MODELLING OF DYNAMICS OF COMBUSTION OF BIOMASS IN FLUIDIZED BEDS

by

Jaakko J. SAASTAMOINEN

Original scientific paper
UDC: 662.636/.638:591.876.5-912
BIBLID: 0354-9836, *8* (2004), 2, 107-126

New process concepts in energy production and biofuels, which are much more reactive than coal, call for better controllability of the combustion in circulating fluidized bed boilers. Simplified analysis describing the dynamics of combustion in fluidized bed and circulating fluidized bed boilers is presented. Simple formulas for the estimation of the responses of the burning rate and fuel inventory to changes in fuel feeding are presented. Different changes in the fuel feed, such as an impulse, step change, linear increase and cyclic variation are considered. The dynamics of the burning with a change in the feed rate depends on the fuel reactivity and particle size. The response of a fuel mixture with a wide particle size distribution can be found by summing up the effect of different fuel components and size fractions. Methods to extract reaction parameters form dynamic tests in laboratory scale reactors are discussed. The residence time of fuel particles in the bed and the resulting char inventory in the bed decrease with increasing fuel reactivity and differences between coal and biomass is studied. The char inventory affects the stability of combustion. The effect of char inventory and oscillations in the fuel feed on the oscillation of the flue gas oxygen concentration is studied by model calculation. A trend found by earlier measurements is explained by the model.

Key words: *fluidised beds, combustion, dynamics, biomass, modelling*

Introduction

The knowledge of the dynamics of combustion in circulating fluidized bed (CFB) is important, when adopting once through steam cycle with high steam values in CFB boilers [1, 2]. The knowledge of the dynamics of combustion is also important for optimising load changes [2]. In steady operation, combustion is affected by the disturbances in the feed-rate of the fuel and by the incomplete mixing of the fuel in the bed, which may cause changes in the burning rate, oxygen level and increase CO emissions [2]. This is especially important, when considering the new biomass based fuels, which are increasingly been used to replace coal or in co-firing. These new biofuels are often rather inhomogeneous, which can cause instabilities in the feeding. These fuels are usually also very reactive. Biomass fuels have much higher reactivity compared to coals and the knowledge of the factors affecting the combustion dynamics is important for optimum control.

The particle size distribution (PSD) of char particles in the bed in steady and dynamic conditions has been discussed earlier [3]. Here somewhat more simple models that could be used as aid, when planning the control of the combustion, are presented. The model presented here can be used in parallel with the experiments and the more detailed numerical model [4]. Simplified modelling can give insights on the relevant processes and finding rules of thumb.

Simple dynamic experiments can also be used to extract reaction rate parameters for devolatilization or char combustion. By comparing the measured and calculated response (for example oxygen outlet concentration, PSD, *etc.*) to a dynamic change, the best fit for the parameters can be found. These models with the parameters can then be used in steady and unsteady state simulations and in development of process control strategies.

Biomass fuels contain a greater portion of volatiles than coal. The volatiles from biomass are also released faster than from coal, since they are released at lower temperature (see fig. 1). The density of wood is lower than that of coal, which also makes the devolatilization faster, since wood particles are heated up faster than coal particles. Volatiles are released much faster than the residual char is burned. Then the release and burning of the volatiles is the major reason for the variation of the combustion intensity in the bed, when using biofuels.

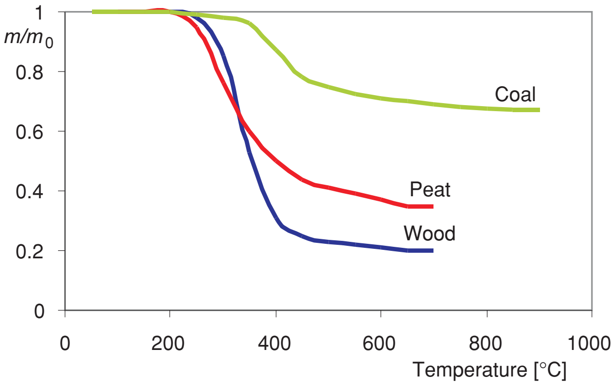

Figure 1. Typical mass loss from small fuel samples during pyrolysis in inert atmosphere with slow heating rate (5 K/s)

Modelling of the dynamics of combustion

Combustion rate of fuel in the bed with constant oxygen level

A simplified method to calculate to volatile mass of fuel and release of volatiles in the bed has been presented earlier [2] for monosized fuel. In the present analysis, the model is extended to a fuel with a particle size distribution (PSD) and the subsequent char combustion regime.

The release of volatiles from fuel particles can be approximated by the equation:

$$\frac{\mathrm{d}m}{\mathrm{d}t} \quad Km \tag{1}$$

where $m = m_v$ is mass of volatiles remaining in the particle and $K = K_v$ is effective rate coefficient for pyrolysis reactions. The time constant of pyrolysis $\tau_v = 1/K_v$ depends on the particle size. The time constant reported in the earlier paper [2] is for the whole period including the heat up time to pyrolysis temperature. In reality the actual release time of volatiles is shorter. The release and the subsequent burning of volatiles consume oxygen, which may lead to increase in CO emissions. The time constant depends on particle size ($\sim d^{1.5}$).

Also the char burning can approximately be described by eq. (1) [1, 5], where $m = m_c$. The experimental coefficient is $K \quad K_c \quad k_c X_{\mathrm{O}_2}^n$, where X_{O_2} is the logarithmic average oxygen mass fraction in the bed. The analysis can be applied to char combustion, if the oxygen concentration does not vary much during the dynamic change, $X_{\mathrm{O}_2}^n$ constant. In reality, the oxygen concentration is changing to some extent, if air rate is constant. If the change in the fuel feed is great, it results in large change in the transient concentration of oxygen, which will be discussed in the next section. The time constant for char burning $\tau_c \quad 1/K_c$ depends on the particle size and this approximation can be applied to different size fractions. It might also be used as a rough estimate to the whole fuel. At relative low temperatures (as it is usual in FBC and CFBC) and when the particle size is small, the pyrolysis and char combustion may be overlapping so that it is difficult to separate these stages [6, 7]. Then the combustion of the whole fuel particle can be described by a similar expression as eq. (1) with $m = m_f$, $K = K_f$ and a time constant $\tau_f = 1/K_f$. Examples of exponetial approximations of burning curves are presented in fig. 2.

