EXACT EXPLICIT EQUATIONS FOR SOME TWO- AND THREE-PASS CROSS FLOW HEAT EXCHANGER EFFECTIVENESS

by

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Original scientific paper
UDC: 536.27:66.045.1=20
BIBLID: 0354–9836, *1* (1997), 1, 29–42

A review of the effectiveness (ε) – number of transfer units (NTU) – capacity rate ratio (ω) relationships for eight possible identical flow arrangements of two- and three-pass cross flow heat exchangers is presented. The flow arrangements are those with weaker fluid unmixed throughout and other stream mixed between the passes and unmixed in each pass, viz. the arrangements from [1] with flow patterns of fluid being interchanged. The results of the exact closed form solutions to the mean mixed temperatures between the passes are given for each configuration as well.

Introduction

When the compact cross-flow heat exchangers with one fluid unmixed throughout are designed as multi-pass units, the passes are usually coupled in inverted order due to the convenience in manufacturing and practical installing. However, in some cases the coupling of passes in identical order may have performance from the thermodynamic standpoint since the irreversibilities that take place due to the overall heat transfer are less pronounced at the smaller local temperature differences that occur in couplings in identical order. In some flow arrangements this fact results in an increase oh the heat exchanger effectiveness under any other condition being unchanged.

The irreversibility caused by mixing the fluid between the passes makes the heat exchanger effectiveness lower as well. If one fluid is unmixed throughout and the other is mixed just between the passes coupled in identical order, an increase in heat exchanger effectiveness may be reached if the flow arrangement is designed in a way that reduces the irreversibilities due the mixing. This can be accomplished if the weaker (with lower capacity rate $W_1 = (\dot{M}c_p)_{\min}$) fluid is mixed between passes. The thermal performances of the two- and three-pass cross-flow heat exchangers, with passes coupled in identical order, when $(\dot{M}c_p)_{max}$ fluid is unmixed throughout and the fluid is mixed between the $(\dot{M}c_p)_{min}$ passes and unmixed in each pass are presented in our previous report [1]. To complete the report on the subject as well as to point out the losses in heat exchanger effectiveness caused by mixing the $(\dot{M}c_p)_{max}$ fluid between passes, this paper gives the final results of the analytical solutions for the flow arrangements from [1] with flow patterns of fluids being interchanged.

The types of flow arrangements in [1] are differentiated by denoting alphabetically the order which the stream W_1 is entering some pass. The arrangement from [1] in which W_1 is replaced by W_2 we call the "transposed" one and denote by simply imposing a bar over the notation of the type of flow arrangement. Fig. 1 illustrates this for the two-pass exchanger with overall cross-parallel flow. Note that the W_1 fluid steams always in the x-direction so that the transposed arrangement can mathematically be strictly distinguished from the untransposed one.

Performing the procedure as described in Fig. 1 for each flow arrangement considered in [1] one is led to the eight transposed arrangements whose thermal performances are presented in the present report.

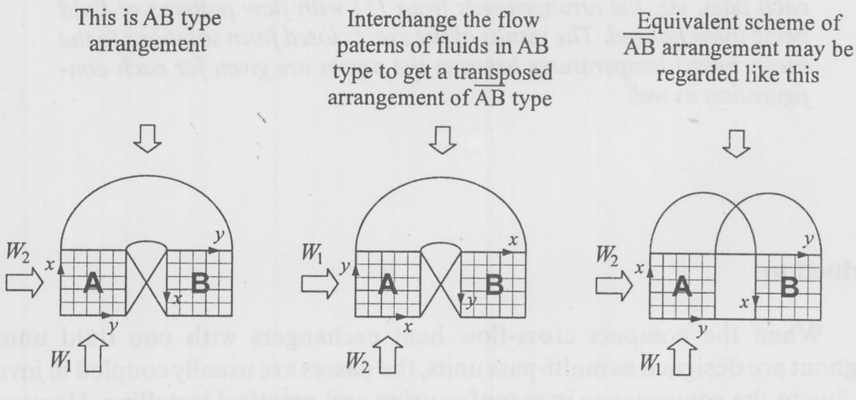


Figure 1. What is meant by a "transposed" flow arrangement

Ineffectiveness of the transposed flow arrangements

The problem of establishing ε-NTU-ω relationship is in fact that of evaluating the mean exit temperatures of the unmixed flow. To reach this goal one needs the solution to the differential equations that govern the crossflow heat transfer. When the solution to the problem is established, as discussed in [1], the explicit formula for the heat exchanger effectiveness is obtainable by simple integration, *i. e.* by averaging the exit temperature distribution over the flow cross-section area.

For a unified treatment of various flow arrangements it is useful to adopt the dimensionless temperatures θ_i $(i = 1, 2)$ in the mathematical mode in such way that weaker fluid enters the exchanger at $\theta_1' = 1$, and the stronger fluid enters $\theta_2' = 1$. For this case, the exit temperatures are related the heat exchanger effectiveness as

$$\theta_1'' = 1 - \varepsilon \tag{1}$$

$$\theta_2'' = \omega\varepsilon \tag{2}$$

It can be seen that the dimensionless mean exit temperature of the weaker stream (W_1) is equal to the complementary value of the effectiveness and thus may be termed as the heat exchanger ineffectiveness

$$\nu = 1 - \varepsilon = \theta_1'' \tag{3}$$

Since any calculation involving the exit temperatures is in fact the calculation of the ineffectivenes, it is useful to introduce the complementary value of the dimensionless mean exit temperature of the stronger stream as well, *i. e.*,

$$1 - \theta_2'' = 1 - \omega\varepsilon \tag{4}$$

It is not difficult to show that

$$\bar{\nu} = 1 - \omega\varepsilon = 1 - \omega(1 - \nu) \tag{5}$$

expresses the final result for the effectiveness of the transposes flow arrangements as does n for untransposed arrangements.

