luikov drying theory. A control volume numerical method is used for discretization of a system of the coupled partial differential equations, while the obtained linearized system of algebraic equations is solved by means of TDMA (TriDiagonal-Matrix Algorithm). Results of numerical simulation are compared with the available corresponding experimental data. The mathematical model is successfully verified.

Introduction

Due to development of numerical mathematics, a great part of contemporary research studies concerned with various processes is involved in modeling and numerical simulation. This is also the case with investigations of drying processes. Results produced by modeling and numerical simulation allow prediction of the residence time of a material or dimensions of a dryer required to dry a particular material to a final target moisture. A good model (numerical simulation) can successfully substitute lengthy and expensive experimental investigations of the process.

Much work has been done on experimental and theoretical investigations of convective drying processes in fixed, fluidized and vibrofluidized beds by the Laboratory for Thermal Engineering and Energy at the VINČA Institute of Nuclear Sciences (Belgrade, Yugoslavia) during past years. Mathematical models describing heat and mass transfer in drying of grained and powdery real (colloidal capillary-porous) materials in fixed [3, 9, 11, 12], fluidized [10, 13] and vibrofluidized [8] beds were developed and successfully verified. Common concept of the mentioned models is that drying process is calculated for elementary beds of immobile particles, and that mixing of particles occurring in the case of mobile beds (fluidized and vibrofluidized) is taken into account by the diffusion term in the differential equations, using the available particle diffusion coefficients. Development of this models, being used and checked constantly for new materials, permanently innovated and improved in order to obtain higher precision of calculated data, is enabled with a new approach in defining the basic drying equation.

This paper presents a review of the obtained results in numerical simulation of drying the fine-grained (poppy seed) and grained materials (corn) in fixed bed, with the new approach in defining the drying equation, realized in the Laboratory for Thermal Engineering and Energy during past years.

Mathematical model

Consideration is given to convective drying as a nonstationary problem of heat and mass transfer caused by fluid flow (drying agent) through a bed of particles (grains). Differential equation for conservation of the general dependent variable ϕ for this problem can be written in the following form:

$$\frac{\partial}{\partial \tau}(\rho\phi) + div(\rho\vec{U}\phi) = div(\Gamma_\phi \, grad\phi) + S_\phi \tag{1}$$

The terms in the differential equation on the left side are the unsteady-state term and the convection term, while on the right side are the diffusion term, where Γ_ϕ represents the diffusion coefficient, and the source term. All kinds of the transport laws must be incorporated inside the source term, which represents generation and dissipation of the variable ϕ. Forms of the variables Γ_ϕ and S_ϕ depend on the physical meaning of the variable ϕ. For the case of the continuity equation $\phi = 1$ and $\Gamma_\phi = 0$.

In the case of the nonstationary one-dimensional problem, equation (1) has the following form:

$$\frac{\partial}{\partial \tau}(\rho\phi) + \frac{\partial}{\partial x}(\rho U\phi) = \frac{\partial}{\partial x}\left(\Gamma_\phi \frac{\partial \phi}{\partial x}\right) + S_\phi \tag{2}$$

In the convective drying theory, the moisture (humidity) content and temperature of a material to be dried and of a drying agent represent particular cases of the general dependent variable ϕ.

The basic assumptions adopted in the mathematical model describing the nonstationary simultaneous one-dimensional heat and mass transfer between a gas phase and a material during convective drying in fixed bed are:

- each phase (gas and solid phase) is considered as a quasi-homogeneous phase with effective transport coefficients;
- one-dimensional model is used (changes of the parameters are discussed in the vertical direction only, while those in the other directions are neglected);

- all solids are of the same size, shape and density at one moment of time;
- heat and mass transfer between a solid phase and a gas phase are taking place at the surface of the solids, assuming the state of thermodynamic equilibrium;
- moisture transfer inside a dried material is calculated using an original technique based on the modified Luikov equation from the drying theory;
- heat transfer inside a dried material is neglected;

On the basis of mentioned assumptions a system of partial differential equations of heat and mass balance for the case of convective drying in fixed bed (the simplest case) can be defined in the following form:

- Gas humidity and enthalpy conservation:

$$\rho_g V_g \left(\frac{\partial g}{\partial \tau} + U_g \frac{\partial g}{\partial x} \right) = S_M \tag{3}$$

$$\rho_g V_g C_g \left(\frac{\partial T_g}{\partial \tau} + U_g \frac{\partial T_g}{\partial x} \right) = h_{pg} a_p V (T_p - T_g) + h_{wg} a_w V (T_w - T_g) + (C_v T_p - C_v T_g) S_M \tag{4}$$

where $U_g = U_0/\varepsilon$ is a real gas velocity, while $S_M = \rho_g k_{pg} a_p V_p (g_s - g)$ defines a mass source.