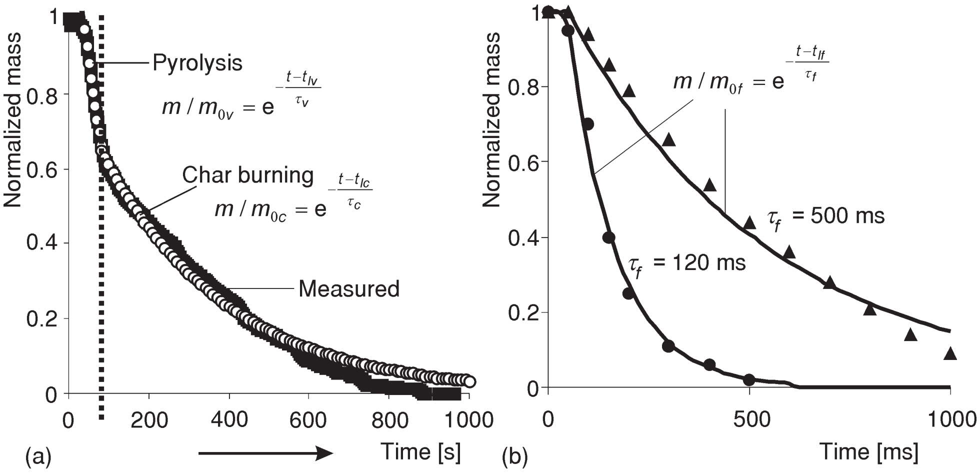

Figure 2. Exponential approximation. (a) Sequential pyrolysis and char burning on one coal, (b) Overlapping pyrolysis and char burning (two coals with a narrow size fraction $d \quad 250\ \mu$m)

These different reaction time constants τ (pyrolysis, char oxidation, or both united to a constant) can be measured by using a thermobalance or a batch fluidised bed for different size fractions and components of the fuel mixture or approximately for the whole fuel mixture.

The system studied is shown in fig. 3. Fuel is fed in the reactor and burned. There is a time lag t_l between the actual time of feeding and the time at which the fuel is ready for burning due to delay cased by the heat-up. The balance for the volatile matter, char or fuel in the bed accounting the time lag t_l needed to prepare the fuel to burning state (devolatilization or char burning) is:

$$\frac{\mathrm{d}m_b}{\mathrm{d}t} \quad \dot{m}_0(t \quad t_l) \quad \frac{m_b}{\tau} \tag{2}$$

The term on the left-hand side describes the accumulation of fuel, the first term on the right hand-side denotes the fuel feed accounting for the time lag needed for reac-

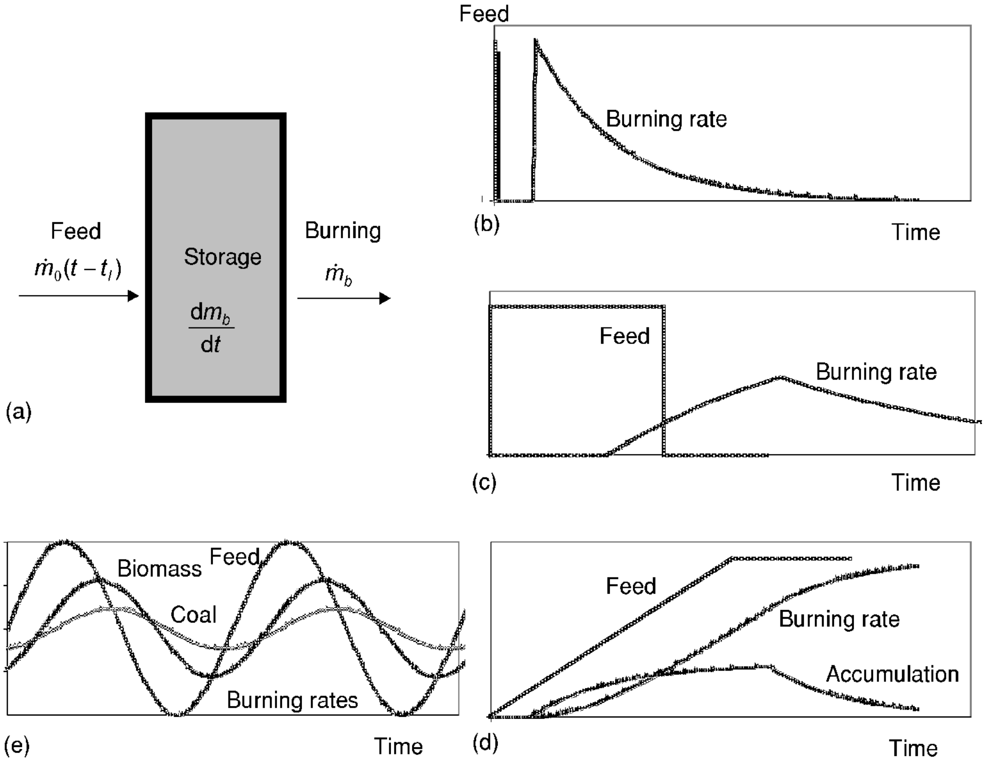

Figure 3. (a) 0-dimensional model of FB or CFB reactor, (b) Impulse feed and its response in burning rate, (c) A square pulse and its response in burning rate, (d) A ramp after which a constant fuel feed and its responses in burning rate and accumulated fuel, (e) Sinusoidal vibration in the fuel feed and its responses in burning rates of coal and biomass

tions to start after the feeding instant. The second term accounts for the mass loss from the fuel bed due to burning. This equation can be used separately for volatile matter, char or the whole fuel (= volatiles + char) in the bed. For char $\dot{m}_0 = (1 - X_v)\dot{m}_{0f}$ and for volatiles $\dot{m}_0 = X_v \dot{m}_{0f}$. The devolatilization of particles does not start at once after they are introduced to the furnace. At first they must be heated up to the devolatilization temperature, which is taken in the account by the time lag t_l. Also char combustion takes place after devolatilization and the char oxidation could be defined to start at the moment, when a certain amount (for example 90%) of volatiles has been released. Thus, the zero point in the time scale for the actual reaction reactions is after the fuel feeding with a time lag t_l. If the fuel has a wide PSD, this equation is applied separately to each size fraction (of the feed) using the time constants τ and lag times t_l (heat up, pyrolysis) of different size fractions. If the fuel is a mixture, then the burning of each fuel component can be described separately by eqs. (1) and (2).

The above discussion is summarised. Equation (2) can be applied to volatile matter (matter that can later be devolatilized) in the bed. Then time lag t_l accounts for the time needed to heat up the particles to pyrolysis to start, $t_l = t_h$. Then the second term accounts for the mass loss by release of volatiles. It can also be applied to char in the bed. Then the time lag $t_l = t_h + t_v$ accounts the times of heat up t_h and devolatilization t_v. If one considers pyrolysis and char oxidation to be successive and the char oxidation to start after release of mass fraction X_{rel} of volatiles, then $t_v = -\tau_v \ln X_{\mathrm{rel}}$. One can consider the pyrolysis and char oxidation united as a single combustion process and use eq. (2). Then the time lag is the same as in case 1.