The effectiveness of all possible combinations of two- and three-pas identical order flow arrangements with W_2 unmixed throughout and W_1 mixed between passes are just algebraic combinations of the ineffectiveness of the form

$$\nu_{\alpha/\beta} = \nu\left(\frac{NTU}{\beta}, \frac{\alpha\omega NTU}{\beta}\right) = e^{-(1+\alpha\omega)\frac{NTU}{\beta}}\left[I_0\left(\frac{2NTU}{\beta}\sqrt{\alpha\omega}\right) + \right.$$
$$\left. + \sqrt{\alpha\omega}\, I_1\left(\frac{2NTU}{\beta}\sqrt{\alpha\omega}\right) - \left(\frac{1}{\alpha\omega} - 1\right)\sum_{n=2}^{\infty}(\alpha\omega)^{\frac{n}{2}} I_n\left(\frac{2NTU}{\beta}\sqrt{\alpha\omega}\right)\right] \tag{6}$$

as it was shown in [1], with $\alpha = 1, 2, 3$ and $\beta = 2, 3$.

For the transposed flow arrangements presented in this paper it is useful to introduce the ineffectiveness function of the form

$$\bar{\nu}_{\alpha/\beta} = \bar{\nu}\left(\frac{\omega NTU}{\beta}, \frac{\alpha NTU}{\beta}\right) = e^{-(\alpha+\omega)\frac{NTU}{\beta}}\left[I_0\left(\frac{2NTU}{\beta}\sqrt{\alpha\omega}\right) + \right.$$
$$\left. + \sqrt{\frac{\alpha}{\omega}}\, I_1\left(\frac{2NTU}{\beta}\sqrt{\alpha\omega}\right) - \left(\frac{\omega}{\alpha} - 1\right)\sum_{n=2}^{\infty}\left(\frac{\alpha}{\omega}\right)^{\frac{n}{2}} I_n\left(\frac{2NTU}{\beta}\sqrt{\alpha\omega}\right)\right] \tag{7}$$

The effectiveness of all possible combinations of two-pass and three-pass identical order flow arrangements with W_1 unmixed throughout and W_2 mixed between passes can be expressed in the terms of $\bar{\nu}_{\alpha/\beta}$ functions. The structure of explicit formulas for the ineffectiveness of the transposed flow arrangements $\bar{\nu}$ in the terms of $\bar{\nu}_{\alpha/\beta}$ is exactly the same as the structure of ν in the terms of $\nu_{\alpha/\beta}$ for untransposed arrangements. This may be verified by comparing the results given in this paper with the results for corresponding untransposed configurations in [1].

Results

Two possible two-pass and six possible three-pass combinations of the transposed flow arrangements from [1] are presented in figures 2 thruogh 9 by ascheme in the upper-left corners. The same figures contain a sketch in a coordinate sistem $0xy$ as well. These are given to provide the necessary notation os well as to recognize the difference in coupling in identical order. A transposed identical order, for mathematical description, means that the strem which is mixed between passes enter each pass at $y = 0$. The fluid 1 is always assumed to be unmixed throughout, while the fluid 2 is one that is mixed between passes.

Each figure 2–9 contains the explicit formulas for the effectiveness and the mean temperatures in the interpasses of the correspondin type of exchanger. It may be seen that all of the formulas involve the ineffectiveness functions $\bar{\nu}_{\alpha/\beta}$.

Notation adopted in Figs. 2–9 for mean mixed (fluid 2) dimensionless temperatures between passes is as follows. One bar denotes the first interpass (always between passes A and B): $\bar{\theta}_2 = (\bar{T}_2 - T_2')/(T_1' - T_2')$. Two bars denote the second interpass (always between B and C): $\bar{\bar{\theta}}_2 = (\bar{\bar{T}}_2 - T_2')(T_1' - T_2')$. Note that structure of the results for $1-\bar{\theta}_2$ and $1-\bar{\bar{\theta}}_2$ in terms of $\bar{\nu}_{\alpha/\beta}$ is the same as in the relations of $\bar{\theta}_1$ and $\bar{\bar{\theta}}_1$ in terms od $\nu_{\alpha/\beta}$ [1].

Studying the figures one can draw many conclusions on usefulness of some particular arrangements. The curves for the mixed temperatures in the interpasses may indicate the operation conditions under which some scheme may be used in practice. The duty of some passes, as well as the possible casse of heat flow inversion may be evaluated from the same curves. Thus, the region of senseless cuopling of some passes and even of use of entire arrangement may be recognised.

Figure 9 in [1] gives the comparison between the effectiveness curves for well balanced flows ($\omega = 1$) in all types of two- and three-pass arrangements under consideration. The same curves hold for the trahspose configurations as well, since the corresponding transposed and untransposed arrangements are equivalent dor $\omega = 1$.

