ε-NTU-ω RELATIONSHIPS FOR INVERTED ORDER FLOW ARRANGEMENTS OF TWO-PASS CROSSFLOW HEAT EXCHANGERS

by

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A review of the effectiveness (ε) – number of transfer units (NTU) – capacity rate ration (ω) relationships for eighteen possible inverted order flow arrangements of two-pass crossflow heat exchangers is presented. The exact closed form solutions to the arrangements with unmixed-unmixed passes are given for the first time. When one fluid is mixed between the passes, the exact explicit formulas for the mean mixed fluid temperatures int he interpasses are given as well. The new relations are particularly helpful for computer-aid-ε-NTU-ω charts for each arrangement are also provided to facilitate hand calcualtions.

Introduction

Two crossflow units can be coupled in a two-pass heat exchanger in either an overall co-current or in an overall counter-current scheme. When both fluids are mixed in the interpass, the resulting flow arrangement can be differentiated only by the flow arrangements in each pass (unmixed-unmixed, unmixed-mixed or mixed-mixed). However, if one fluid is unmixed throughout, the order in which the streams enter the next pass must be differentiated. The coupling is referred to an identical order if the stream leaving one pass enters the next pass from the same side as in previous pass. The coupling is considered in an inverted order if the stream leaving one pass enters the next pass from the side opposite the previous pass. Inverted order coupling is very convenient for manufacturing and installation of heat exchangers. As a result, it is the preferred order used in practice. It is the aim of this paper to present the performance of all possible combinations of two-pass crossflow heat exchangers with passes coupled in an inverted order.

The analytical solutions for the effectiveness of inverted order two-pass heat exchangers are given by Stevens *at al.*[1], but only for the limiting cases of one fluid unmixed throughout and the other fluid mixed throughout. For the flow arrangements involving both passes of the unmixed-unmixed type, numerical solutions are presented in [1]. No solution is reported for the cases when one pass is unmixed-unmixed and theother is mixed-unmixed. Stevens *et al.* [1] further stated that the cases of crossflow

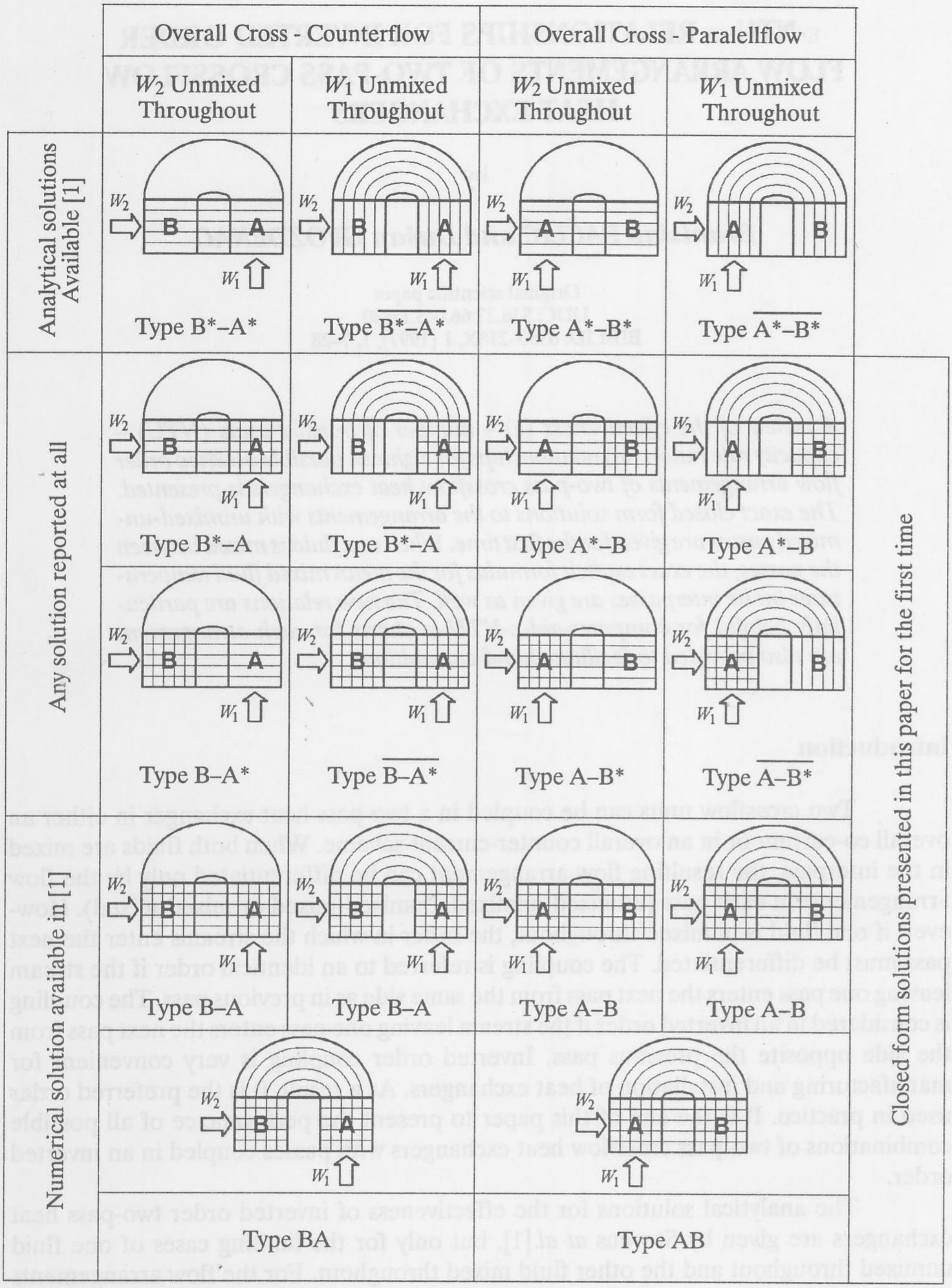


Figure 1. Complete list of two-pass crossflow heat exchangers coupled in inverted order

heat exchangers with unmixed-unmixed passes are "not susceptible to analytical solution" if one fluid is unmixed between passes. That is not the case. This paper presents the analyticla solutions for all the unsolved cases of two-pass crossflow heat exchangers with passes coupled in an inverted order. Explicit formulas for the effectiveness and the mean mixed fluid temperatures between passes are given as functions of *NTU* and ω.

List of possible two-pass inverted order arrangements

A complete list of eighteen different flow arrangements of two-pass crossflow heat exchangers coupled in an inverted order is presented in Fig. 1. In order to distinguish between the various combinations, we will follow the flow of the stream W_1 throughout the exchanger. The order in which fluid W_1 enters a pass is labeled by the alphabetic order of the letters A and B. A preceding B always denotes the overall scheme is co-current, while B preceding A denotes an overall counter-current arrangement. If fluid W_1 is mixed in a pass, the corresponding letter has a superscripted star. If W_1 between the passes is denoted by a dash between the letters. This notation is used in the first and the third column in Fig. 1.

Now, if one interchanges the flow patterns of W_1 and W_2 in the flow arrangements of the first and the third column, the resulting schemes would be those presented in the second and the fourth column, respectively. These arrangements we call the "transposed" configurations and will be denoted simply by placing a bar over the notation of the type of arrangement in which W_1 has been replaced by W_1 has been replaced by W_2.

Heat exchanger ineffectiveness and ineffectiveness functions

This section summarized the definition necessary for a concise and simplified presentation of the results.

The problem of establishing the ε-*NTU*-ω relationship is in fact that of evaluating the mean exit temperatures of the unmixed flow. To reach this goal one needs the solution to the differential equations that govern the crossflow heat transfer. Making the usual idealizations for analysis of any heat exchanger flow arrangement [2], one is led to the mathematical formulation of the problem of determining the spatial temperature distributions in the crossflow heat exchanger core. By means of Laplace transform, the solution to the problems is obtainable for any particular flow arrangement by taking into account the corresponding boundary and coupling conditions. The explicit formula for the heat exchanger effectiveness is then obtained by simple integration, *i. e.* by averaging the exit temperature distribution over the flow cross-sectional area.

In a two-fluid heat exchanger, one of the streams will usually undergo a greater temperature change than the other. The first stream is said to be the "weak" stream having a lower thermal capacity rate $(\dot{M}c_p)_{\min} = W_1$, while the other, with $(\dot{M}c_p)_{\max} = W_1$, is the "strong" stream. The heat exchanger effectiveness is defined as the ratio of the overall temperature drop of the weaker stream to the maximum possible temperature difference, *i. e.* the difference between the fluid inlet temperatures [3]. For a unified treatment of various flow arrangements it is useful to adopt the dimensionless temperatures θ_i ($i = 1, 2$) in the mathematical model in such a way that the weaker fluid enters the exchanger at $\theta_1' = 1$, and the stronger fluid enters at $\theta_2' = 0$. For this case, the exit temperatures are related to the heat exchanger effectiveness as

$$\theta_1'' = 1 - \varepsilon \tag{1}$$

$$\theta_2'' = \omega\varepsilon \tag{2}$$

It can be seen that the dimensionless mean exit temperature of the weaker stream (W_1) is equal to the complementary value of the effectiveness and thus may be termed as the heat exchanger ineffectiveness

$$\nu = 1 - \varepsilon = \theta_1'' \tag{3}$$

Since any calculation involving the exit temperatures is in fact the calculation of the ineffectiveness, it is useful to introduce the complementary value of the dimensionless mean exit temperature of the stronger stream as well, *i. e.*,

$$1 - \theta_2'' = 1 - \omega\varepsilon \tag{4}$$

It can be shown that

$$\bar{\nu} = 1 - \omega\varepsilon = 1 - \omega(1 - \nu) \tag{5}$$

expresses the final results for the effectiveness of the transposed flow arrangements as does ν for the untransposed arrangements.