Differential eq. (2) can be solved for example by applying the Laplace transform:

$$\bar{f}(s) = \int_0^{\infty} \mathrm{e}^{-st} F(s)\mathrm{d}t \tag{3}$$

The transformed solutions (denoted by the bar) for the mass of fuel and burning rate can be obtained:

$$\overline{m}_b = \overline{\dot{m}}_0 \mathrm{e}^{-\frac{t_l s}{s + \frac{1}{\tau}}}, \quad \overline{\dot{m}}_b = \frac{\overline{\dot{m}}_0}{\tau} \tag{4}$$

respectively. The mass of fuel in the bed and burning rate of the fuel are obtained as inverse transforms using the tabulated properties of Laplace transforms found in text books:

$$m_b = \int_0^{t - t_l} \mathrm{e}^{-\frac{t - t_l - u}{\tau}} \dot{m}_0(u)\mathrm{d}u, \quad \dot{m}_b = \frac{m_b}{\tau} \tag{5}$$

The need for oxygen to regulate O_2 level to be constant in the reactor can be estimated from the stoichiometric relation assuming complete combustion. Then:

$$\dot{m}_{O_2} = \frac{\dot{m}_b}{f_s} = \frac{m_b}{f_s \tau} \tag{6}$$

and the corresponding air rate is $\dot{m}_a = \dot{m}_{O_2} / X_{O_2,a}$, where $X_{O_2,a}$ is the mass fraction of oxygen in air (~0.23). Analytical approximations for the mass (volatiles, char, or char + volatiles) and burning rate of volatiles in the bed as function of time for some specific changes in the fuel feed are summarised in tab. 1. This table is compilation of analytical solutions presented in a previous paper [2].

Table 1. Responses of the fuel mass (volatile matter or char) in the bed and of the burning rate (release of mass) to changes in the fuel feed

Type of change in the fuel feed	Mass in bed	Rate of mass release from bed
	$m_b = f(t - t_l, \tau)$	$\dot{m}_b = g(t - t_l, \tau)$
	$f(t) =$	$g(t) =$
Impulse $m_0 \delta(t)$	$m_0 \mathrm{e}^{-\frac{t}{\tau}}$	$\frac{m_0}{t} \mathrm{e}^{-\frac{t}{\tau}}$
Step change $\dot{m}_0 U(t)$	$\dot{m}_0 \tau (1 - \mathrm{e}^{-\frac{t}{\tau}})$	$\dot{m}_0 (1 - \mathrm{e}^{-\frac{t}{\tau}})$
Square pulse	By superposition of two step changes	By superposition of two step changes
Ramp ct	$ct[t - \tau(1 - \mathrm{e}^{-\frac{t}{\tau}})]$	$c[t - \tau(1 - \mathrm{e}^{-\frac{t}{\tau}})]$
Ramp after which a constant level	By superposition of two ramps	By superposition of two ramps
Sinusoidal variation $\hat{\dot{m}}_0 \sin(2\pi ft)$	$\hat{\dot{m}}_0 \frac{\sin[2\pi ft - \mathrm{arctg}\ 2\pi ft]}{\sqrt{\left(\frac{1}{\tau}\right)^2 + (2\pi f)^2}}$	$\frac{\hat{\dot{m}}_0}{\tau} \frac{\sin[2\pi ft - \mathrm{arctg}\ 2\pi ft]}{\sqrt{\left(\frac{1}{\tau}\right)^2 + (2\pi f)^2}}$

The time lags and time constants can be measured using a thermobalance or a batch fluidised bed reactor. It is also possible to estimate theoretically their dependence on particle size. Fragmentation may take place during pyrolysis and char combustion stage, which affects the burning rate. The effect of fragmentation is included in the experimental time constants for different size fractions of the feed. The heat-up of a particle can be described by $(T - T_0)/(T_e - T_0) = 1 - \mathrm{e}^{-t/\tau_h}$ [8, 9]. The time lag for heating can be estimated from the relation $t_l = -\tau_h \ln[(T_e - T_v)/(T_e - T_0)]$, where T_e is the effective temperature of the reactor and τ_h is time constant for heating. For moist fuel the drying gives an additional time lag (see for example [10]).

Some typical dynamic cases for single fuel and single component (volatiles, char or their combination) are presented in fig. 3. Since biomass is more reactive than coal, its burning rate has shorter phase lag and it is less damped (see fig. 3e).

If devolatilization and char oxidation are treated as successive, then the fuel mass in the bed and burning rate are obtained as a sum of these by summing up the contributions of different size fractions and different fuels:

$$m_b \quad \overset{N}{\underset{i\;\;1}{}}(m_{b,v,i} \quad m_{b,c,i}), \quad \dot{m}_b \quad \overset{N}{\underset{i\;\;1}{}} \frac{m_{b,v,i}}{\tau_{v,i}} \quad \frac{m_{b,c,i}}{\tau_{c,i}} \tag{7}$$

The pyrolysis and oxidation of the smallest size fraction could be treated as a single combustion process and the above equations are easily modified to account for this. The summation index i in eq. (9) accounts for the contributions of different size fractions (of the feed) and different fuels, if the feed is a fuel mixture.

The solutions can be applied to two cases: (a) air rate is constant, oxygen level varies, and (b) air rate is changed so that the oxygen level remains constant. In the case a the oxygen concentration will also vary. Then also the reaction coefficient for char oxidation will vary, but the theory can be applied, if the disturbances or changes in the fuel feed are small causing only small changes in X_{O_2} . Then K_c remains roughly constant.

The consumption of oxygen assuming stoichiometric combustion with no losses as unburned fuel is obtained by summing up the different contributions:

$$\dot{m}_{O_2} \quad \overset{N}{\underset{i\;\;1}{}} \frac{m_{b,v,i}}{f_{s,v,i}\tau_{v,i}} \quad \frac{m_{b,c,i}}{f_{s,c,i}\tau_{c,i}} \tag{8}$$

where f_s is the stoichiometric fuel/oxygen ratio. So in case a eq. (8) gives the decrease of the oxygen in flue gases as function of time. This equation can also be used to estimate how the air rate, $\dot{m}_a \quad \dot{m}_{O_2} \big/ X_{O_{2,a}}$, should be varied in time to meet the changes in the burning rate and to keep the oxygen level constant in order to reduce CO emissions (case b). This is important under rapid load changes, when the fuel feed is increasing or decreasing to reduce CO emissions and to improve efficiency, when using complex fuel mixtures.