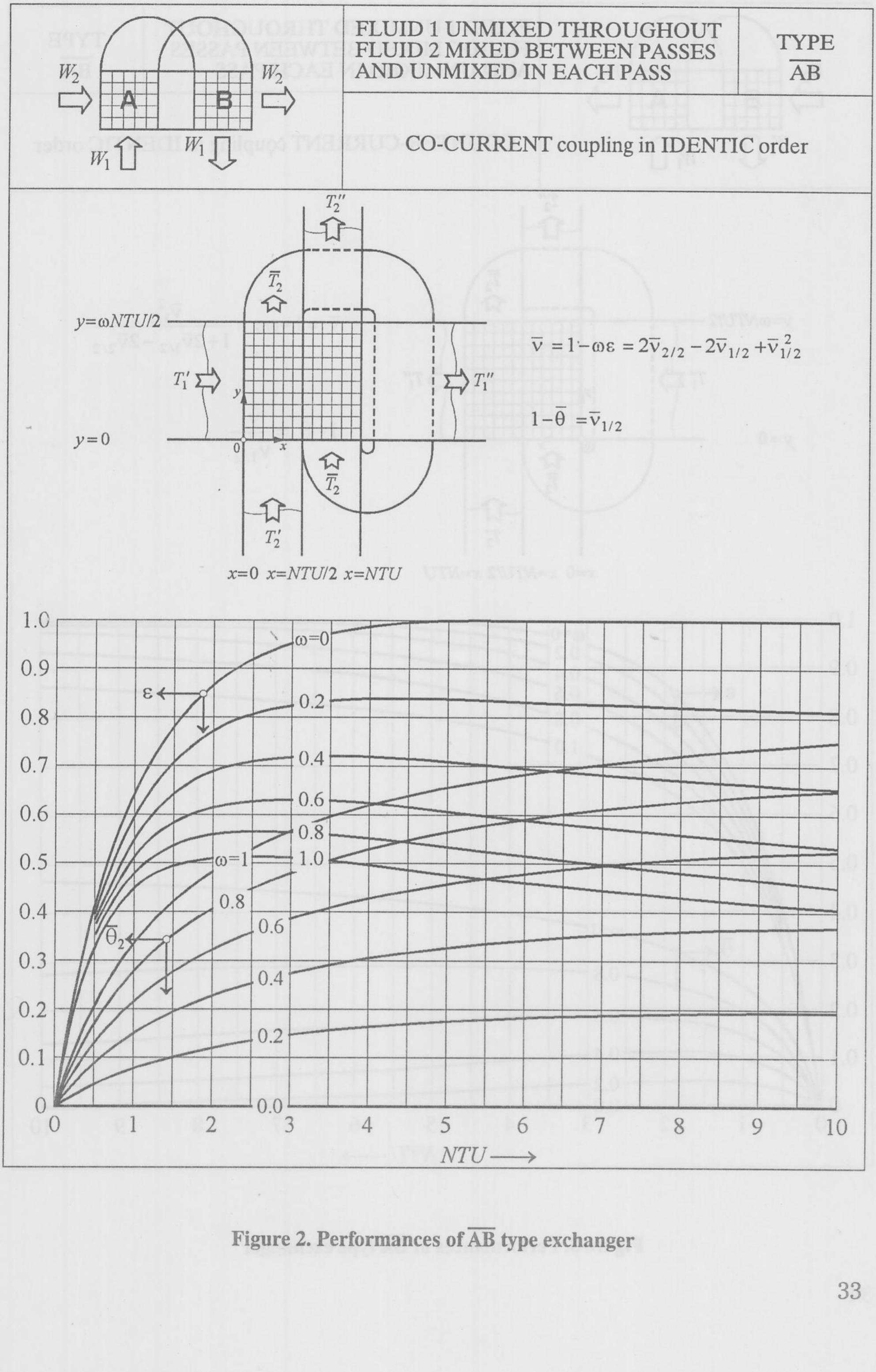


Figure 2. Performances of $\overline{AB}$ type exchanger

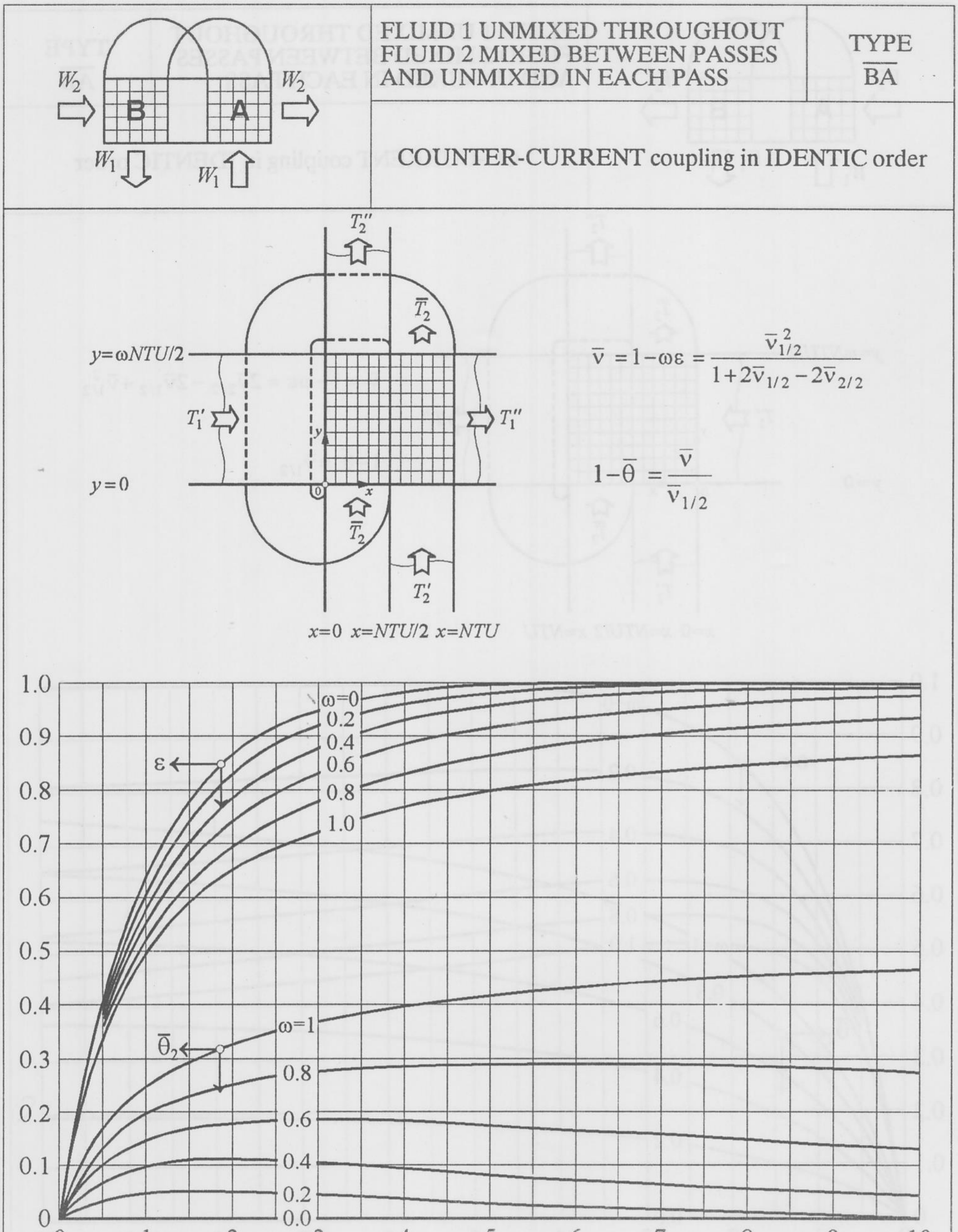


Figure 3. Performances of $\overline{BA}$ type exchanger