Let us consider two examples of single pass crossflow heat exchangers. The effectiveness of the single pass exchanger with weaker fluid (W_1) mixed and W_2 unmixed is [3]

$$\varepsilon = 1 - e^{-\frac{1 - e^{-\omega NTU}}{\omega}} \tag{6}$$

i. e. the ineffectiveness is

$$\nu = 1 - \varepsilon = e^{-\frac{1 - e^{\omega NTU}}{\omega}} \tag{7}$$

If W_1 and W_2 are interchanged in such an arrangement (W_1 becomes unmixed and W_2 mixed) which means a transposition of the arrangement as discussed in the previous section, the effectiveness is [3]:

$$\varepsilon = \frac{1 - e^{-\omega(1 - e^{-NTU})}}{\omega} \tag{8}$$

This result can be written in a from containing the same functional relationship as Eq. (7) for the untransposed arrangement ineffectiveness:

$$\bar{\nu} = 1 - \omega\varepsilon = 1 - e^{-\omega(1-e^{-NTU})} \tag{9}$$

It may be shown that the results of Eqs. (7) and (9) are just a special case of two ineffectiveness functions arising in the problems of multipass crossflow heat exchangers with passes of the mixed-unmixed type. Let us define the ineffectiveness function

$$\nu^*_{\alpha/\beta} = \nu^*(a,b) = e^{-\frac{a}{b}(1-e^{-b})} \tag{10}$$

where the arguments are of the form

$$a = \frac{NTU}{\beta} \tag{11}$$

$$b = \frac{\alpha\omega NTU}{\beta} \tag{12}$$

Integers α and β appearing in Eqs. (11) and (12) indicate that $\nu_{\alpha/\beta}$ may be seen as the ineffectiveness of a single pass exchanger whose number of transfer units is equal to one β-th part of total *NTU* of multipass exchanger and whose capacity rate ratio is α times greater that the corresponding ω of multipass exchanger.

Note that when we write ν without fractional subscripts as well as without any argument, namely as it is given by Eq. (3), it refers to the entire heat exchanger arrangement. However, when we write $\nu_{\alpha/\beta}$ it should be regarded as a function defined by Eqs. (10), (15) or (18) and (19). Note also that a star superscript indicates the mixing of W1 in the pass, while a bar over ν and $\nu_{\alpha/\beta}$ means the transposition.

Using Eq. (10), Eq. (7) may be written concisely as:

$$\nu = \nu^*_{1/1} = \nu^*(NTU, \omega NTU) \tag{13}$$

For two-pass exchangers, an ineffectiveness function of the following form is useful.

$$\nu^*_{1/2} = \nu^*\left(\frac{NTU}{2}, \frac{\omega NTU}{2}\right) = e^{-\frac{1-e^{-\frac{\omega NTU}{2}}}{\omega}} \tag{14}$$

The definition of the ineffectiveness function for the transposed arrangement (with W_2 mixed in the pass) is

$$\bar{\nu}^*_{\alpha/\beta} = \bar{\nu}^*(a,b) = e^{-\frac{b}{a}(1-e^{-a})} \tag{15}$$

Where *a* and *b* given by Eqs. (11) and (12). Note that the arguments have interchanged positions in Eq. (15) when compared with Eq. (10). Using Eq. (15), the ineffectiveness of the single-pass exchanger with W_2 mixed, Eq. (9), may be written as

$$\bar{\nu} = \bar{\nu}^*_{1/1} = \bar{\nu}^*(NTU, \omega NTU) \tag{16}$$

In the case of two-pass crossflow exchangers involving passes with W_2 mixed one needs the function

$$\bar{\nu}^*_{1/2} = \bar{\nu}^*\left(\frac{NTU}{2}, \frac{\omega NTU}{2}\right) = e^{-\omega\left(1-e^{-\frac{NTU}{2}}\right)} \tag{17}$$

Any arrangement having at least one pass with one fluid mixed and the other unmixed involves the functions of Eq. (14) or (17) in the final results for its effectiveness. As seen from Fig. 1, there are twelve cases involving at least one pass with one fluid mixed and the other unmixed.

The presence of the unmixed-unmixed passes in a multipass crossflow heat exchangers requires an ineffectiveness function of the form:

$$\nu_{\alpha/\beta} = \nu(a,b) = e^{-(a+b)}\left[I_0(2\sqrt{ab}) + \sqrt{\frac{b}{a}}I_1(2\sqrt{ab}) - \left(\frac{a}{b}-1\right)\sum_{n=2}^{\infty}\left(\frac{b}{a}\right)^{\frac{n}{2}} I_n(2\sqrt{ab})\right] =$$

$$= \nu\left(\frac{NTU}{\beta}, \frac{\alpha\omega NTU}{\beta}\right) = e^{-(1+\alpha\omega)\frac{NTU}{\beta}}\left[I_0\left(\frac{2NTU}{\beta}\sqrt{\alpha\omega}\right) + \sqrt{\alpha\omega}\, I_1\left(\frac{2NTU}{\beta}\sqrt{\alpha\omega}\right) - \right.$$

$$\left. -\left(\frac{1}{\alpha\omega}-1\right)\sum_{n=2}^{\infty}(\alpha\omega)^{\frac{n}{2}} I_n\left(\frac{2NTU}{\beta}\sqrt{\alpha\omega}\right)\right] \tag{18}$$

and

$$\bar{\nu}_{\alpha/\beta} = \nu\left(\frac{\omega NTU}{\beta}, \frac{\alpha NTU}{\beta}\right) = e^{-\left(1+\frac{\alpha}{\omega}\right)\frac{\omega NTU}{\beta}}\left[I_0\left(\frac{2NTU}{\beta}\sqrt{\alpha\omega}\right) + \sqrt{\frac{\alpha}{\omega}}I_1\left(\frac{2NTU}{\beta}\sqrt{\alpha\omega}\right) - \right.$$

$$\left. -\left(\frac{\omega}{\alpha}-1\right)\sum_{n=2}^{\infty}\left(\frac{\alpha}{\omega}\right)^{\frac{n}{2}} I_n\left(\frac{2NTU}{\beta}\sqrt{\alpha\omega}\right)\right] \tag{19}$$

These functions arise from averaging the exit temperatures of the unmixed flows. For example, $\nu_{\alpha/\beta}$ results from

$$\nu_{\alpha/\beta} = \frac{1}{\frac{\alpha\omega NTU}{\beta}} \int_0^{\frac{\alpha\omega NTU}{\beta}} \theta_1\left(\frac{NTU}{\beta}, y\right) dy \tag{18*}$$

where

$$\theta_1\left(\frac{NTU}{\beta}, y\right) = e^{-\frac{NTU}{\beta} - y} \sum_{n=0}^{\infty} \left(\frac{y}{NTU/\beta}\right)^{\frac{n}{2}} I_n\left(2\sqrt{y\frac{NTU}{\beta}}\right) \tag{18**}$$

is the temperature distribution of the weaker fluid at the exit edge ($x = NTU/\beta$) of the α-th pass core of a β-pass exchanger. Similarly, $\nu_{\alpha/\beta}$ results from averaging the exit temperature of the stronger fluid in the transposed flow arrangements.

The ineffectiveness of a single pass unmixed-unmixed heat exchanger may be interpreted as

$$\nu = 1 - \varepsilon = \nu_{1/1} = \nu(NTU, \omega NTU) \tag{20}$$

which coincides with the result presented in [4].

The effectiveness of all possible combinations of 263-pass identical order flow arrangements with W_2 unmixed throughout are just algebraic combinations of the functions $\nu_{1/2}$, $\nu_{2/2}$, $\nu_{1/3}$, $\nu_{2/3}$ and $\nu_{3/3}$ as is shown in [5]. The same relations hold, but in terms of $\bar{\nu}_{1/2}, \bar{\nu}_{2/2}, \bar{\nu}_{1/3}, \bar{\nu}_{2/3}$ and $\bar{\nu}_{3/3}$ [6] when the arrangements from [5] are transposed (W_1 unmixed throughout).