Any cyclic feed rate can be presented as Fourier series and the response is obtained by summing up the responses to each term. In principle, it is possible to damp totally the vibration in the burning rate due to sinusoidal cyclic fuel feeding (constant rate plus sinusoidal vibration) by using a mixture of two fuels. In such a case, the fuels should have a phase lag in the burning rates between them and mixed in such mass proportions that the sum of the cyclic parts of the burning rates is zero. In this case, which seems rather hypothetical we have:

$$2\pi f t_{l,b} \quad arctg(2\pi f \tau_b) \;\; 2\pi f t_{l,a} \quad arctg(2\pi f \tau_a) \quad \pi \tag{9}$$

$$\frac{\dot{m}_{0,b}}{\dot{m}_{0,a}} \quad \sqrt{\frac{1 \quad (2\pi f \tau_b)^2}{1 \quad (2\pi f \tau_a)^2}} \tag{10}$$

for two fuels *a* and *b*, which can be derived by summing two responses (tab. 1, sinusoidal case). So in principle, if there are vibrations in the feeding and fuel *a* that has time constant τ_a is burned, a stabilising fuel *b* could be used. The first of these relations gives the necessary time lag $t_{l,b}$ and time constant τ_b (which could be fixed properly by the particle size) for this fuel. The second condition gives the necessary mass flow rate for total damping. If some characteristic cyclic mixing phenomena cause cyclic burning, the burning might be stabilised using a suitable fuel mixture or by using cyclic feeding.

Combustion rate or char in the bed with constant air rate

Here the case, in which the air rate is constant, is considered. This subject has shortly been discussed earlier [1]. Here a more general analysis is given. The change in the fuel feed results in a change in the oxygen concentration. This analysis is restricted to char combustion. The devolatilization, in contrast to char combustion, is not affected much by the oxygen concentration in the bed, but depends mainly on the temperature. Thus the previous analysis can be used for the devolatilization stage even the oxygen concentration does not remain constant. Only the case of impulse char feed is considered. The bed is treated as one-dimensional along co-ordinate x in the direction of gas flow and there will be a distribution in the oxygen concentration in the bed.

The char balance in the bed can be described by the equation:

$$\frac{\mathrm{d}m_c}{\mathrm{d}t} \quad \dot{m}_0 \quad \dot{m}_p \tag{11}$$

which applies to the whole amount of char or the char of a specific size fraction *i*. m_c is the amount of char in the combustion chamber, $\dot{m}_0$ is the steady mass flow rate of char in the feed and $\dot{m}_p$ is the mass flow rate of char consumed in burning. m_f is the amount of char feed as an impulse.

The burning/length of the fuel bed in the direction of gas flow (kg m^{-1}s^{-1}) is described by:

$$r_{c,i} \quad k_i m_{c,i} X_{\mathrm{O}_2}^{n_i} \tag{12}$$

where $m_{c,i}$ is mass of fuel/length. Then the loss of carbon due to burning in the reactor can be calculated by:

$$\dot{m}_{p,i} \quad {}_{0}^{L} k_i m_{c,i} X_{\mathrm{O}_2}^{n_i} \mathrm{d}x \tag{13}$$

Oxygen is consumed in the direction of the flow (co-ordinate x) due to combustion, which is described by:

$$ {}_{j} \frac{r_{c,j}}{f_{s,j}} \quad \frac{\mathrm{d}\dot{m}_{\mathrm{O}_2}}{\mathrm{d}x} \quad \dot{m}_g \frac{\mathrm{d}X_{\mathrm{O}_2}}{\mathrm{d}x} \tag{14}$$

If there are several fuels $j > 1$. Equations (13) and (14) give

$$\dot{m}_{p,i} = \int_{X_{O_2,x=0}}^{X_{O_2,x=L}} \frac{k_i m_{c,i} X_{O_2}^{n_i} \dot{m}_g}{\sum_j \frac{r_{c,j}}{f_{s,j}}} \mathrm{d}X_{O_2} \tag{15}$$

The distribution of fuel in the height direction is $m_{c,j} = F_j(x) m_{c,j} / L$, where the distribution function is normalised so that $\int_0^L F_j(u)\mathrm{d}u / L = 1$. If we assume the distribution function and n_i to be the same for all fuels, $F_j(x) = F(x)$, $n_i = n$, eq. (15) can be integrated giving:

$$\dot{m}_{p,i} = \frac{k_i m_{c,i} \dot{m}_g (X_{O_2,x=0} - X_{O_2,x=L})}{\sum_j \frac{k_j m_{c,j}}{f_{s,j}}} \tag{16}$$

The distribution of oxygen concentration (and the level at the top, when $x = L$, $\int_0^L F_j(u)\mathrm{d}u = L$) can be solved by integrating eq. (14), if we assume n_i to be same for all fuels in the bed ($n_i = n$):

$$n \neq 1, \quad x_{O_2,x} = \left[X_{O_2,x=0}^{1-n} - \frac{1}{L\dot{m}_g} \sum k_j m_{c,j} \int_0^x \frac{F_j(u)\mathrm{d}u}{f_{s,j}} \right]^{\frac{1}{1-n}} \tag{17}$$

$$n = 1, \quad X_{O_2,x} = X_{O_2,x=0} \mathrm{e}^{-\frac{1}{L\dot{m}_g} \sum_j k_j m_{c,j} \int_0^x \frac{F_j(u)du}{f_{s,j}}} \tag{18}$$

If the fuel is evenly distributed, $F_j(u) = 1$ and $\int_0^x F_j(u)\mathrm{d}u = x$. If n_i are not the same for different fuels, eqs. (14) and (15) must be integrated numerically.