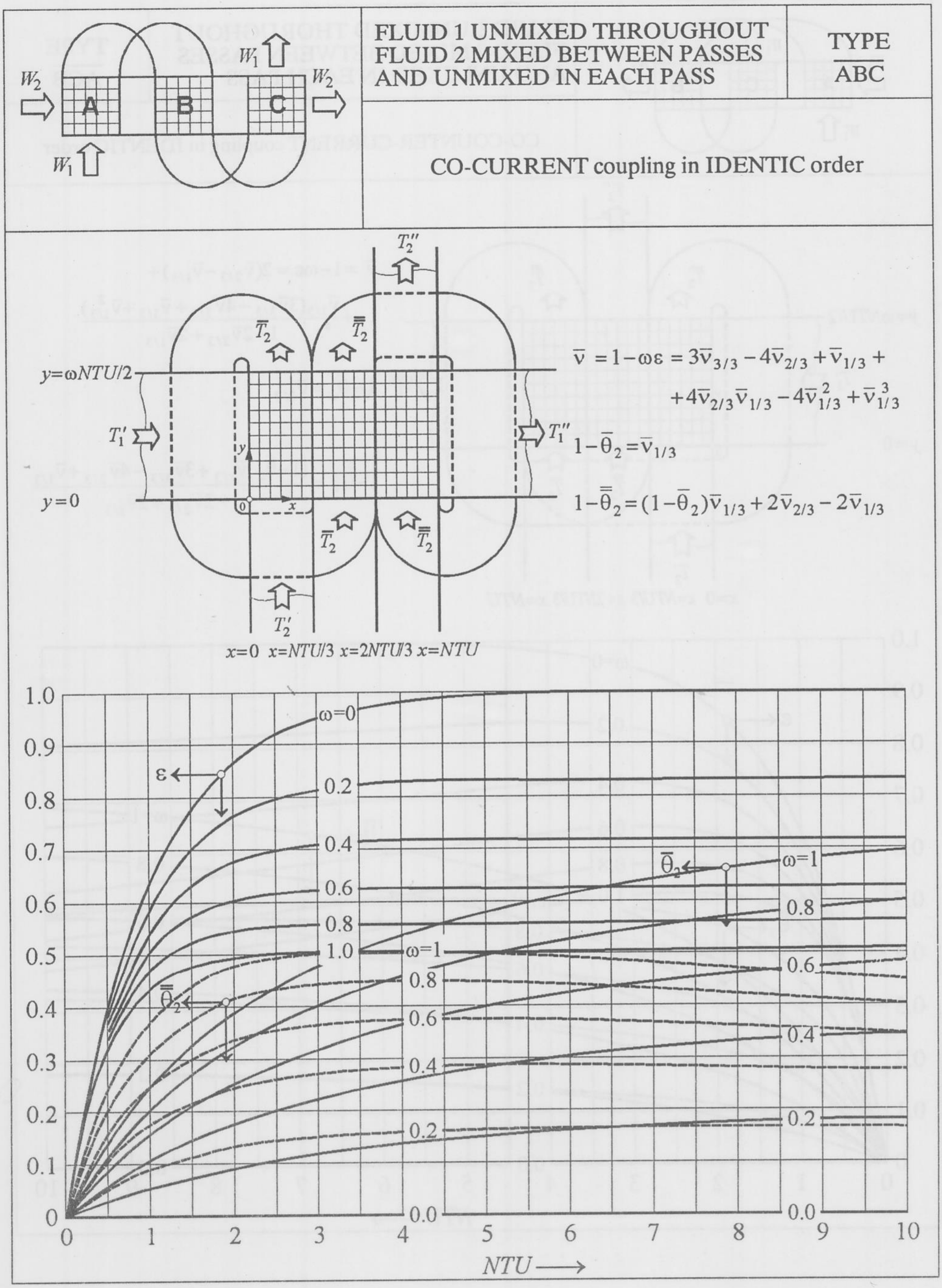
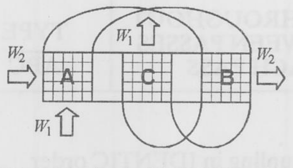


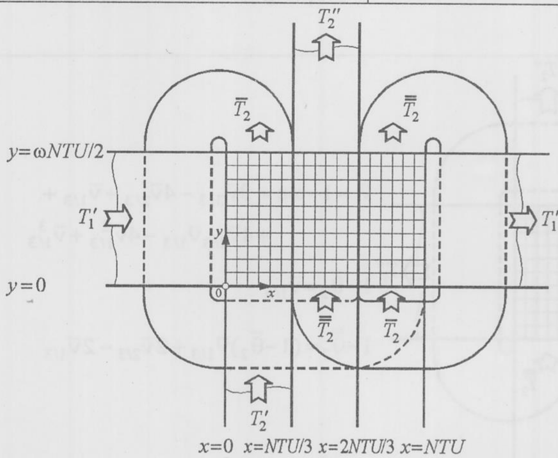
Figure 4. Performances of $\overline{ABC}$ type exchanger