- Material moisture and enthalpy conservation:

$$\rho_{pd} V_p \left(\frac{\partial u}{\partial \tau} + U_p \frac{\partial u}{\partial x} \right) = -S_M \tag{5}$$

$$\rho_{pd} V_p C_p \left(\frac{\partial T_p}{\partial \tau} + U_p \frac{\partial T_p}{\partial x} \right) = h_{pg} a_p V (T_g - T_p) + h_{wp} a_w V (T_w - T_p) - r S_M \tag{6}$$

- Continuity equation:

$$\frac{\partial \rho_g}{\partial \tau} + \frac{\partial}{\partial x} (\rho_g U_0) = S_M^* \tag{7}$$

The defined system of coupled partial differential equations (3–7) has to be completed with the drying equation. It is not easy to determine exactly a drying rate for a particular convective drying process. In the scope of the original approach [6, 7] developed for describing a drying process of an arbitrary bed for any real material, the transport phenomena inside and outside a material must be discussed separately because of the fact that during convective drying of any real material two different types of resistance exist due to:

- moisture transport from material interior to its surface,
- moisture convection from material surface to a drying agent.

The internal moisture transport is a more complex problem, as it is a result of several different mechanisms of moisture transport [5], namely:

- liquid transport due to the existence of capillary forces,

- liquid transport due to the existence of a moisture concentration gradient (diffusion of liquid phase in the case of solution existence in a material),
- liquid transport at pore surface due to surface tensions,
- vapor transport due to concentration gradients (vapor diffusion),
- liquid and vapor transport due to the existence of a temperature gradient (thermal diffusion),
- liquid and vapor transport due to the existence of the total pressure gradient.

On the basis of the simplified differential equation for drying rate, used from the Luikov drying theory, a drying rate for moisture transport inside a dried material can be expressed in the general form as:

$$-\frac{\partial u}{\partial \tau} = K_i(u - u_{eq,s}) \tag{8}$$

The coefficient "K_i" (internal moisture transport coefficient) includes resistance of moisture transport inside a material, and can be defined on the basis of material characteristics, separately from drying conditions (drying agent velocity and temperature). This coefficient depends on the material type, the size and shape of solids, material temperature, and a depth of the evaporation front. A detailed analysis of the results obtained by the experimental investigation of the drying process for different materials in fixed bed [2, 5] shows that internal moisture transport coefficient (defining moisture transport from the material internal space to its surface) can be sufficiently exactly expressed using an empirical function depending on the mean material moisture and temperature, initial material moisture and condition of material surface. It is suitable to present mentioned dependence in the following form:

$$K_i = A_k \left(\frac{u}{u_0}\right)^{n_u}, \quad A_k = A_1 t_m^2 + A_2 t_m + A_3 \tag{9}$$

Expression (9) is possible to obtain for every explored material using the original technique based on the experimental investigating the drying kinetics for an elementary fixed bed of a given material [6, 7], assuming a bed of particles having linear and differentially small changes of the parameters along the bed height as an elementary bed.

On the other hand, it is assumed that all amount of moisture transported from the material interior to its surface is transferred in to the surrounding gas, permanently keeping the material surface and thin layer of gas close to that surface at the state of thermodynamic equilibrium. Drying rate for the case of external moisture transport (between a gas boundary layer at material surface and a surrounding gas) can be expressed as:

$$\frac{\partial u}{\partial \tau} = k_{pg} a_p (g - g_s) \frac{\rho_g}{\rho_p} \tag{10}$$