One fuel in steady operation

Then in eq. (11) $\mathrm{d}m_c/\mathrm{d}t = 0$. Equations (11) and (15) give:

$$\dot{m}_0 = \dot{m}_p = f_s \dot{m}_g (X_{O_2,x=0} - X_{O_2,x=L}) \tag{19}$$

The oxygen concentration on the top depends on mass flow rate of fuel:

$$X_{O_2,x=L} = X_{O_2,x=0} - \frac{\dot{m}_0}{\frac{\dot{m}_g}{f_s}} \tag{20}$$

On the other hand, the oxygen concentration is obtained from eq. (17) or (18), which depend on reaction rate coefficient k. Then k can be solved and we get

$$n \quad 1, \quad \frac{km_c}{f_s \dot{m}_g} \quad X^{1 \quad n}_{\mathrm{O}_2,x \quad 0} \quad X^{1 \quad n}_{\mathrm{O}_2,x \quad L} \quad X^{1 \quad n}_{\mathrm{O}_2,x \quad 0} \quad X_{\mathrm{O}_2,x \quad 0} \quad \frac{\frac{\dot{m}_0}{\dot{m}_g}}{f_s}^{1 \quad n} \tag{21}$$

$$n \quad 1, \quad \frac{km_c}{f_s \dot{m}_g} \quad \frac{X_{\mathrm{O}_2,x \quad L}}{X_{\mathrm{O}_2,x \quad 0}} \quad \ln \; 1 \quad \frac{\dot{m}_0}{f_s \dot{m}_g X_{\mathrm{O}_2,x \quad 0}} \tag{22}$$

In principle, the value of k can be calculated from these, but then one should measure the char mass (inventory) in the bed m_c in stationary situation.

If the value of the reactivity coefficient k is known, the char inventory m_c can be calculated by using equations:

$$n \quad 1, \quad m_c \quad \frac{\dot{m}_g f_s}{k} (X^{1 \quad n}_{\mathrm{O}_2,x \quad 0} \quad X^{1 \quad n}_{\mathrm{O}_2,x \quad L}) \tag{23}$$

$$n \quad 1, \quad m_c \quad \frac{\dot{m}_g f_s}{k} \ln \frac{X_{\mathrm{O}_2,x \quad 0}}{X_{\mathrm{O}_2,x \quad L}} \tag{24}$$

One fuel and its response to an impulse

An impulse m_f is fed in the reactor at time $t = 0$ (accounting for the time lag to heat up the particle). Then eqs. (11) and (19) give:

$$\frac{\mathrm{d}m_c}{\mathrm{d}t} \quad \dot{m}_0 \quad \dot{m}_p \quad \dot{m}_0 \quad f_s \dot{m}_g (X_{\mathrm{O}_2,x \quad 0} \quad X_{\mathrm{O}_2,x \quad L}) \tag{25}$$

$$n \quad 1, \quad \frac{\mathrm{d}m_c}{\mathrm{d}t} \quad \dot{m}_0 \quad f_s \dot{m}_g X_{\mathrm{O}_2,x \quad 0} \; 1 \quad \frac{km_c X^{n \quad 1}_{\mathrm{O}_2,x \quad 0}}{f_s \dot{m}_g}^{\frac{1}{1 \quad n}} \tag{26}$$

$$n \quad 1, \quad \frac{\mathrm{d}m_c}{\mathrm{d}t} \quad \dot{m}_0 \quad f_s \dot{m}_g X_{\mathrm{O}_2,x \quad 0} \; 1 \quad \mathrm{e}^{\frac{km_c}{f_s \dot{m}_g}} \tag{27}$$

Equation (26) can be integrated numerically giving m_c as function of time. Equation (27) can be integrated analytically:

$$m_c \quad \frac{f_s\dot{m}_g}{k}\ln \frac{1}{1 \quad R} \quad \mathrm{e}^{\;kX_{\mathrm{O}_2,x\quad 0}(1 \quad R)t} \quad \mathrm{e}^{\frac{km_{c,0}}{f_s\dot{m}_g}} \quad \frac{1}{1 \quad R} \tag{28}$$

where $R \quad \dot{m}_0 \,/\, (f_s\dot{m}_g X_{\mathrm{O}_2,x\quad 0})$.The char inventory at time $t = 0$ is

$$m_{c,0} = m_{c,\infty} + m_f \tag{29}$$

where $m_{c,\infty}$ is the char inventory of the base fuel in stationary situation obtained from eqs. (23) or (24).

Reaction rate coefficient k can be determined by choosing its value so that the oxygen concentration calculated using eq. (17) or (18), when m_c known as function of time, is closest to the measured value. In the case $n = 1$, we get:

$$\frac{X_{\mathrm{O}_2,x\quad L}}{X_{\mathrm{O}_2,x\quad 0}} \quad \frac{1 \quad R}{1 \quad \mathrm{e}^{\;kX_{\mathrm{O}_2,x\quad 0}(1 \quad R)t} \quad (1 \quad R)\mathrm{e}^{\frac{km_{c,0}}{f_s\dot{m}_g}} \quad 1} \tag{30}$$

In special case, when there is no base fuel (an electrically heated laboratory scale reactor) $\dot{m}_0 = \; 0$ and $n = 1$:

$$m_c \quad \frac{f_s\dot{m}_g}{k}\ln 1 \quad \mathrm{e}^{\;kX_{\mathrm{O}_2,x\quad 0}t} (\mathrm{e}^{\frac{km_f}{f_s\dot{m}_g}} \quad 1) \tag{31}$$

$$\frac{X_{\mathrm{O}_2,x\quad L}}{X_{\mathrm{O}_2,x\quad 0}} \quad \frac{1}{1 \quad \mathrm{e}^{\;kX_{\mathrm{O}_2,x\quad 0}t} (\mathrm{e}^{\frac{km_f}{f_s\dot{m}_g}} \quad 1)} \tag{32}$$

One fuel and shut down

This is a special solution of the previous case, where $mf = 0$. Then $m_{c,0} = m_{c,\infty}$ and $\dot{m}_0 \quad 0$. In the special case $n = 1$ the char inventory can be solved analytically from eq. (28):

$$m_c \quad \frac{f_s\dot{m}_g}{k}\ln 1{+}\mathrm{e}^{\;kX_{\mathrm{O}_2,x\quad 0}t} (\mathrm{e}^{\frac{km_{c,0}}{f_s\dot{m}_g}} \quad 1) \tag{33}$$

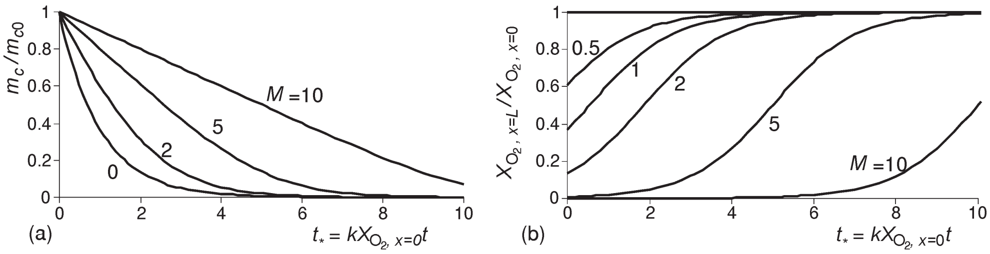

Figure 4. Char mass/initial mass in the reactor (a) and oxygen concentration at the top of the reactor (b) as function of dimensionless time with impulse fuel feed or when the fuel feed is closed (the temperature level is maintained with electric energy) with impulse of initial dimensionless char mass *M* as a parameter

and oxygen concentration from eq. (30):

$$\frac{X_{O_2,x=L}}{X_{O_2,x=0}}=\frac{1}{1+\mathrm{e}^{-kX_{O_2,x=0}t}\left(\mathrm{e}^{\frac{km_{c,0}}{f_s\dot{m}_g}}-1\right)} \tag{34}$$