FLUID 1 UNMIXED THORUGHOUT
FLUID 2 MIXED BETWEEN PASSES
AND UNMIXED IN EACH PASS

TYPE $\overline{\text{ACB}}$

CO-COUNTER-CURRENT coupling in IDENTIC order



$$\bar{v} = 1-\omega\varepsilon = 2(\bar{v}_{2/3}-\bar{v}_{1/3}) + \frac{\bar{v}_{1/3}(3\bar{v}_{3/3}-4\bar{v}_{2/3}+\bar{v}_{1/3}+\bar{v}_{1/3}^{2})}{1-2\bar{v}_{2/3}+2\bar{v}_{1/3}}$$

$$1-\bar{\theta}_2 = \bar{v}_{1/3}$$

$$1-\bar{\bar{\theta}}_2 = \frac{(1-\bar{\theta}_2)\bar{v}_{1/3}+3\bar{v}_{3/3}-4\bar{v}_{2/3}+\bar{v}_{1/3}}{1-2\bar{v}_{2/3}+2\bar{v}_{1/3}}$$

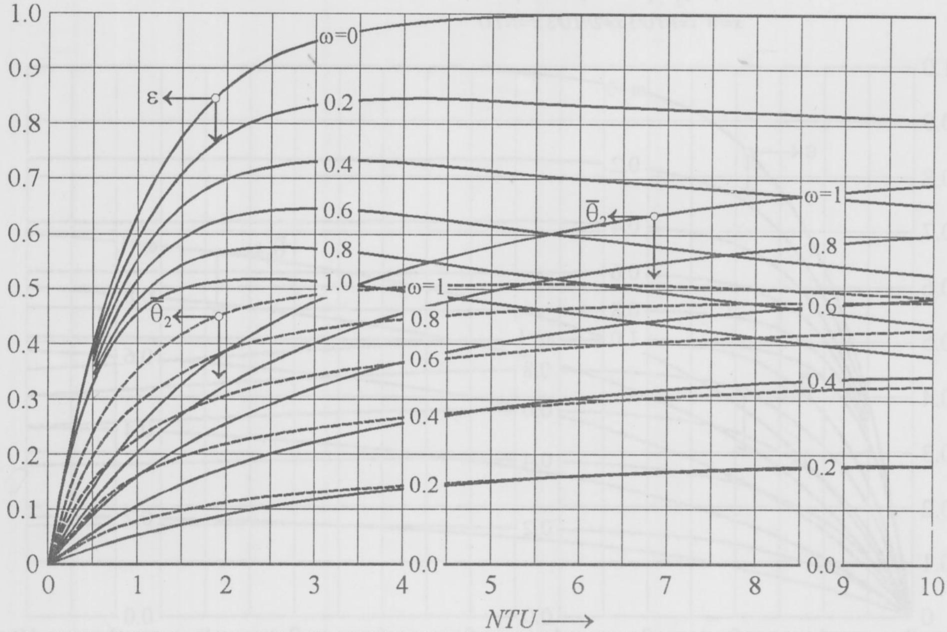


Figure 5. Performances of $\overline{\text{ACB}}$ type exchanger

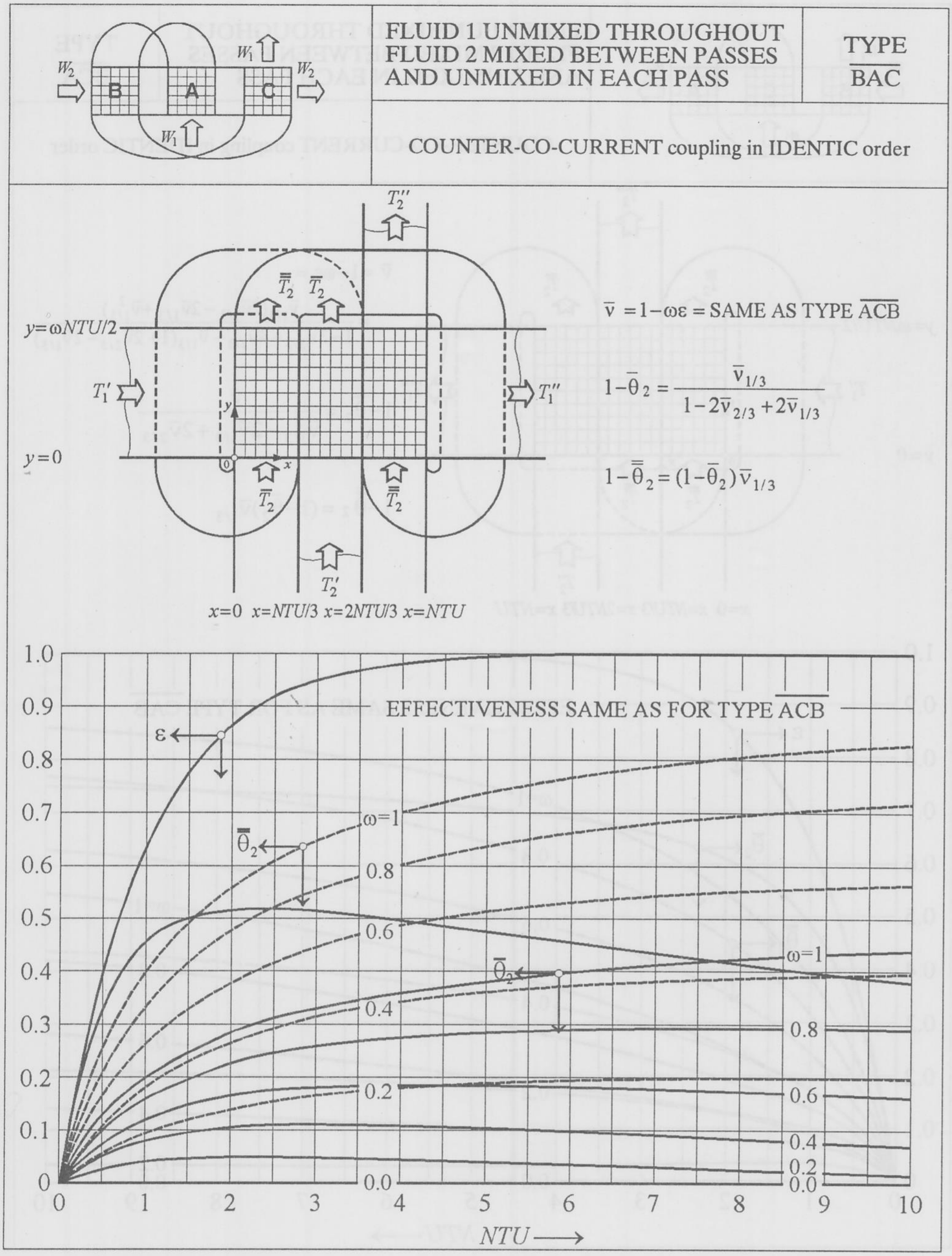
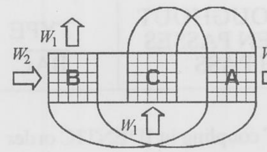