For the two-pass arrangements considered in the present paper, one needs only two functions of the type expressed in Eqs. (18) and (19):

$$\nu_{1/2} = \nu\left(\frac{NTU}{2}, \frac{\omega NTU}{2}\right) \tag{21}$$

$$\bar{\nu}_{1/2} = \bar{\nu}\left(\frac{\omega NTU}{2}, \frac{NTU}{2}\right) \tag{22}$$

It should be noted that the following relation holds in this particular case:

$$\bar{\nu}_{1/2} = 1 - \omega(1 - \nu_{1/2}) \tag{23}$$

The functions $\bar{\nu}^*_{1/2}, \bar{\nu}^*_{1/2}, \nu_{1/2}$ and $\bar{\nu}_{1/2}$, defined by Eqs. (14), (17), (21) and (23) respectively, form a set that is fundamental for simplified presentation of the results for all flow arrangements under consideration. Some other functions arising from the peculiarities of coupling the passes are presented in the next section.

Special functions arising from coupling in an inverted order

Heat exchangers with flow arrangements involving one mixed-unmixed and one unmixed-unmixed pass coupled in an inverted order like those in the second and the third row of Fig. 1, require evaluation of the function

$$g(\xi,\eta)=1+\sum_{j=2}^{\infty}\left(\frac{1-e^{-\xi}}{\xi}\right)^{j-1}V_j(\xi,\eta) \tag{24}$$

for determining the effectiveness. This function takes two forms that are of interest for two-pass heat exchangers with equal passes. By equal passes we mean heat exchangers in which the number of transfer units in each pass is equal to *NTU*/2. Two forms of function of Eq. (24) are to be distinguished by interchanging the arguments for the transposed arrangements as follows:

$$g=g\left(\frac{NTU}{2},\frac{\omega NTU}{2}\right)=1+\sum_{j=2}^{\infty}\left(\frac{1-e^{-\frac{NTU}{2}}}{\frac{NTU}{2}}\right)^{j-1}V_j\left(\frac{NTU}{2},\frac{\omega NTU}{2}\right) \tag{25}$$

$$\overline{g}=g\left(\frac{\omega NTU}{2},\frac{NTU}{2}\right)=1+\sum_{j=2}^{\infty}\left(\frac{1-e^{-\frac{\omega NTU}{2}}}{\frac{\omega NTU}{2}}\right)^{j-1}V_j\left(\frac{\omega NTU}{2},\frac{NTU}{2}\right) \tag{26}$$

The flow arrangements from the fourth row of Fig. 1, viz., those with both passes unmixed-unmixed, require another class of functions for the exact explicit relations for the effectiveness. Since one has again to distinguish the cases with W_1 unmixed throughout from those with W_2 unmixed throughout, one has to define two functions whose form differ by interchanged arguments of *NTU*/2 and ω*NTU*/2:

$$\mu_{1/2}=\frac{1}{\frac{\omega NTU}{2}}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^m(n+m)!}{n!m!}V_{m+2}\left(\frac{\omega NTU}{2},\frac{NTU}{2}\right)V_{n+2}\left(\frac{NTU}{2},\frac{\omega NTU}{2}\right) \tag{27}$$

$$\overline{\mu}_{1/2}=\frac{1}{\frac{NTU}{2}}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^m(n+m)!}{n!m!}V_{m+2}\left(\frac{NTU}{2},\frac{\omega NTU}{2}\right)V_{n+2}\left(\frac{\omega NTU}{1},\frac{NTU}{2}\right) \tag{28}$$

These two functions are sufficient, for example, for a concise notation of the final result for effectiveness of the AB type exchanger (both fluids unmixed throughout, both coupled in an inverted order in overall cross-parallel flow):

$$\varepsilon=\mu_{1/2}+\overline{\mu}_{1/2}\frac{1}{\omega} \tag{29}$$

The $V_j(\varepsilon, \eta)$, (j = 2, 3,...) functions appearing in Eqs. (24–28) are of the form:

$$V_j(\xi,\eta) = \underbrace{\int_0^n \cdots \int_0^n}_{j-1} \left[\mathrm{e}^{-(\xi+\eta)} \sum_{n=0}^{\infty} \left(\frac{\eta}{\xi}\right)^{\frac{n}{2}} I_n(2\sqrt{\xi\eta})\,(d\eta) \right]^{j-1} \tag{30}$$

and can be expressed explicitly in the form of series of modified Bessel functions as described below.

Let

$$V_0(\xi,\eta) = \mathrm{e}^{-(\xi+\eta)} I_0(2\sqrt{\xi\eta}) \tag{31}$$

$$V_1(\xi,\eta) = \mathrm{e}^{-(\xi+\eta)} \sum_{n+0}^{\infty} \left(\frac{n}{\xi}\right)^{\frac{n}{2}} I_n(2\sqrt{\xi\eta}) \tag{32}$$

and

$$V_2(\xi, \eta) = \eta\nu(\xi, \eta) \tag{33}$$

where $\nu(\xi, \eta)$ is defined by Eq. (18). Then, for j = 2, 3,..., it can be proved that the following recursion formula holds:

$$V_{j+1}(\xi,\eta) = \frac{1}{j}\left\{ \eta[V_j(\xi,\eta) + V_{j-1}(\xi,\eta)] - (j-1)V_j(\xi,\eta) - \xi \sum_{i=0}^{j} (-1)^{i+j} V_i(\xi,\eta) \right\} \tag{34}$$

together with the property

$$V_0(\xi,\eta) = \sum_{j=1}^{\infty} (-1)^{i+j} V_j(\xi,\eta) \tag{35}$$

The notation V_j is adopted from [7].

The functions presented under this and the previous section are sufficient for a simplified presentation of the results for the eighteen crossflow configurations under consideration.

Results

Eighteen possible two-pass inverted order flow arrangements are presented in Figs. 2 through 19 with a schematic in the upper-left corners. The same figures contain a sketch in a coordinate system *0xy* as well.

These are given to provide the necessary notation as well as to demonstrate the differences in various couplings in an inverted. For mathematical description, an inverted order means he corresponding axes are directed opposite in the second pass for the stream that is inverted. Note that it is assumed that the weak fluid W_1 always flow in the x-direction and W_2 and in y-direction.

Each of Figs. 2–19 constrains the explicit formulas for the effectiveness and the mean temperatures in the interpasses of the corresponding type of exchanger. It may be seen that all of these formulas are very simple when the functions described in two preceding sections are used. Although these functions appear complicated, they are rapidly convergent and easily programmable even on a desk-top computer or a programmable pocket calculator.

Also note that the formulas for the ineffectiveness of the transposed arrangements are exactly the same as the formulas corresponding to the untransposed type of exchanger, but they are in the terms of the functions having a bar (the arguments are interchanged). The same statement applies for the mean mixed temperatures in the interpasses θ_1 and $1 - \theta_2$.

On the basis of these results, the corresponding ε-*NTU* charts are given in each figure. These might help designers with hand calculations, particularly in the search for a more effective configuration of a two-pass exchanger. This is possible because the order in which Figs. 2 through 19 are arranged is not the same as in the list in Fig. 1, but rather in an order that gives each subsequent configuration better performance than the previous. A better exchanger is the one that has higher ε for given *NTU* and ω. When two arrangements have the same effectiveness, the preference is given according to the $(1-\bar{\theta}) - NTU$ and $\bar{\theta}_1 - NTU$ curves that can be regarded as a measure of the effectiveness of the first pass.

The analysis of the figures indicate that any of the inverted order co-current crossflow arrangements should be avoided in practice due to the shapes of the ε-*NTU* curves at the higher values of *NTU*. Further analysis is left to the reader since the figures are self explanatory and because the primary aim of this paper is to present the exact explicit formulas.

Using the exact solutions, a check was made of the correction factors given by Stevens *et al.* [1] for the corresponding flow arrangements. The agreement was found to be within the tolerances acceptable in drawing the curves. The polynomial representations of ε-*NTU* relations proposed in [8] have also been checked. A maximum relative error in those approximations was found to be within 1% for $B^* - A^*$, $\overline{B^* - A^*}$ and $\overline{A^* - B^*}$ type exchangers for the range of parameters presented in the figures of the present paper. However, the relative error $(\varepsilon_{appr}/\varepsilon_{exact} - 1) \times 100$ for A*–B* exchanger varies from –2% up to –7.19% for $\omega > 0$ and $NTU > 6$. It should be noted that the results for any two-pass exchangers with unmixed-unmixed passes were obtained by [8]. However, having the exact analytical solutions given here, a procedure for establishing approximate ε-*NTU* relations as proposed in [8] can be performed for all configurations listed in Fig. 1.

The flow arrangement BA (both fluids unmixed throughout and both coupled in inverted order in an overall cross-counter flow) is the only arrangement that does not have an exact closed form formula for effectiveness. At present we are not able to provide even an approximate reliable analytical solution. For that reason, the results presented in Fig. 17 are based on the numerical solution that was obtained by a method similar to that used by Stevens *et al.* [1].

The main difficulty in establishing the exact analytical solution for the BA type exchanger may be summarized as follows.