These are of the same form as eqs. (31) and (32) for an impulse fuel feed, when there is no base fuel. Figure 4 [1] present the mass/initial mass and oxygen concentration in the flue gases as function of dimensionless time $t_* = kX_{O_2,x=0}t$ and dimensionless initial mass $M = km_{c,0}/f_s\dot{m}_g$ or impulse mass $M = km_f/f_s\dot{m}_g$ as the parameter. The initial mass is obtained from eq. (24). The oxygen concentration at the exit is:

$$\frac{X_{O_2,x=L}}{X_{O_2,x=0}}=\frac{1}{1+\mathrm{e}^{-kX_{O_2,x=0}t}\left(\dfrac{X_{O_2,x=0}}{X_{O_2,x=L}(t=0)}-1\right)} \tag{35}$$

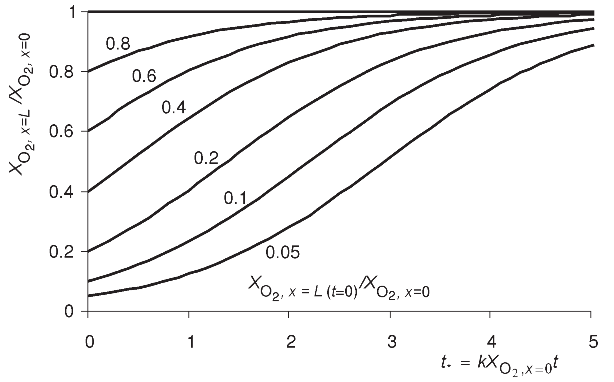

Figure 5. Oxygen concentration at the top of the reactor as function of dimensionless time and concentration ratio of oxygen at *t* = 0 as a parameter, when the fuel feed is closed

which is presented in fig. 5. When k is solved from this equation, we obtain :

$$k \quad \frac{f_s \dot{m}_g}{X_{\mathrm{O}_2,x\ 0}\, t} \ln \frac{\dfrac{X_{\mathrm{O}_2,x\ 0}}{X_{\mathrm{O}_2,x\ L}(t \quad 0)} \quad 1}{\dfrac{X_{\mathrm{O}_2,x\ 0}}{X_{\mathrm{O}_2,x\ L}} \quad 1} \tag{36}$$

Thus k is obtained as function of time. It should be noted when analysing values of k obtained from shut down and impulse test, the situations do not fully correspond to each other, since in the impulse test, the initial particle size at the start is larger than that in the shutdown.

Two fuel and dynamic operation

We consider a case, where the base fuel 1 is burned with steady feed rate $\dot{m}_{0,1}$ = = constant and a char mass impulse 2 $m_{f,2}$, is fed into the reactor. Then $m_{c,2}(t = 0) = m_{f,2}$. Equation (11) gives:

$$\frac{\mathrm{d}m_{c,1}}{\mathrm{d}t} \quad \dot{m}_{0,1} \quad \dot{m}_{p,1}, \quad \frac{\mathrm{d}m_{c,2}}{\mathrm{d}t} \quad \dot{m}_{p,2} \tag{37}$$

where from eq. (16):

$$\dot{m}_{p,i} \quad \frac{k_i m_{c,i} \dot{m}_g \left(X_{\mathrm{O}_2,x\ 0} \quad X_{\mathrm{O}_2,x\ L}\right)}{\dfrac{k_1 m_{c,1}}{f_{s,1}} \quad \dfrac{k_2 m_{c,2}}{f_{s,2}}} \tag{38}$$

for fuels i = 1 and i = 2. From eqs. (17) and (18):

$$n \quad 1, \quad X_{\mathrm{O}_2,x\ L} \quad X_{\mathrm{O}_2,x\ 0}^{1\ n} \quad \frac{\dfrac{k_1 m_{c,1}}{f_{s,1}} \quad \dfrac{k_2 m_{c,2}}{f_{s,2}}}{\dot{m}_g}^{\frac{1}{1\ n}} \tag{39}$$

$$n \quad 1, \quad X_{\mathrm{O}_2,x\ L} \quad X_{\mathrm{O}_2,x\ 0}\mathrm{e}^{\frac{\frac{k_1 m_{c,1}}{f_{s,1}} \quad \frac{k_2 m_{c,2}}{f_{s,2}}}{\dot{m}_g}} \tag{40}$$

It is assumed the value of k_1 (and n) for fuel 1 is known. Equations (37) can be integrated numerically, when some value for k_2 is assumed. After this $X_{\mathrm{O}_2,x\ L}$ is calculated and compared to measurements. Then a new value is chosen for k_2 and the measured and calculated oxygen responses are compared. This procedure is repeated. This iteration is continued to seek the situation, where the measured and calculated oxygen responses correspond well giving the value k_2.

Effect char inventory on stability of combustion

The effect of reactivity on the mass of bed can be seen from the eq. [3]:

$$\frac{m}{\dot{m}_0} \quad \frac{\dfrac{\mu^2 \mathrm{e}^{2(\ln\sigma)^2}}{10 f_s \mathrm{Sh} \rho_g D_{\mathrm{O}_2}} \quad \dfrac{\mu \mathrm{e}^{\frac{(\ln\sigma)^2}{2}}}{8 k_s}}{X_{\mathrm{O}_2}} \rho_c \tag{41}$$

when the PSD of the fuel feed is lognormal. For monosized particles ($\sigma \quad 0$) of the fuel feed the expression for the char inventory is simplified into:

$$\frac{m}{\dot{m}_0} \quad \frac{\dfrac{d_0^2}{10 f_s \mathrm{Sh} \rho_g D_{\mathrm{O}_2}} \quad \dfrac{d_0}{8 k_s} \rho_c}{X_{\mathrm{O}_2}} \tag{42}$$