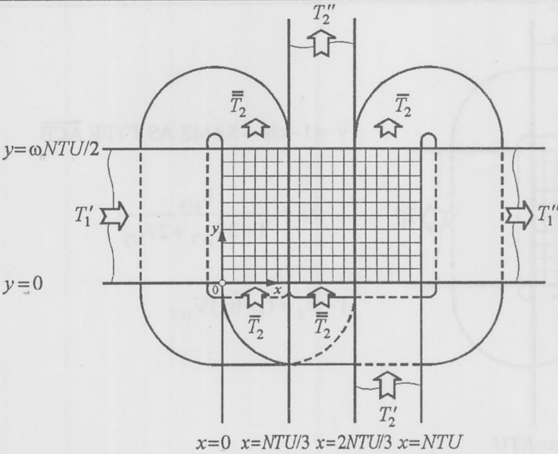
Figure 6. Performances of $\overline{BAC}$ type exchanger



FLUID 1 UNMIXED THROUGHOUT
FLUID 2 MIXED BETWEEN PASSES
AND UNMIXED IN EACH PASS

TYPE $\overline{\text{BCA}}$

COUNTER-CO-CURRENT coupling in IDENTIC order



$$\bar{\nu} = 1-\omega\varepsilon = \frac{\bar{\nu}_{1/3}(2\bar{\nu}_{2/3}-2\bar{\nu}_{1/3}+\bar{\nu}_{1/3}^{2})}{1-3\bar{\nu}_{3/3}+4\bar{\nu}_{2/3}-\bar{\nu}_{1/3}(1+2\bar{\nu}_{2/3}-2\bar{\nu}_{1/3})}$$

$$1-\bar{\theta}_2 = \frac{\bar{\nu}}{\bar{\nu}_{1/3}^{2}-2\bar{\nu}_{1/3}+2\bar{\nu}_{2/3}}$$