Mass transfer coefficients between a material and a gas in fixed bed (k_{pg}) are well known (detailed investigated, generalized in the form of suitable empirical expressions and reported by great number of the authors) mainly for the case of two groups of materials: 1) powdery and fine-grained materials (milk powder, poppy seed, *etc.*), 2) grained and coarse-grained materials (corn grain with and/or without the corncob, coal, *etc.*). Variable g_s (gas humidity in a boundary layer) have to be defined from the state of equilibrium for the system formed by moist material in contact with humid gas, attained when partial pressure of water vapor at the material surface equalizes with partial pressure of water vapor in the surrounding gas. Defining the state of equilibrium for different materials and gases at certain temperature, can be done by means of experimental method only, because of very complex mechanisms for moisture to material bonds. This is based on determination of the moisture content of the material being in thermal and hydrological equilibrium with the surrounding gas:

$$u_{eq} = f(\varphi), \; at \;\; T = const., \quad and \quad p = const. \tag{11}$$

Equation (11) is known as sorption and/or desorption (moisture) isotherm (depending on the manner of achieving the equilibrium; by sorption or desorption of the moisture). For the purpose of numerical simulation of the drying process it is necessary to generalize the experimental data for desorption isotherms in the form of an empirical expression. The great similarity of the desorption isotherm curves with one side of the probability distribution functions enables presentation of the $1 - \varphi$ as a function of the material moisture u in the form of a normal distribution function:

$$1 - \varphi = \mathrm{e}^{\mathrm{B}}, \quad B = -(B_1 t_m^2 + B_2 t_m + B_3) u_{eq}^n \tag{12}$$

All the coefficients (B_1, B_2, B_3, and n) have to be defined on the basis of the experimentally obtained equilibrium values u_{eq} and φ for several corresponding temperatures T.

Numerical procedure

Analysis of the partial differential equations (3–7) shows that all the dependent variables of interest obey a generalized conservation principle. Numerical procedure based on computer program TEACH [4] has been used for solving the partial differential equations. Each partial differential equation is discretized by means of the control-volume method (a flow field is divided by means of a vertical grid into a finite number of control volumes). Iterative line-by-line method is used for solving the obtained linearized algebraic equations, using recurrence formula during calculation of the variable's values for every line, following the procedure for all the lines in one direction. This method is called the Thomas algorithm or the TDMA (TriDiagonal-Matrix Algorithm).

Results and discussion

According to the procedure described in [6, 7], the coefficients in equation (9) and (12) were defined for the case of corn grain, obtaining the following values:

$$A_1 = 0.0707 \cdot 10^{-6},\ A_2 = 2.6745 \cdot 10^{-6},\ A_3 = 107.67 \cdot 10^{-6},\ n_u = 1.0, \tag{13}$$

$$B_1 = 0.0397,\ B_2 = 1.2585,\ B_3 = 160.5678,\ n = 8/3. \tag{14}$$

Validation of described mathematical model defining the drying process and/or numerical simulation based on the model was done on the basis of the available experimental data (our own and from literature). Data obtained by the numerical simulation on the basis of the model were compared with the data obtained by the experimental investigation of the corn grain drying in fixed bed, as well as with the data obtained on the basis of the model developed by D. Milojević [3]. One example of mentioned comparisons is shown on Fig. 1.

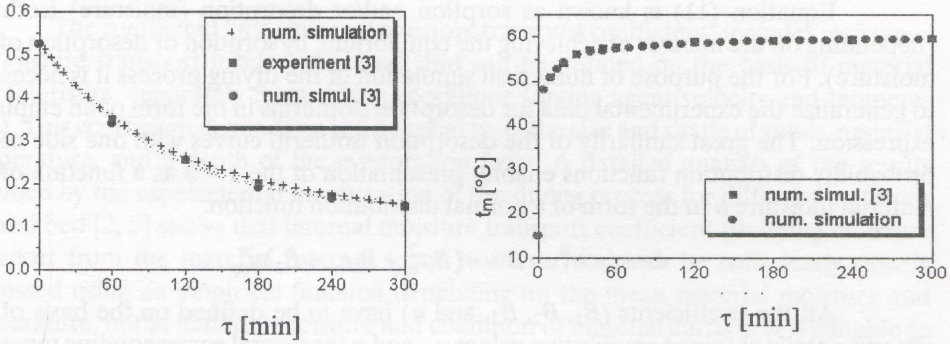

Figure 1. Comparison of numerically simulated and experimental drying kinetics in the case of corn grain drying in fixed bed;
$d_p = 0.00517$ [m], $u_0 = 0.526$ [kg/kg], $t_{p0} = 15$ [°C], $t_{a0} = 60$ [°C], $x_0 = 0.0077$ [kg/kg], $U_{a0} = 0.85$ [m/s], $H_0 = 0.08$ [m]

It can be seen that deviations between the results of the calculations by D. Milojević as well as the calculations on the basis of the presented model and the experimental data are nearly negligible. It is important to underline that almost identical results were obtained on the basis of the numerical simulation using two different mathematical models:

– the model developed by D. Milojević is explicit one, and the differential equation system was solved by means of the finite difference method,
– the model presented in this paper is implicit one; defined system of partial differential equations is solved by means of the control volume method.