The effectiveness is to be found from

$$\varepsilon = 1 - \frac{1}{\frac{\omega NTU}{2}} \int_0^{\frac{\omega NTU}{2}} f(y) e^{-\left(\frac{NTU}{2} + \frac{\omega NTU}{2} - y\right)} \sum_{n=0}^{\infty} \left(\frac{\frac{\omega NTU}{2} - y}{\frac{NTU}{2}} \right)^{\frac{n}{2}} \cdot I_n \left[2\sqrt{\frac{NTU}{2}\left(\frac{\omega NTU}{2} - y\right)} \right] dy \tag{36}$$

where $f(y)$ can be fond from

$$f(y) = e^{-\left(\frac{NTU}{2} + \frac{\omega NTU}{2} - y\right)} \sum_{n=0}^{\infty} \left(\frac{\frac{\omega NTU}{2} - y}{\frac{NTU}{2}} \right)^{\frac{n}{2}} I_n \left[2\sqrt{\frac{NTU}{2}\left(\frac{\omega NTU}{2} - y\right)} \right] + \int_0^{\frac{NTU}{2}} \int_0^{\frac{\omega NTU}{2}} f(\tau) e^{-\left(x + \frac{\omega NTU}{2} - y\right)} \cdot I_0 \left[2\sqrt{x\left(\frac{\omega NTU}{2} - y\right)} \right] e^{-\left(x + \frac{\omega NTU}{2} - \tau\right)} I_0 \left[2\sqrt{x\left(\frac{\omega NTU}{2} - \tau\right)} \right] d\tau dx \tag{37}$$

expressing the condition of coupling both unmixed streams in an inverted order. Several elaborate methods for solving integral equations were tried on this problem, but none provided a solution that was simple for practical purposes.

Concluding remarks

The exact explicit formulas for the effectiveness and interpass temperatures for two-pass crossflow heat exchangers coupled in inverted order are presented in this paper. These will be useful to designers for computer-based design producers. Examination of the results reveals that the order in which a more effective flow arrangement is to be chosen is the sequence of Figs. 2–19.

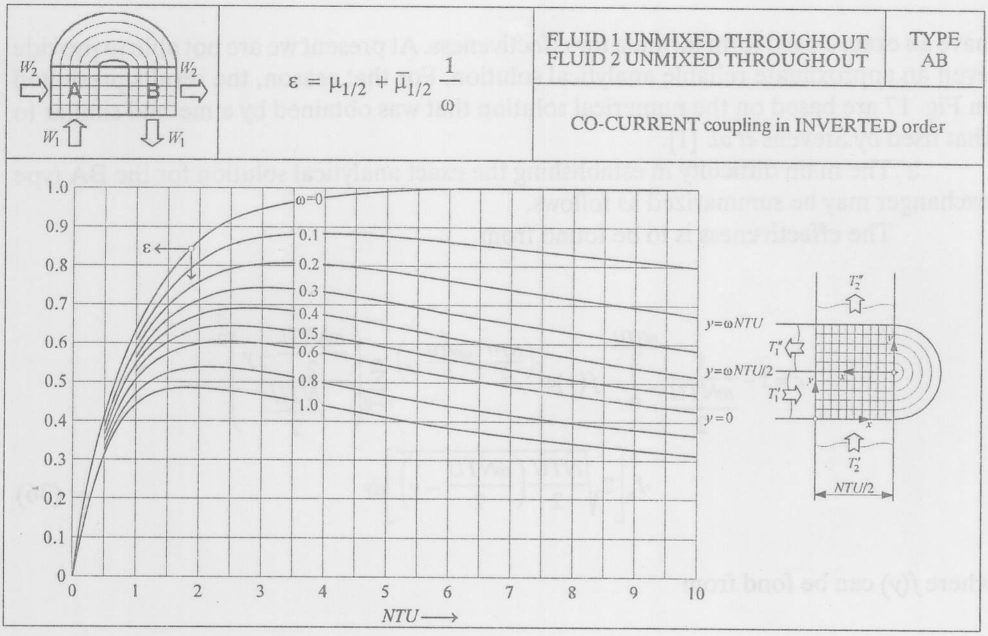


Figure 2. ε-*NTU*-ω relationship for AB type exchanger

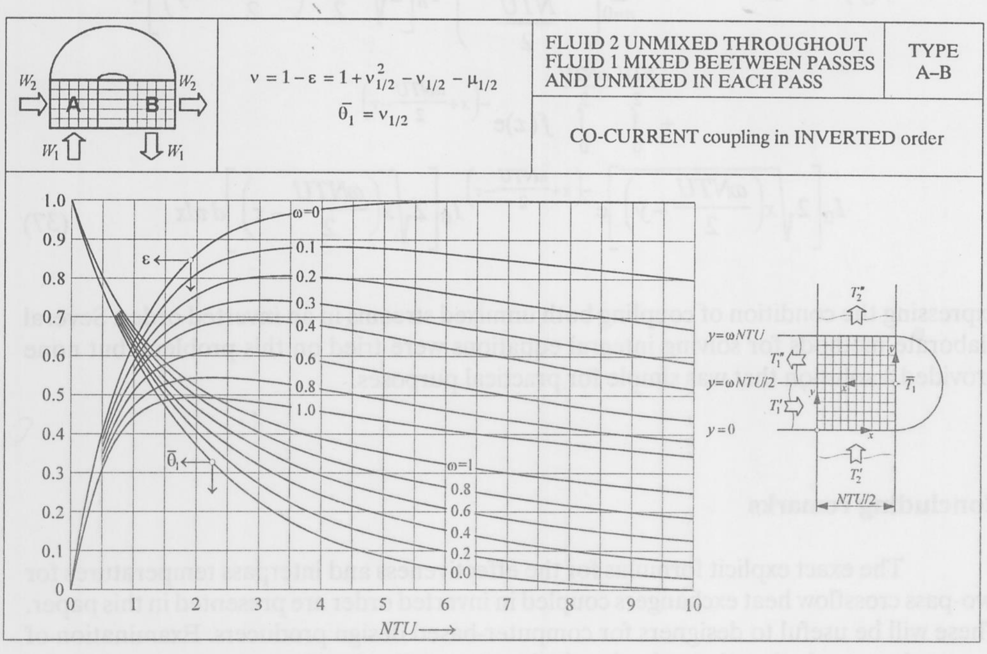


Figure 3. ε-*NTU*-ω and $\bar{\theta}_1$-*NTU*-ω relationships for A–B type exchanger

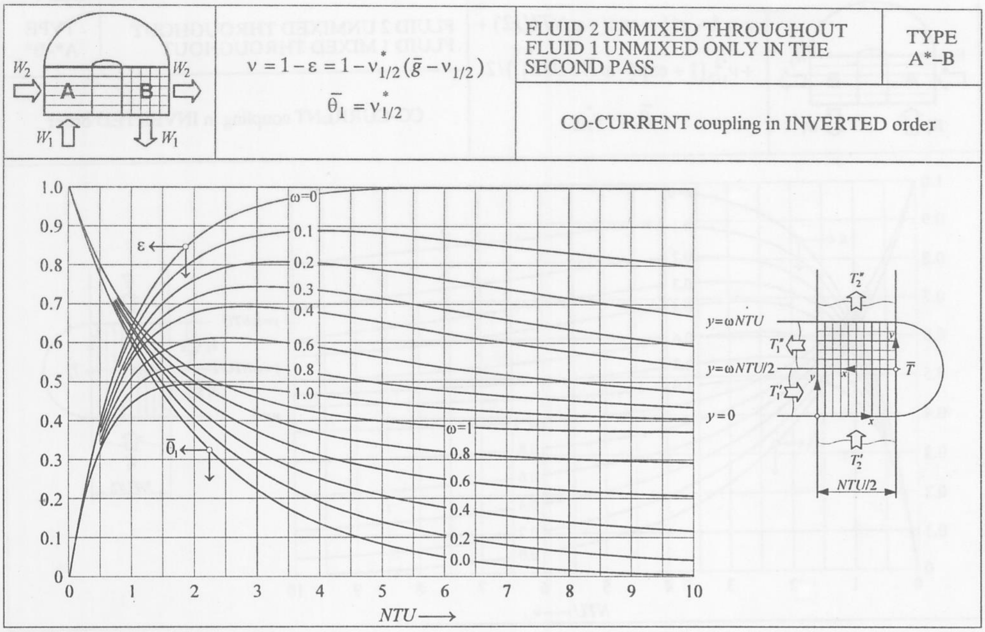


Figure 4. ε-*NTU*-ω and $\bar{\theta}_1$-*NTU*-ω relationships for A*–B type exchanger