$$1-\bar{\bar{\theta}}_2 = (1-\bar{\theta}_2)\bar{\nu}_{1/3}$$

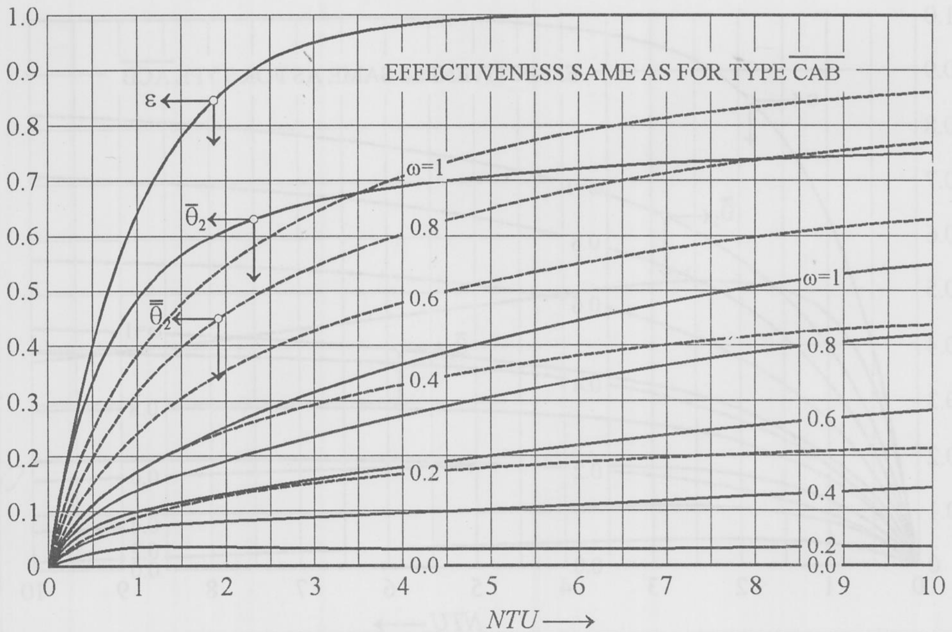


Figure 7. Performances of $\overline{\text{BCA}}$ type exchanger

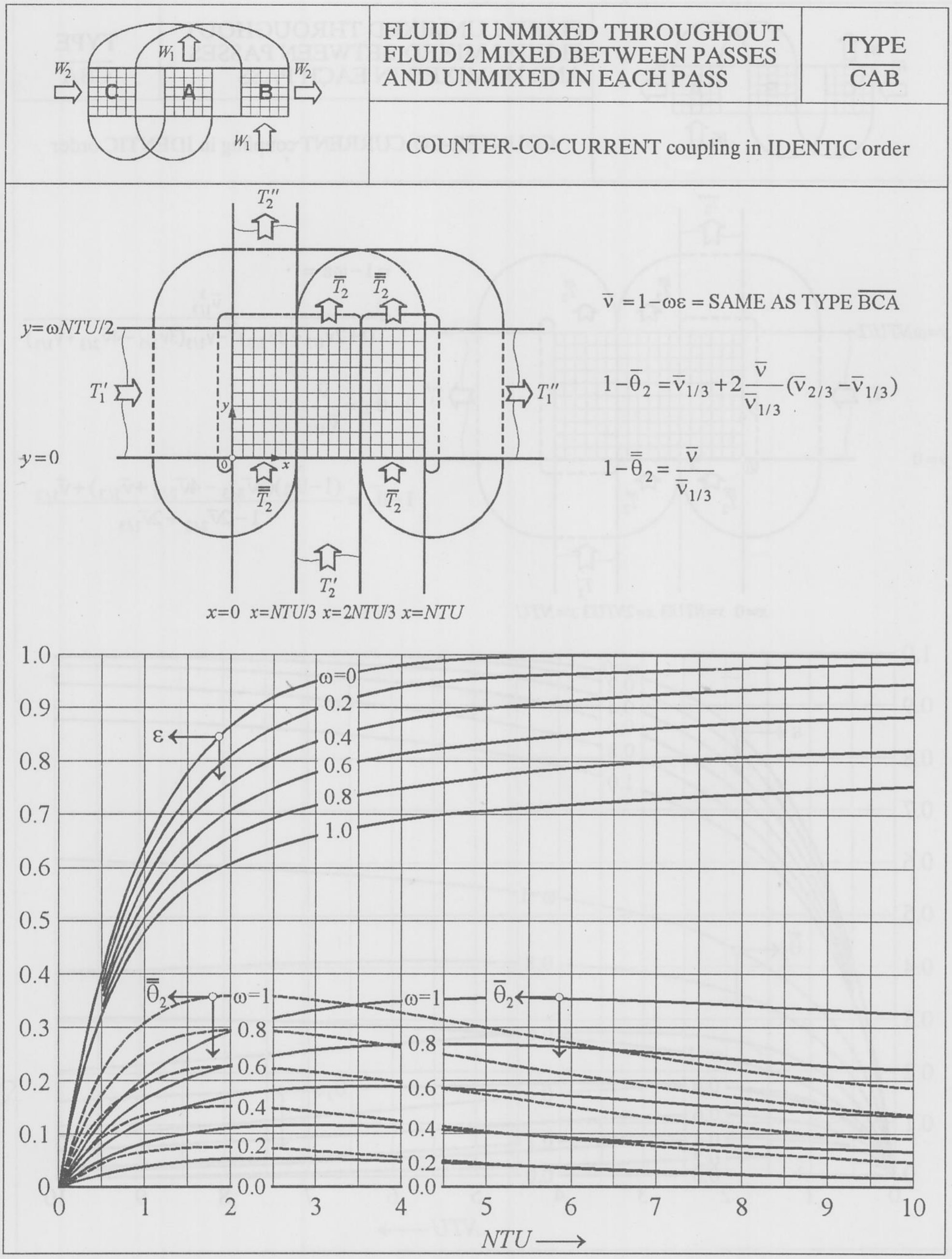
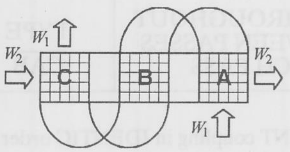
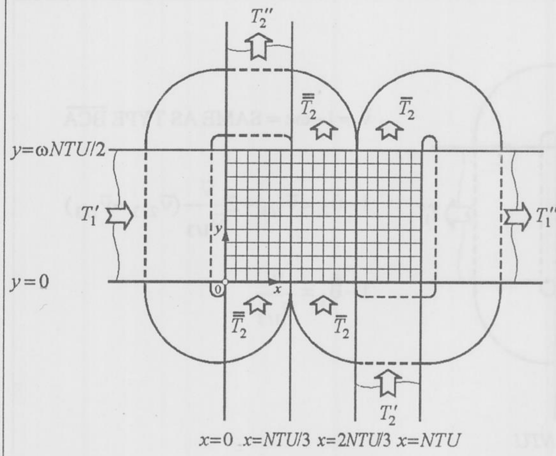


Figure 8. Performances of $\overline{\mathrm{CAB}}$ type exchanger



FLUID 1 UNMIXED THROUGHOUT FLUID 2 MIXED BETWEEN PASSES AND UNMIXED IN EACH PASS	TYPE $\overline{\text{CBA}}$
COUNTER-CO-CURRENT coupling in IDENTIC order	



$$\bar{v} = 1-\omega\varepsilon = \frac{\bar{v}_{1/3}^{3}}{(1+2\bar{v}_{1/3}-2\bar{v}_{2/3})^{2}-\bar{v}_{1/3}(3\bar{v}_{3/3}-4\bar{v}_{2/3}+\bar{v}_{1/3})}$$

$$1-\bar{\bar{\theta}}_2 = \frac{\bar{v}}{\bar{v}_{1/3}}$$

$$1-\bar{\theta}_2 = \frac{(1-\bar{\bar{\theta}}_2)(3\bar{v}_{3/3}-4\bar{v}_{2/3}+\bar{v}_{1/3})+\bar{v}_{1/3}}{1-2\bar{v}_{2/3}+2\bar{v}_{1/3}}$$