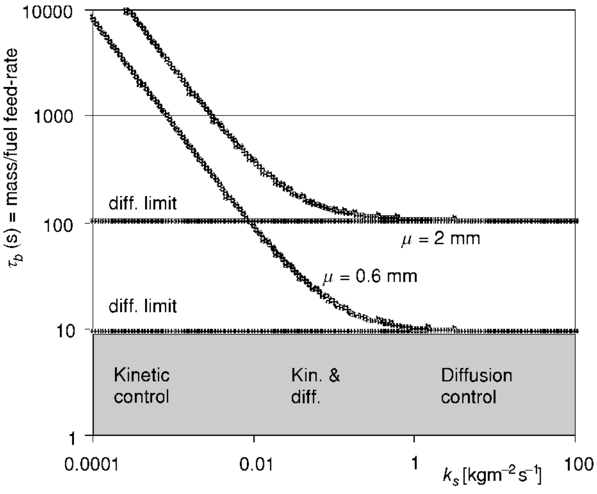

Figure 6. Effect of char reactivity on residence time (or on char inventory) in the bed for two average particle sizes. The arrow shows the direction when changing to more reactive char

The implications of eq. (41) are shown in fig. 6. When fuel is changed to a more reactive one (shown by the arrow) the residence time of the fuel in the bed decreases. Low reactive coals burn in regime of kinetic control and the most reactive coal chars in the regime of diffusion control.

The char inventory for wood fuel becomes small due to two reasons. The first reason is that for reactive biomass particles k_s is large and the contribution of the last term in eq. (41) is small compared to coal. The second reason is that the yield of char is much lower for wood than that for coal. Also the density of wood is much less than that of coal. Then the char from wood is much less dense (order of 100 kg m^{-3}) compared that of coal (1000 kg m^{-3}). In diffusion controlled combustion, the burning time of a particle is directly proportional to the char density, since the amount of oxygen required and transferred by diffusion from surroundings is directly proportional to the amount of char. Figure 7, which is based on eq. (44), shows schematically the difference between biomass and coal combustion in fluidised bed, if other conditions remain the same. The dashed line corresponds to diffusion control.

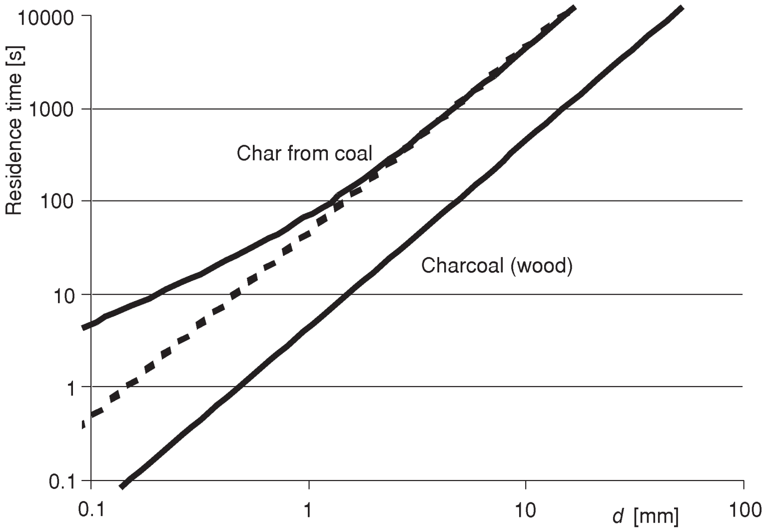

Figure 7. Residence time of char particle in the bed (density ratio char/charcoal is 10)

The char inventory is related to the residence time t_r by:

$$m_c = (1 - X_v)\dot{m}_0 t_r \tag{43}$$

Then the ratio of char inventories for biomass and coal is, if other conditions are same:

$$\frac{m_{c,\,\text{bio}}}{m_{c,\,\text{coal}}} \quad \frac{(1 \quad X_{v,\,\text{bio}})\dot{m}_{0,\,\text{bio}}t_{r,\,\text{bio}}}{(1 \quad X_{v,\,\text{coal}})\dot{m}_{0,\,\text{coal}}t_{r,\,\text{coal}}} \tag{44}$$

It is seen that in addition to the shorter residence time of biomass, the great proportion of volatiles reduces the char inventory, when biomass is used as the fuel. On the other hand, larger particle size is usually used for biomass, also the mass flow rate of biomass fuel must be greater than that of coal, if the same firing power effect is needed, which increase the char inventory to some extent.

Another form for the fuel inventory is obtained from the step response (tab. 1) in the steady state $t \quad \infty$, giving $m/\dot{m}_0 \quad \tau$. Then for a fuel mixture we get:

$$m_{b,c} \quad \underset{i\;\;1}{\overset{N}{}}\tau_{c,i}(1 \quad X_{v,i})\dot{m}_{0f,i} \quad \text{and} \quad m_{b,v} \quad \underset{i\;\;1}{\overset{N}{}}\tau_{v,i}X_{v,i}\dot{m}_{0f,i} \tag{45}$$

for the char inventory and for the inventory of the volatile matter (matter that will be devolatilized) in the bed, respectively. X_v is the volatile matter content of the size fraction or fuel i in the feed.

We consider the case with steady and unsteady components of the fuel feed using single fuel, $\dot{m}_0 \quad \dot{m}_{0,ave} \quad \hat{\dot{m}}_0\sin(2\pi ft)$. Then the burning rate of the bed and the corresponding fluctuation in the oxygen concentration in the flue gases (assuming complete combustion) are:

$$\dot{m}_b \quad \dot{m}_{0,ave} \quad \frac{\hat{\dot{m}}_o\sin[2\pi f(t \quad t_l) \quad \mathrm{arctg}(2\pi f\tau)]}{\sqrt{1 \quad (2\pi f\tau)^2}} \tag{46}$$

$$X_{\mathrm{O}_{2,1}} \quad X_{\mathrm{O}_{2,0}} \quad \frac{\dfrac{\hat{\dot{m}}_0\sin[2\pi f(t \quad t_l) \quad \mathrm{arctg}(2\pi f\tau)]}{\sqrt{1+(2\pi f\tau)^2} \quad \dot{m}_{0,ave}}}{f_s\dot{m}_g} \tag{47}$$

respectively. It is seen that the fluctuation amplitude $\hat{X}_{\mathrm{O}_2} \quad \hat{\dot{m}}_0/[f_s\dot{m}_g\sqrt{1 \quad (2\pi f\tau)^2}\,]$ of the oxygen concentration decreases with increasing time constant τ or increasing fuel inventory $m_b \quad \tau\dot{m}_{0,ave}$. This is the theoretical explanation for the measured result [2, 4] between the char inventory and standard deviation of flue gas oxygen concentration. The dependence of the amplitude $\hat{X}_{\mathrm{O}_2}$ on the residence time is shown in fig. 8a, when $\hat{\dot{m}}_0/f_s\dot{m}_g = 0.03$. Since the residence time of biofuels is shorter, the amplitude of the oxygen concentration will increase compared to coal.