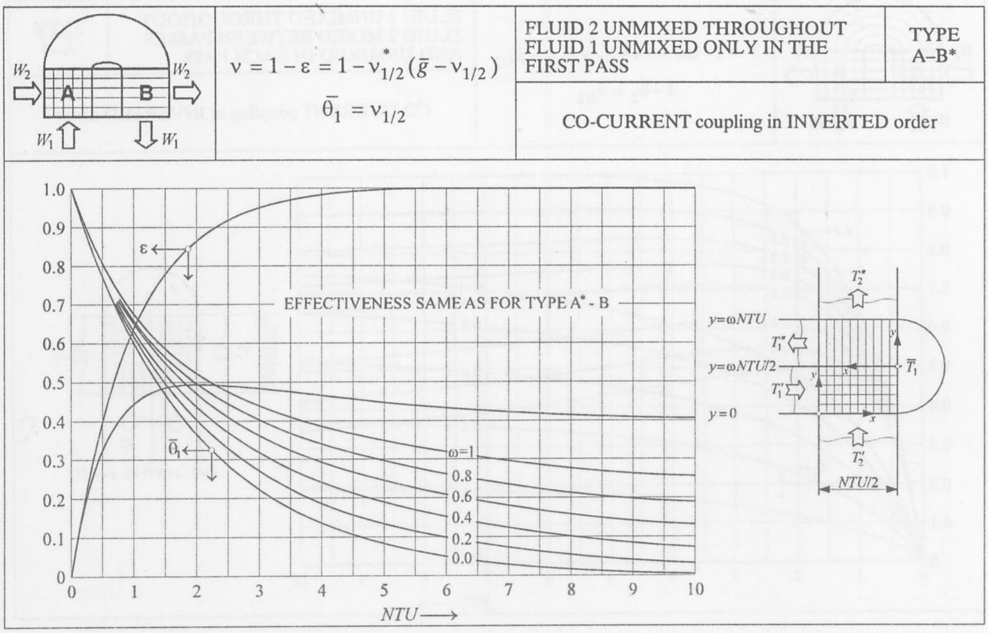


Figure 5. ε-*NTU*-ω and $\bar{\theta}_1$-*NTU*-ω relationships for A–B* type exchanger

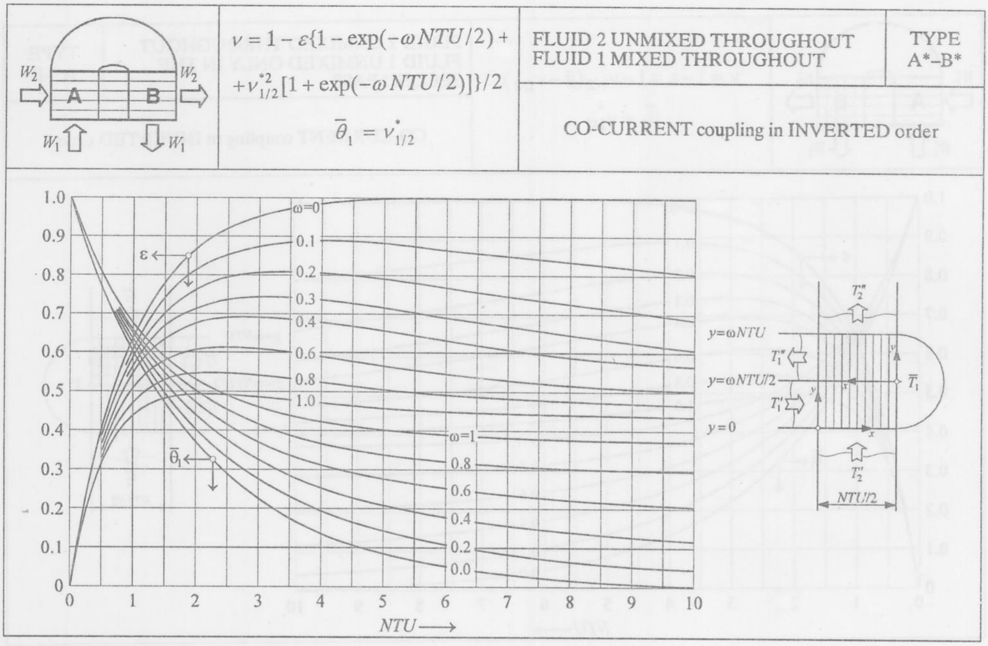


Figure 6. ε-*NTU*-ω and $\overline{\theta}_1$-*NTU*-ω relationships for A*–B* type exchanger

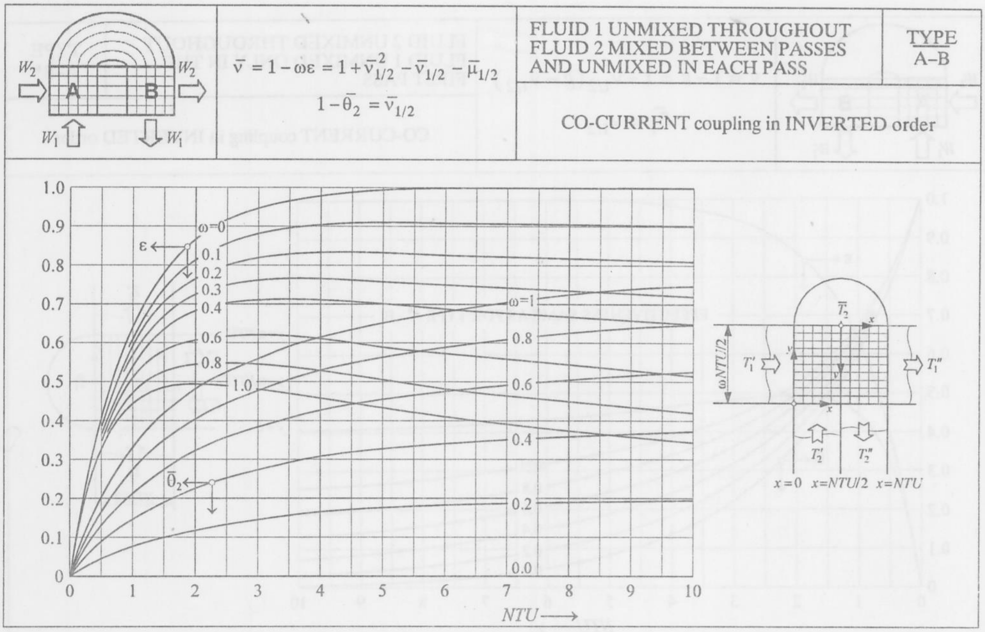


Figure 7. ε-*NTU*-ω and $\overline{\theta}_2$-*NTU*-ω relationships for $\overline{\text{A–B}}$ type exchanger

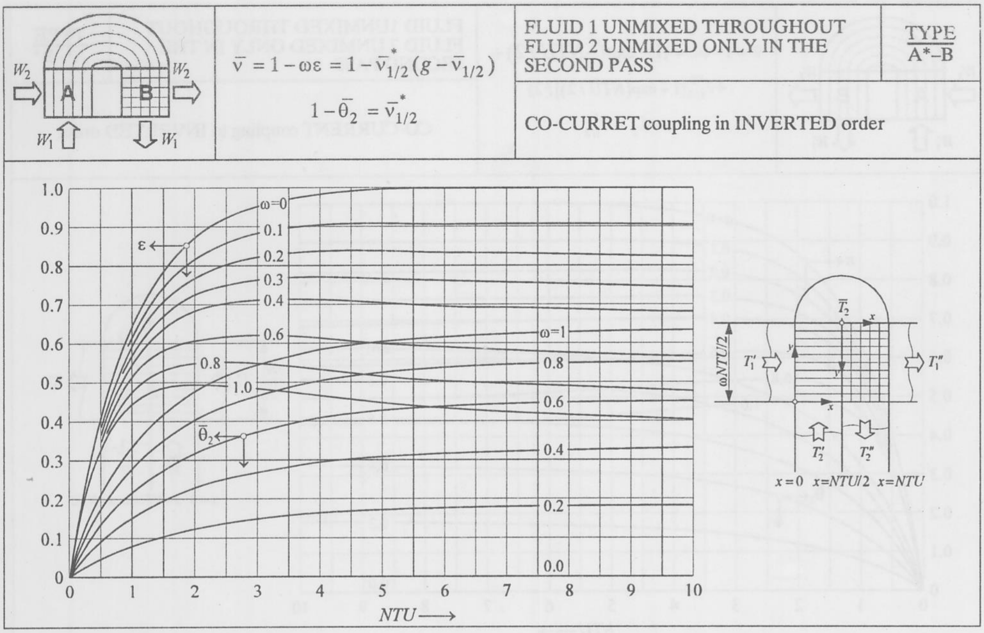


Figure 8. ε-*NTU*-ω and $\bar{\theta}_2$-*NTU*-ω relationships for $\overline{A^*–B}$ type exchanger