Comparisons of the data obtained using the numerical simulation and experimental investigation of poppy seed drying in fixed bed can be seen on Fig. 2. Results are in a very good agreement, and the same agreement of the data was obtained for all the materials that were used.

Just a very small part of the results that can be obtained using numerical simulation for the case of corn grain drying in a fixed bed is shown, as an example, on

Figs. 3. and 4. Numerical simulation (developed software) can give a very interesting and useful analysis of the drying process, showing the influence of the curtain parameters on to the process, which can be clearly seen from the shown examples. This is in fact the main reason for making, developing and using the mathematical models describing behavior of the relevant parameters for the case of different processes.

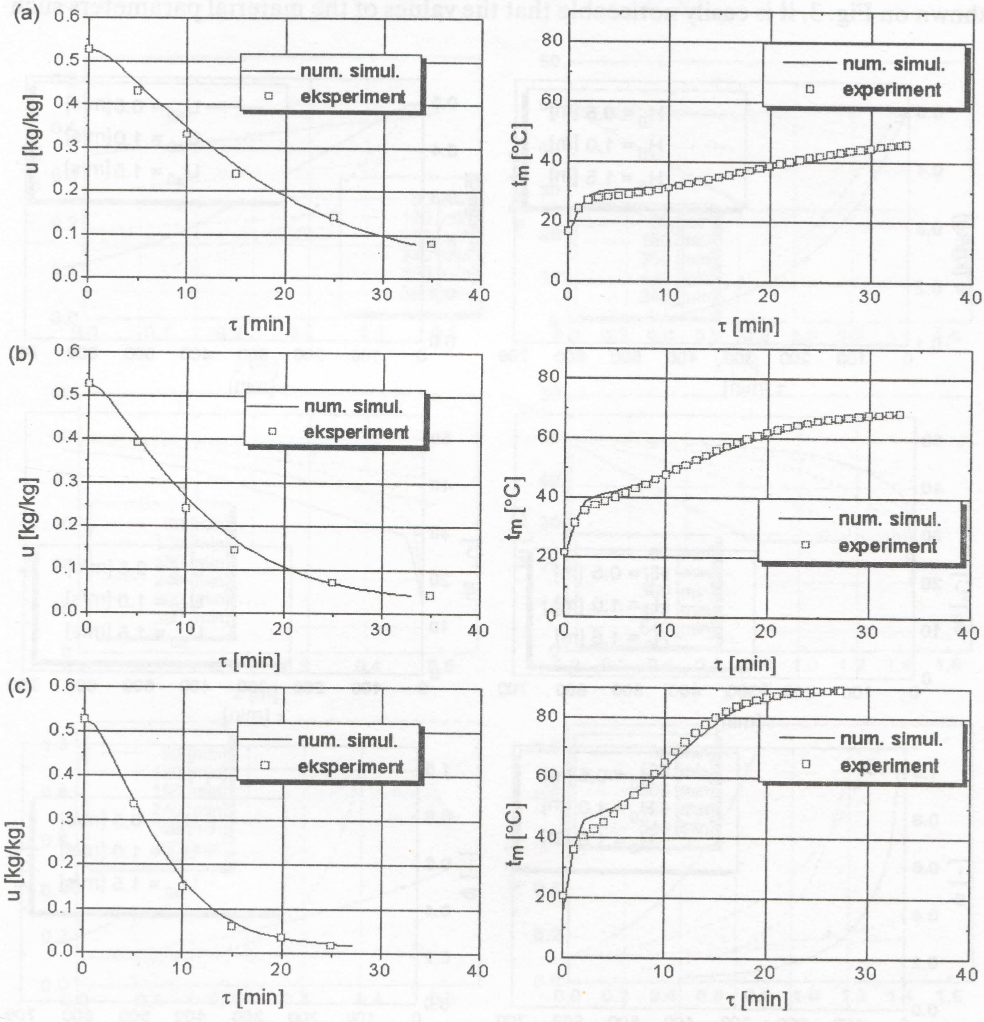

Figure 2. Comparison of numerically simulated and experimental drying kinetics in the case of poppy seed drying in fixed bed
$d_p = 0.755$ [mm], $u_0 = 0.527$ [kg/kg]