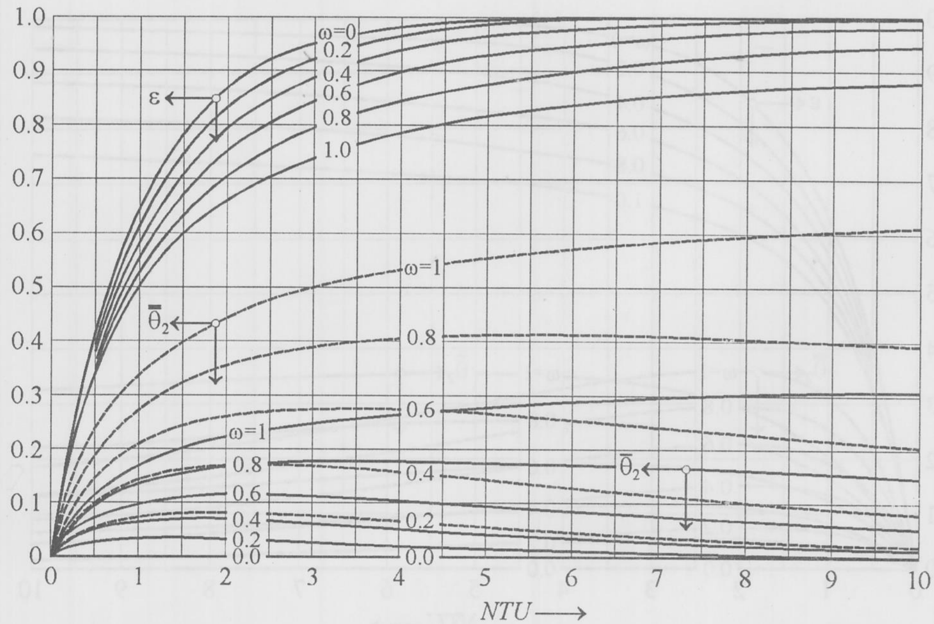


Figure 9. Performances of $\overline{\text{CBA}}$ type exchanger

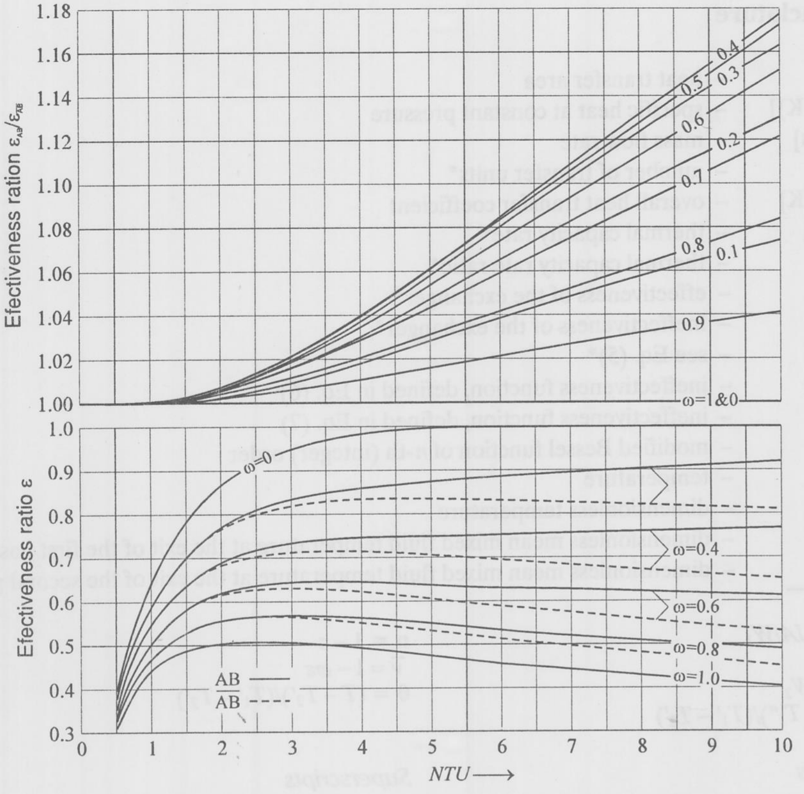


Figure 10. Comparison of AB & $\overline{AB}$ effectiveness

It is interesting to compare ε-*NTU*-ω curves of transposed and untransposed flow arrangements of the same type for $0 < \omega < 1$. Fig. 10 gives such a comparison for AB and $\overline{AB}$ exchanger. A pronounced deviation (indicating that the preference should not be given to the transposed arrangement) is evident.

By superimposing the corresponding ε-*NTU*-ω charts for ABC & $\overline{ABC}$, ACB & $\overline{ACB}$, BAC & $\overline{BAC}$, BCA & $\overline{BCA}$ and CAB & $\overline{CAB}$ types of arrangements one finds the similar situation, viz., the effectiveness of transposed arrangement is always lower for given *NTU* and ω. The exceptions are pure counter-current configurations (BA & $\overline{BA}$ and CBA & $\overline{CBA}$). The ε-*NTU*-ω curves for these arrangements practically coincide regardless W_1 or W_2 is unmixed throughout.

Conclusion

The identical order two- and three-pass crossflow arrangements with weaker fluid unmixed throighout involving co-curent coupling of passes should be avoided in practice.

Nomenclature

A [m^2] – heat transfer area
c_p [J/(kg K)] – specific heat at constant pressure
$\dot{M}$ [kg/s] – mass flow rate
NTU – number of transfer units*
U [W/m^{2}K] – overall heat transfer coefficient
W [W/K] – thermal capacity rate*
ω – thermal capacity rate ratio*
ε – effectiveness of the exchanger*
ν – ineffectiveness of the exchanger*
$\bar{\nu}$ – see Eq. (5)*
$\nu_{\alpha/\beta}$ – ineffectiveness function, defined in Eq. (6)
$\bar{\nu}_{\alpha/\beta}$ – ineffectiveness function, defined in Eq. (7)
$I_n(\cdot)$ – modified Bessel function of n-th (integer) order
T [K] – temperature
θ – dimensionless temperature
$\bar{\theta}$ – dimensionless mean mixed fluid temperature at the exit of the first pass
$\bar{\bar{\theta}}$ – dimensionless mean mixed fluid temperature at the exit of the second pass

*

$NTU = UA/W_1$
$W = \dot{M}c_p$
$w = W_1/W_2$
$\varepsilon = (T_1' - T_1'')/(T_1' - T_2')$

$n = 1 - \varepsilon$
$\bar{\nu} = 1 - \omega\varepsilon$
$\theta = (T - T_2')/(T_1' - T_2')$

Subscripts

1 – refers to fluid with $(\dot{M}c_p)_{\min}$
2 – refers to fluid with $(\dot{M}c_p)_{\max}$

Superscripts

' – at exchanger inlet
'' – at exchanger outlet

References

[1] Bačlić, B. S., Gvozdenac, D. D., Exact Explicit Equations for some Two- and Three-Pass Cross-Flow Heat Exchangers Effectiveness, in *Heat Exchangers – Thermohydraulic Fundamentals and Design* (Eds. S. Kakac, A. E. Bergles and F. Mayinger), Hemisphere Publishing Corp., Washington, D.C., 1981

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