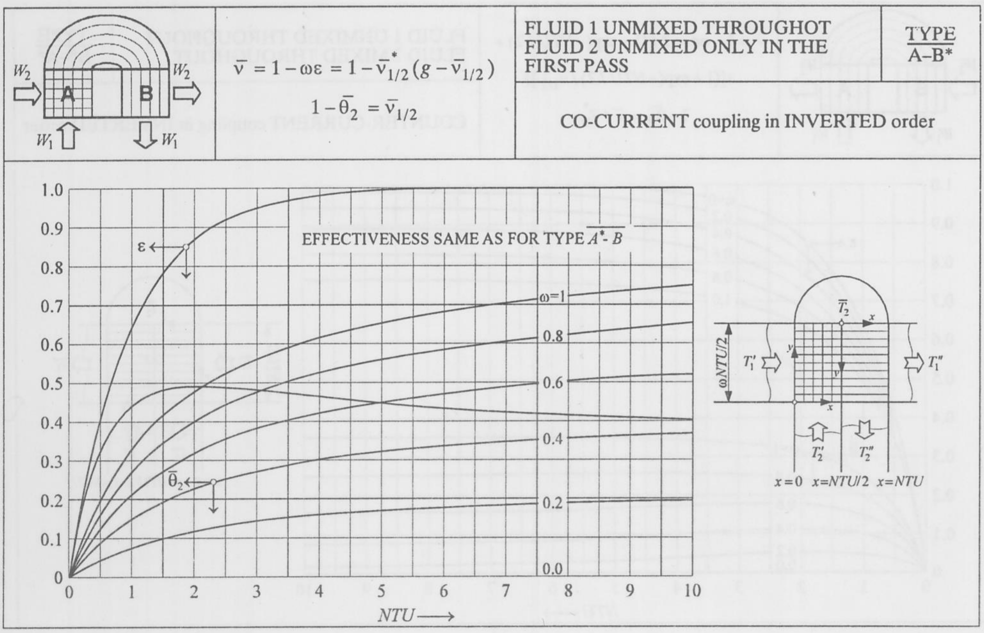
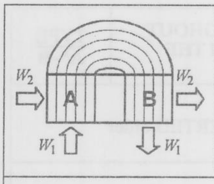


Figure 9. ε-*NTU*-ω and $\bar{\theta}_2$-*NTU*-ω relationships for $\overline{A–B^*}$ type exchanger



$$\bar{v} = 1-\omega\varepsilon = \{1-\exp(-NTU/2) + \bar{v}^{*2}_{1/2}[1+\exp(NTU/2)]/2\}$$

$$1-\bar{\theta}_2 = \bar{v}^{*}_{1/2}$$

FLUID 1UNMIXED THROUGHOUT
FLUID 2 UNMIXED ONLY IN THE SECOND PASS

TYPE $\overline{A^*\text{–}B^*}$

CO-CURRENT coupling in INVERTED order

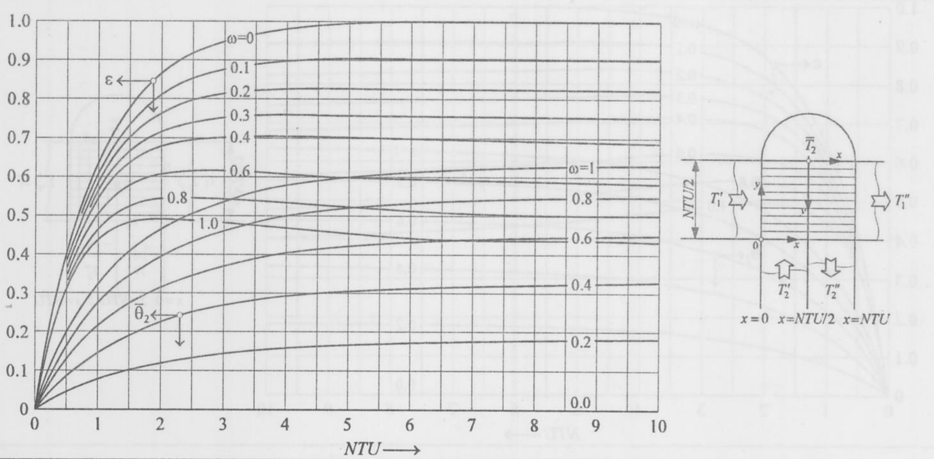
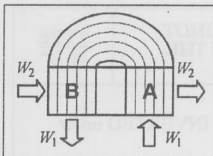


Figure 10. ε-*NTU*-ω and $\bar{\theta}_2$-*NTU*-ω relationships for $\overline{A^*\text{–}B^*}$ type exchanger



$$\bar{v} = 1-\omega\varepsilon = 2/\{1-\exp(-NTU/2) + [(1+\exp(-NTU/2)/\bar{v}^{*2}_{1/2}]\}$$

$$1-\bar{\theta}_2 = \bar{v}/\bar{v}^{*}_{1/2}$$

FLUID 1 UNMIXED THROUGHOUT
FLUID 2 MIXED THROUGHOUT

TYPE $\overline{B^*\text{–}A^*}$

COUNTER-CURRENT coupling in INVERTED order

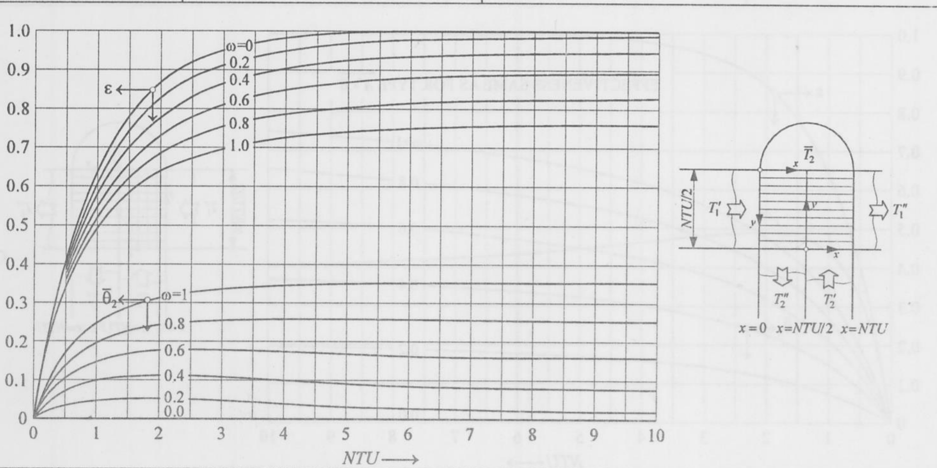
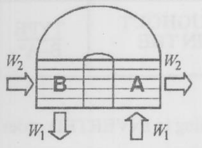


Figure 11. ε-*NTU*-ω and $\bar{\theta}_2$-*NTU*-ω relationships for $\overline{B^*\text{–}A^*}$ type exchanger

	$\nu = 1-\varepsilon = 2/\{1-\exp(-\omega NTU/2) + [(1+\exp(-\omega NTU/2))/\nu_{1/2}^{*2}]\}$ $\bar{\theta}_1 = \nu/\nu_{1/2}^{*}$	FLUID 2 UNMIXED THROUGHOUT FLUID 1 MIXED THROUGHOUT COUNTER-CURRENT coupling in INVERTED order	TYPE B*–A*

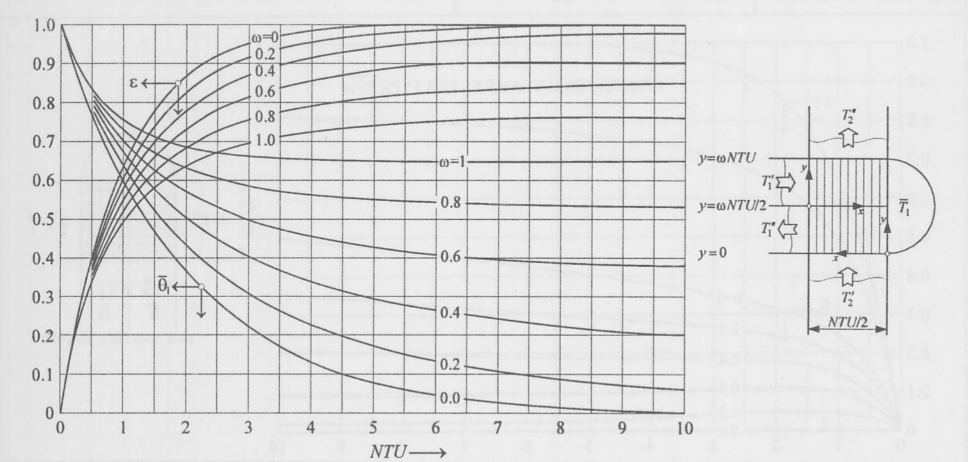
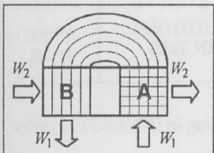


Figure 12. ε-*NTU*-ω **and** $\bar{\theta}_1$-*NTU*-ω **relationships for B*–A* type exchanger**

	$\nu = 1-\omega\varepsilon = \frac{\bar{\nu}_{1/2}}{g}$ $1-\bar{\theta}_2 = \frac{1}{g}$	FLUID 1 UNMIXED THROUGHOUT FLUID 2 UNMIXED ONLY IN THE SECOND PASS COUNTER-CURRENT coupling in inverted order	TYPE $\overline{\text{B*–A}}$