(a) $t_{a0} = 49.6$ [°C], $x_{a0} = 0.095$ [kg/kg], $U_a = 0.188$ [m/s], $t_{p0} = 16.6$ [°C], $H_0 = 0.015$ [m]
(b) $t_{a0} = 70.2$ [°C], $x_{a0} = 0.096$ [kg/kg], $U_a = 0.197$ [m/s], $t_{p0} = 21.7$ [°C], $H_0 = 0.015$ [m]
(c) $t_{a0} = 90.4$ [°C], $x_{a0} = 0.113$ [kg/kg], $U_a = 0.208$ [m/s], $t_{p0} = 20.6$ [°C], $H_0 = 0.015$ [m]

Developed mathematical model gives a lot of the possibilities in predicting the relevant parameters for the drying process. Values of the time-averaged parameters, as well as parameters along the bed height for one moment of time, can be calculated. Just a few of the results that can be obtained are shown on Figs. 3 and 4.

Besides well known differences in drying rates and period of material heating due to the different drying agent parameters (temperature, flow rate and bed height) shown on Fig. 3, it is easily noticeable that the values of the material parameters such

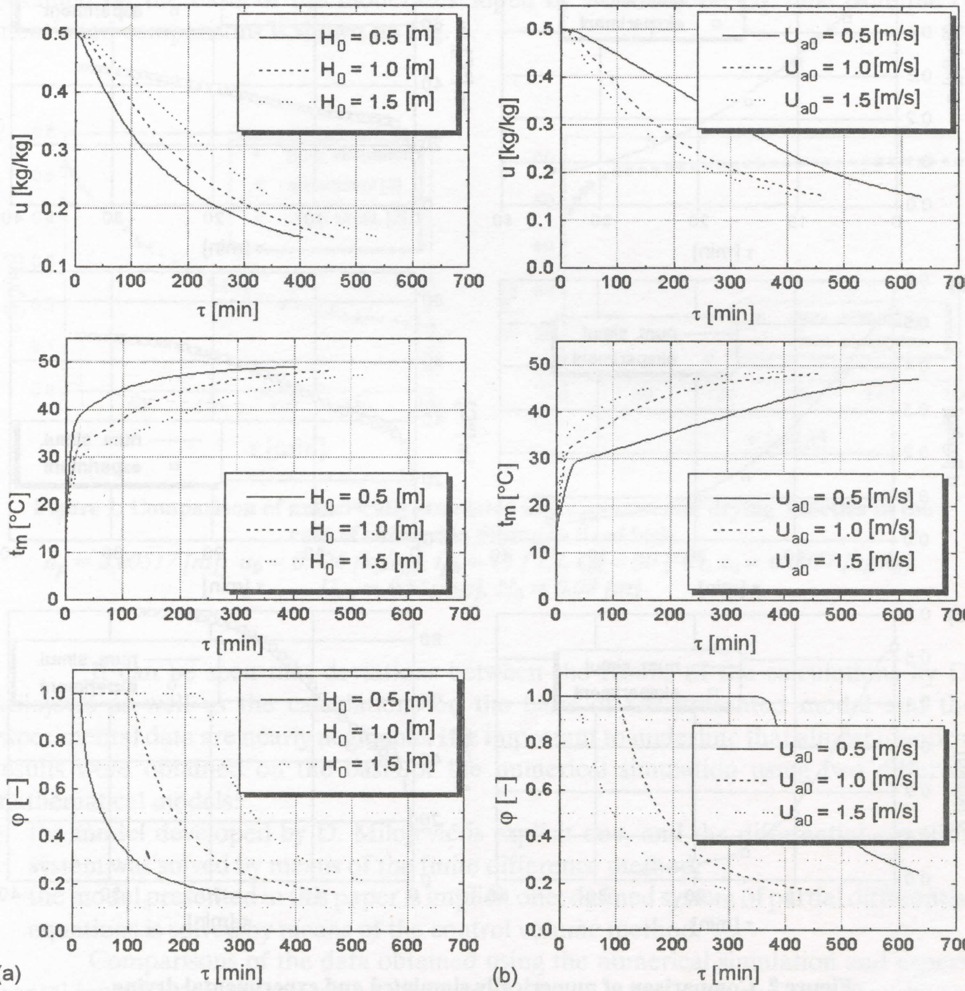

Figure 3. Results of numerical analysis of drying kinetics in the cas of corn grain drying in fixed bed