The residence time (or char inventory) as function of amplitude is shown in fig. 8b. It is seen that the shape of the curve ($f = 1/60\ \mathrm{s}^{-1}$) resembles the one measured [2,4]. The standard deviation of oxygen mass fraction is related to the amplitude by $\sigma \quad \hat{X}_{\mathrm{O}_2}/2^{1/2}$. The vibrations in the fuel feed can be caused by the characteristics of the feeding system; for example the order $f \sim 1\ \mathrm{min.}^{-1}$ was noticed in pilot scale tests [2]. Vi-

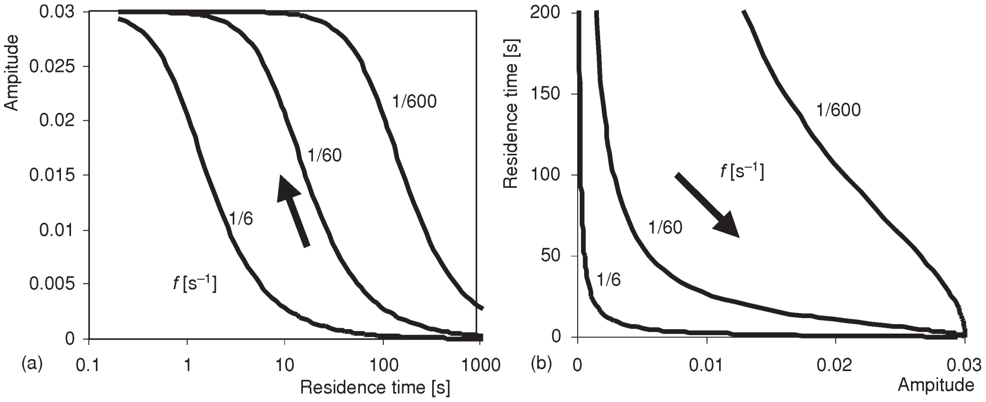

Figure 8. (a) Amplitude of oxygen mass fraction in flue gases as function of residence time τ of fuel (or fuel inventory $m_b \quad \tau\dot{m}_{0,ave}$) in the reactor. (b) Residence time t of fuel (or fuel inventory $m_b \quad \tau\dot{m}_{0,ave}$) as function of amplitude of vibrations in flue gas oxygen concentration. The arrow shows the direction when changing to more reactive conditions (higher temperature and oxygen concentration) or from coal to biomass

brations can also be introduced in purpose. Then it is possible to determine the residence time τ (or the reaction rate coefficient $K = 1/\tau$ (or char inventory $m_b \quad \tau\dot{m}_{0,ave}$) by introducing vibrations in the fuel feed and analysing the responses.

Conclusions

Analytical models for the dynamics of burning of volatiles and char in FB and CFB have been developed
(1) model when oxygen concentration in the reactor is ~ constant [2],
(2) model for char burning when the air rate is constant [1], and
(3) char burning including the evolution of PSD in steady and unsteady situation (non-fragmenting char particles) [3].

Models 1 and 2 are further developed here in the present paper. Model 3 is applied to discuss the effect of biomass on char inventory.

These models can give insight and rule of thumbs for the causes and control strategies related to changes in the burning rate due to disturbances in the fuel feed or due to load changes. Reaction rate parameters can also be determined by applying steady and dynamic models with measurements.

When using biofuels the response times of burning rate to changes in the fuel feed are much shorter than with coal. The burning rate follows also the feed rate much less damped than when using coal. Using biofuels may lead to increased CO emissions, if other conditions remain unchanged. When replacing coal with biofuels (other variables being unchanged) the char inventory of the bed becomes smaller, since the amount of volatiles is higher, reactivity of char from biofuels is higher and their density is much lower (burning time of a particle decreases with char density, fragmentation is more probable). The smaller fuel inventory makes burning more sensitive to disturbances in the fuel feeding. This could be avoided by using larger particle size and lower primary air rate for biofuels.

Nomenclature

c – rate of increase, [kg s^{-2}]
c_p – specific heat capacity of particle, [J kg^{-1}s^{-1}]
D – diffusivity, [m^{-2} s^{-1}]
d – particle diameter, [m]
f – frequency, [s^{-1}]
f_s – stoichiometric ratio of fuel and oxygen, [–]
K – reaction rate coefficient, [s^{-1}]
k – reaction rate coefficient, [s^{-1}]
k_s – surface reaction rate coefficient, [kg m^{-2}s^{-1}]
L – height of reactor, [m]
M – dimensionless initial or impulse mass, [–]
m – mass, [kg]
m' – mass/length of reactor, [kg m^{-1}]
$\dot{m}$ – rate of mass flow, [kg s^{-1}]
$\hat{\dot{m}}$ – amplitude of mass flow rate, [kg s^{-1}]
R – dimensionless parameter, [–],
n – reaction order with respect to oxygen, [–]
r' – reaction rate/length of reactor, [kg s^{-1}m^{-1}]
s – Laplace transform variable, [–]
Sh – Sherwood number, [–]
T – temperature, [K]
t – time, [s]
t_* – dimensionless time, [–], $t_* \quad kX_{O_2,x\ 0}t$
U – Heaviside's unit step function, [–]
X – mass fraction, [–]
$\hat{X}$ – amplitude of mass fraction, [–]
x – co-ordinate along the gas flow, [m]

Greec letters

δ – Dirac's delta function, [–]
μ – average particle diameter in lognormal distribution, [m]
ρ – density, [kg m^{-3}]
σ – standard deviation, [–]
τ – residence time, time constant, [s]

Subscripts

0 – initial, inlet

1 – base fuel, outlet
2 – impulse fuel
∞ – stationary
a – air (or mixture of air and circulated flue gases)
ave – average
b – in bed
c – char
e – effective
f – fuel, char impulse
g – gas
h – heat-up (including drying)
l – lag
O_2 – oxygen
p – char burning
r – residence
s – stoichiometric
v – volatile matter (in partly devolatilized) fuel, devolatilization

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Author's address:

J. J. Saastamoinen
VTT Processes
P.O. Box 1603, FIN-40101 Jyväskylä, Finland

E-mail: jaakko.saastamoinen@vtt.fi

Paper submited: March 6, 2004
Paper revised: May 10, 2004
Paper accepted: May 27, 2004