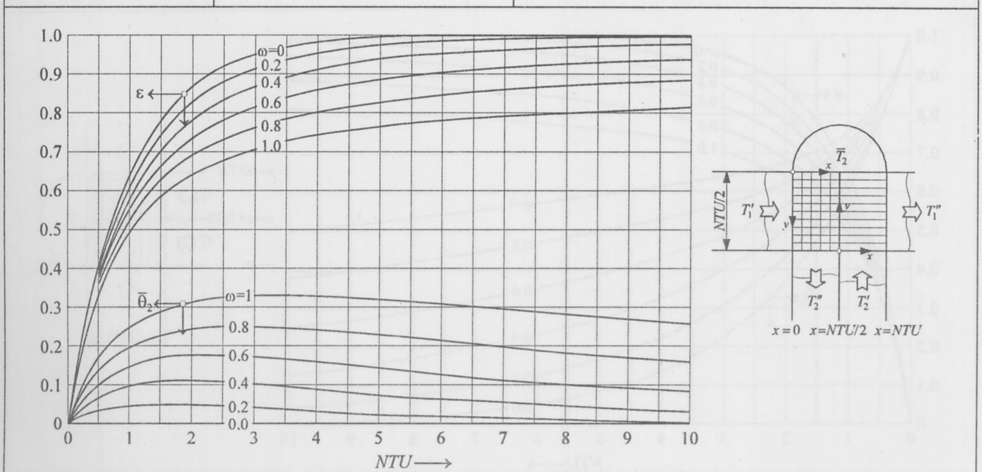


Figure 13. ε-*NTU*-ω **and** $\bar{\theta}_2$-*NTU*-ω **relationships for** $\overline{\text{B*–A}}$ **type exchanger**

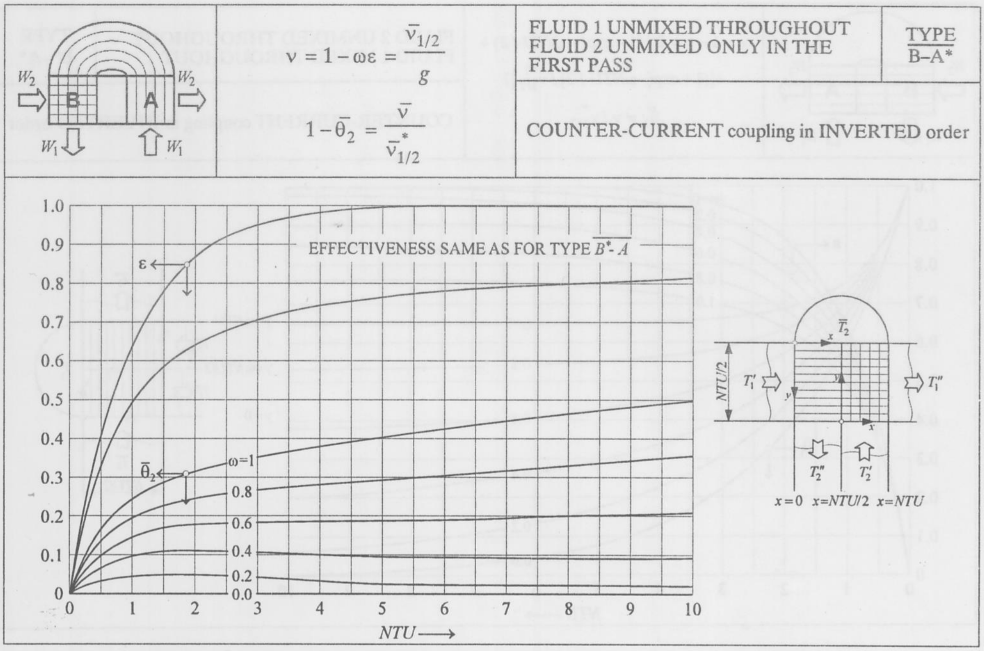


Figure 14. ε-*NTU*-ω and $\bar{\theta}_2$-*NTU*-ω relationships for $\overline{\text{B–A*}}$ type exchanger

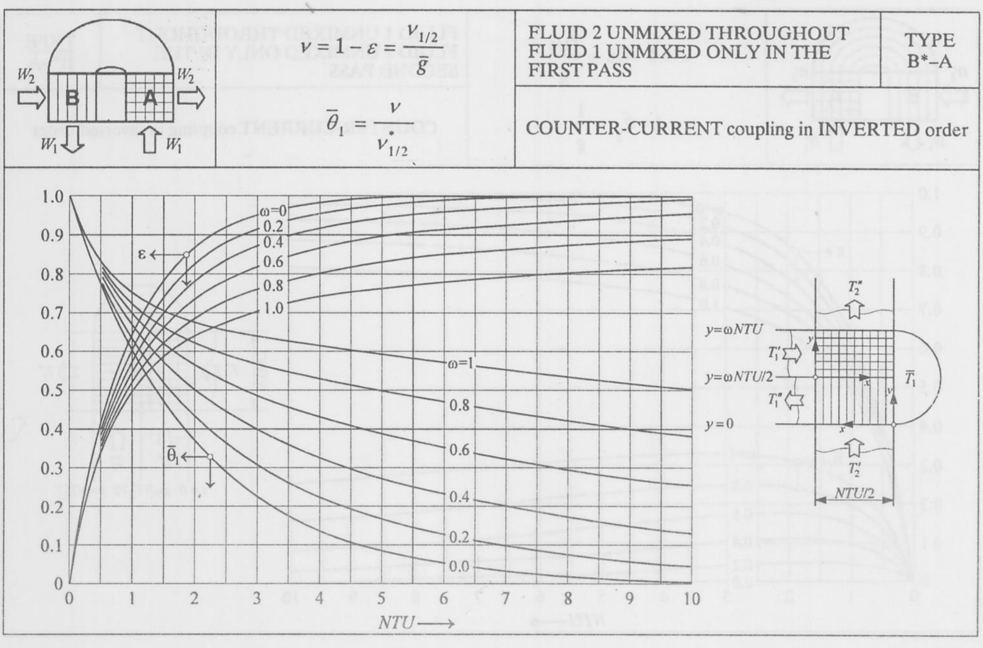


Figure 15. ε-*NTU*-ω and $\bar{\theta}_1$-*NTU*-ω relationships for B*–A type exchanger

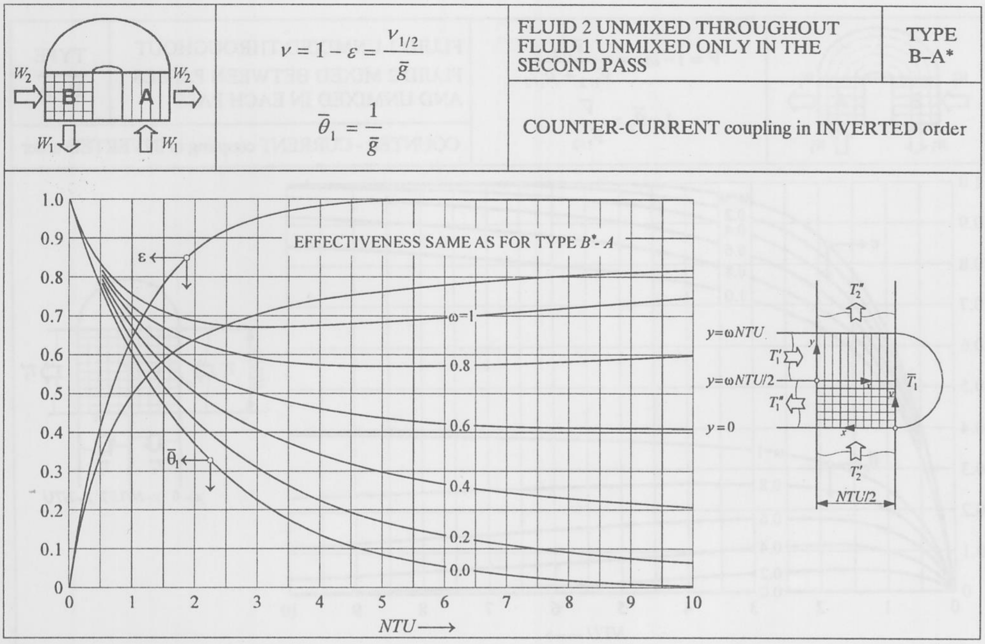


Figure 16. ε-*NTU*-ω and $\overline{\theta}_1$-*NTU*-ω relationships for B–A* type exchanger