$d_p = 0.0057\ [m],\ u_0 = 0.5\ [kg/kg],\ t_{p0} = 15\ [°C],\ x_0 = 0.008\ [kg/kg]$

$(a)\ t_{a0} = 50\ [°C],\ U_{a0} = 1.0\ [m/s],\ (b)\ t_{a0} = 50\ [°C],\ H_0 = 1.0\ [m]$

as temperature and moisture content are different along the bed height for one moment of time, due to non-mixing of the particles in the case of drying process in fixed bed (Fig. 4). At the same time, the influence of the used drying agent flow rate at the drying rate is clearly noticeable (Fig. 3). For the curtain cases, drying process is keeping slower mostly because of the low drying agent flow rate, i.e. because of the insufficient drying agent capacity (saturation of the drying agent). That can be clearly seen for the case of U_{a0} = 0.5 [m/s], especially in the case of larger amount of wet material (higher bed).

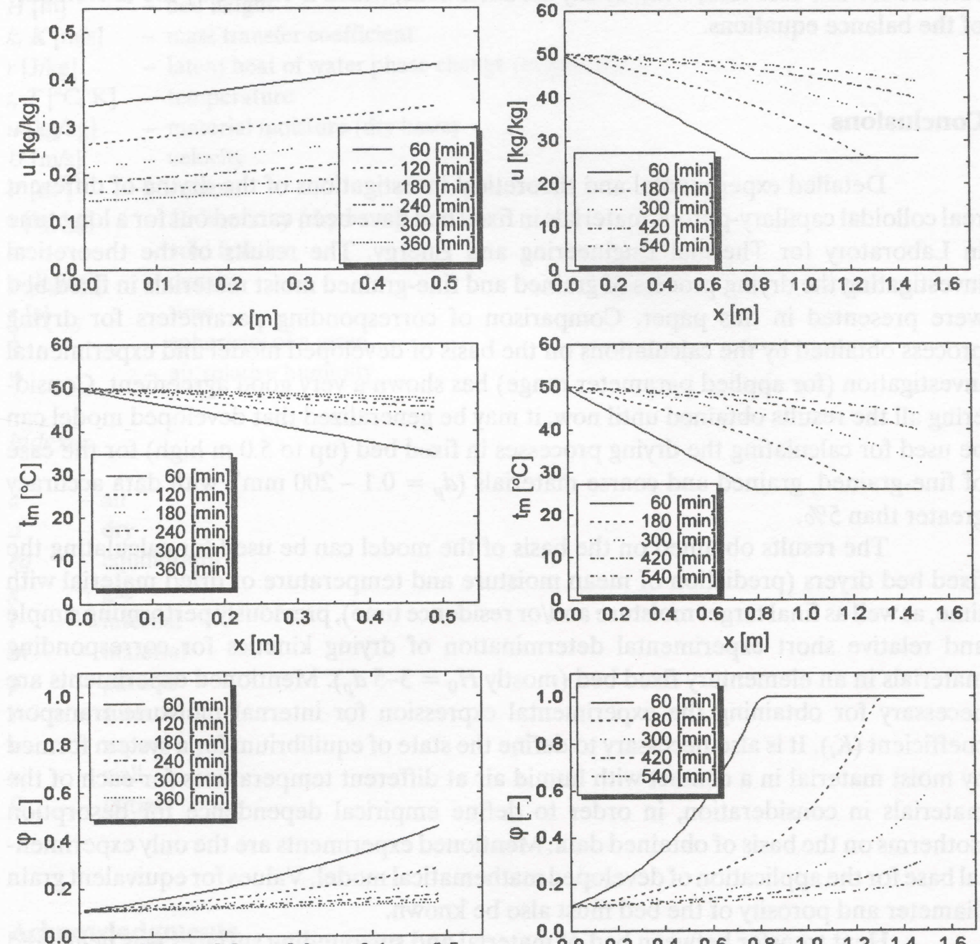

Figure 4. Material moisture content, material temperature and outlet gas relative humidity along the bed height (*x*) during corn grain drying in fixed bed; $d_p = 0.0057$ [m], $u_0 = 0.5$ [kg/kg], $t_{p0} = 15$ [°C], $t_{a0} = 50$ [°C], $x_0 = 0.008$ [kg/kg] (a) $H_0 = 0.5$ [m], $U_{a0} = 0.5$ [m/s], (b) $H_0 = 1.5$ [m], $U_{a0} = 1.0$ [m/s]