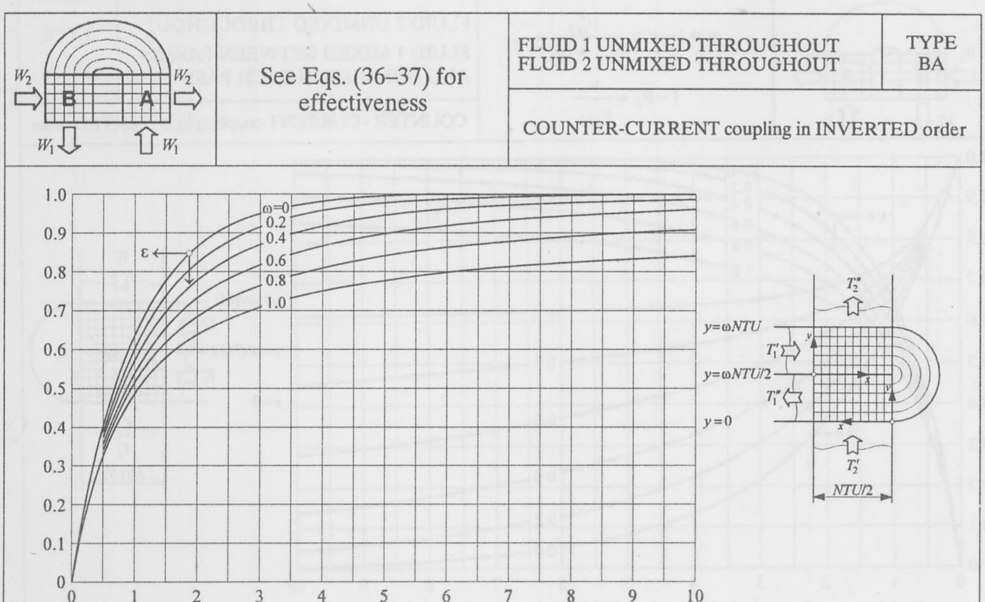


Figure 17. ε-*NTU*-ω relationships for BA type exchanger

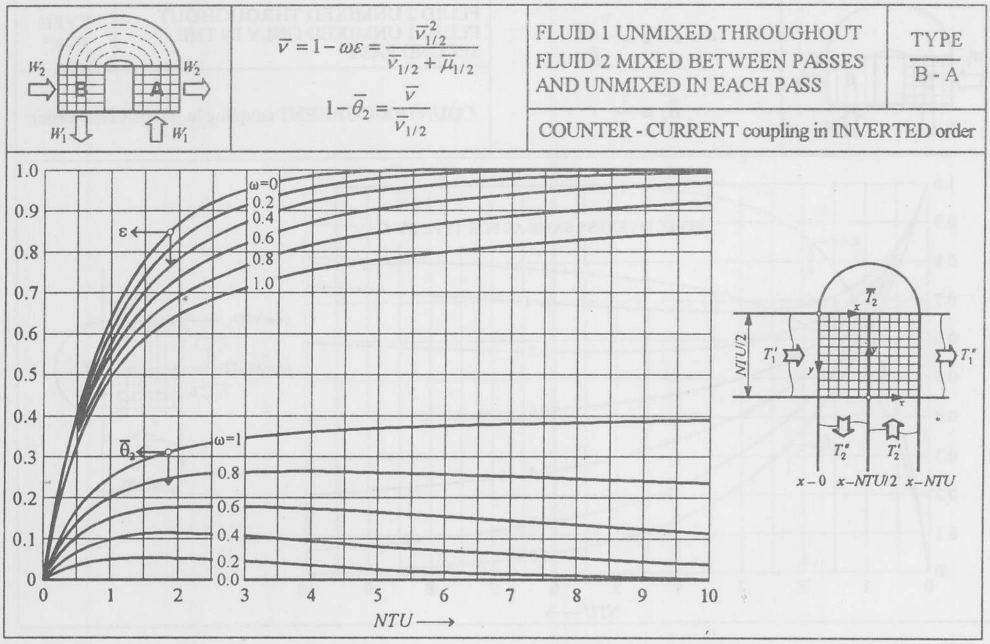


Figure 18. ε-NTU-ω and $\overline{\theta_2}$-NTU-ω relationships for B-A type exchanger

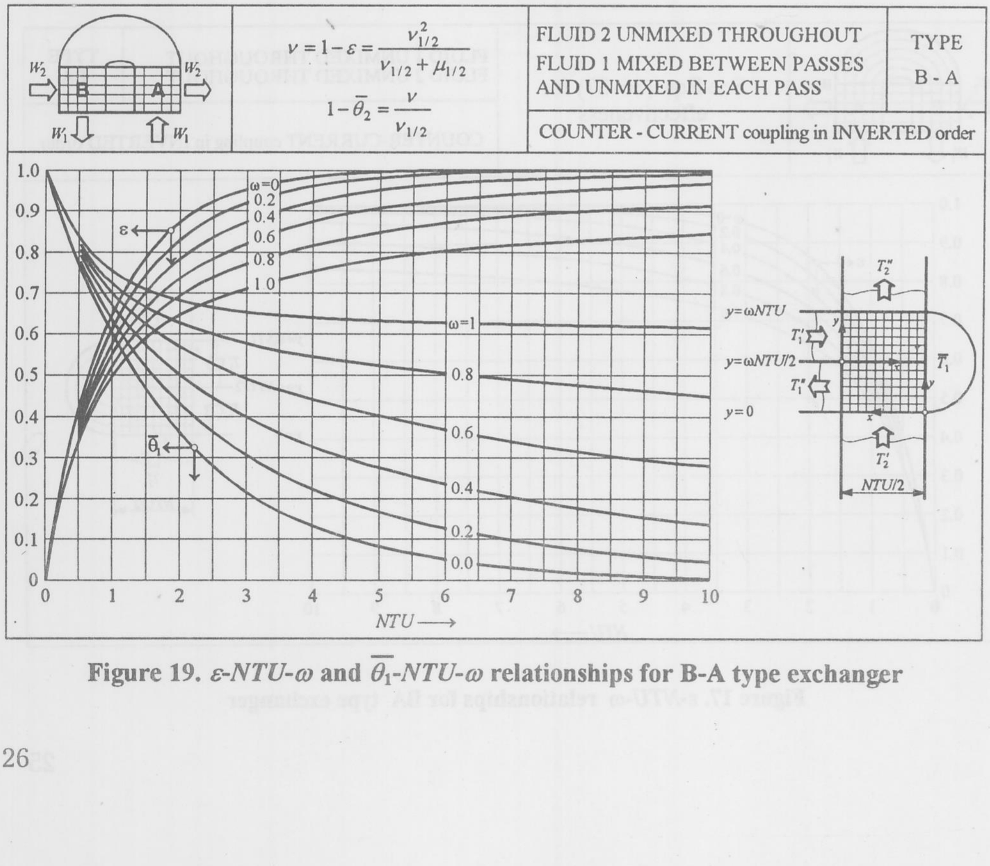


Figure 19. ε-NTU-ω and $\overline{\theta_1}$-NTU-ω relationships for B-A type exchanger

Nomenclature

A [m²]	– heat transfer area
a	– argument of ineffectiveness function, defined in Eq. (11)
b	– argument of ineffectiveness function, defined in Eq. (12)
c_p [J/kg K]	– specific heat at constant pressure
g	– function defined in Eq. (25)
$\bar{g}$	– function defined in Eq. (26)
I_n (·)	– modified Bessel function of n-th (integer) order
$\dot{M}$ [kg/s]	– mass flow rate
NTU	– number of transfer units*
T [K]	– temperature
$\bar{T}$	– mean mixed fluid temperature at the exit of the first pass
U [W/m² K]	– overall heat transfer coefficient
V_j	– functions defined in Eqs. (30–35)
W [W/K]	– thermal capacity rate, $\dot{M}c_p$
α	– integer, see Eq. (12)
β	– integer, see Eqs. (11–12)
ε	– effectiveness of the exchanger*
$\mu_{1/2}$	– function defined in Eq. (27)
$\bar{\mu}_{1/2}$	– function defined in Eq. (28)
ν	– ineffectiveness of the exchanger*
$\bar{\nu}$	– $= 1 - \omega\varepsilon$
$\nu_{\alpha/\beta}$	– ineffectiveness function, defined in Eq. (18)
$\nu^*_{\alpha/\beta}$	– ineffectiveness function, defined in Eq. (10)
$\bar{\nu}_{\alpha/\beta}$	– ineffectiveness function, defined in Eq. (19)
$\bar{\nu}^*_{\alpha/\beta}$	– ineffectiveness function, defined in Eq. (15)
ω	– thermal capacity rate ratio*
θ	– dimensionless temperature*
$\bar{\theta}$	– dimensionless mean mixed fluid temperature at the exit of the first pass

*

$NTU = UA/W_1$ $\omega = W_1/W_2$

$\varepsilon = (T_1' - T_1'')/(T_1' - T_2')$ $\theta = (T - T_2')/(T_1' - T_2')$

$\nu = 1 - \varepsilon$

Subscripts

1	– refers to fluid with $(\dot{M}c_p)_{min}$
2	– refers to fluid with $(\dot{M}c_p)_{max}$

Superscripts

'	– at exchanger inlet
"	– at exchanger outlet
–	– means transposition (flow patterns of fluids are interchanged)
*	– fluid W_1 unmixed in the pass

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