The results obtained on the basis of developed model were also successfully verified using the experimental data obtained in the case of coarse materials: corn on the corncob dried on a specific multi-pass batch dryer at "Agroseme Panonija", Subotica [9], and coal pieces dried at "Kolubara" drying plant near Belgrade [11]. Although described mathematical model is developed for the case of convective drying of cold wet materials using preheated air, during the analyzes of the coal drying, where already heated material is dried and cooled using surrounding air, it was shown that described model is absolutely suitable for this case also, without any modifications, which is caused by the universality of the balance equations.

Conclusions

Detailed experimental and theoretical investigations of the drying of different real colloidal capillary-porous materials in fixed bed have been carried out for a long time in Laboratory for Thermal Engineering and Energy. The results of the theoretical investigating the drying process of grained and fine-grained moist materials in fixed bed were presented in this paper. Comparison of corresponding parameters for drying process obtained by the calculations on the basis of developed model and experimental investigation (for applied parameter range) has shown a very good agreement. Considering all the results obtained until now, it may be generalized that developed model can be used for calculating the drying processes in fixed bed (up to 5.0 m high) for the case of fine-grained, grained and coarse materials (d_p = 0.1 – 200 mm), with data accuracy greater than 5%.

The results obtained on the basis of the model can be used for calculating the fixed bed dryers (prediction of mean moisture and temperature of dried material with time, as well as final target moisture and/or residence time), previously performing simple and relative short experimental determination of drying kinetics for corresponding materials in an elementary fixed bed (mostly H_0 = 3–5 d_p). Mentioned experiments are necessary for obtaining the experimental expression for internal moisture transport coefficient (K_i). It is also necessary to define the state of equilibrium for a system formed by moist material in a contact with humid air at different temperatures for each of the materials in consideration, in order to define empirical dependence for desorption isotherms on the basis of obtained data. Mentioned experiments are the only experimental base for the application of developed mathematical model. Values for equivalent grain diameter and porosity of the bed must also be known.

Heat transfer between bed of material and surrounding surfaces was neglected in this phase of development of presented mathematical model. Inclusion of this problem will further improve existing model, and will certainly enable higher accuracy of mentioned processes' simulation, specially in the case of high drying agent temperature, but qualitative improvement would be the definition of the parameter changes along the other bed coordinates, thus developing unsteady two- and/or three-dimensional mathematical model of heat and mass transfer during drying process.

Nomenclature

a_p [m²/m³] – bed specific surface, $6(1-\varepsilon)/(\phi d_p)$
a_w [m²/m³] – wall specific surface, A_w/V
C [J/kgK] – specific heat capacity
d [m] – diameter
g [kg/kg] – gas humidity (wet basis)
h [W/m²K] – heat transfer coefficient
H [m] – bed height
k, K [m/s] – mass transfer coefficient
r [J/kg] – latent heat of water phase change (evaporation)
t, T [°C, K] – temperature
u [kg/kg] – material moisture (dry basis)
U [m/s] – velocity
V [m³] – volume
x [kg/kg] – air humidity (dry basis)
ε – void fraction
ρ [kg/m³] – density
τ [s] – time
ϕ – sphericity of a grain
φ – air relative humidity

Indexes

a – air
d – dry
eq – equilibrium
g – gas
i – internal
m – material
p – particle
s – surface
v – vapor
w – wall
0 – initial

Acknowledgments

This work was supported by the contract from the Ministry of Science and Technology of the Republic of Serbia (Project No 08M11E1 entitled "Rational Energy Consumption in Industry and Technological Processes; Processes with Coupled Heat and Mass Transfer in a Multiphase Multicomponent Systems"). The author wishes to express his gratitude to the Ministry for their interest in this work.

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Paper submited: December 7, 1997
Paper revised: April 24, 1998
Paper accepted: May 13, 1998